4 Harmonics

4.1 DEFINITION OF HARMONICS

Webster's New World Dictionary defines *harmonics* as pure tones making up a composite tone in music. A pure tone is a musical sound of a single frequency, and a combination of many pure tones makes up a composite sound. Sound waves are electromagnetic waves traveling through space as a periodic function of time. Can the principle behind pure music tones apply to other functions or quantities that are time dependent? In the early 1800s, French mathematician, Jean Baptiste Fourier formulated that a periodic nonsinusoidal function of a fundamental frequency f may be expressed as the sum of sinusoidal functions of frequencies which are multiples of the fundamental frequency. In our discussions here, we are mainly concerned with periodic functions of voltage and current due to their importance in the field of power quality. In other applications, the periodic function might refer to radiofrequency transmission, heat flow through a medium, vibrations of a mechanical structure, or the motions of a pendulum in a clock.

A sinusoidal voltage or current function that is dependent on time t may be represented by the following expressions:

$$\text{Voltage function, } v(t) = V \sin(\omega t) \tag{4.1}$$

$$\text{Current function, } i(t) = I \sin(\omega t \pm \text{Ø}) \tag{4.2}$$

where $\omega = 2 \times \pi \times f$ is known as the angular velocity of the periodic waveform and Ø is the difference in phase angle between the voltage and the current waveforms referred to as a common axis. The sign of phase angle Ø is positive if the current leads the voltage and negative if the current lags the voltage. Figure 4.1 contains voltage and current waveforms expressed by Eqs. (4.1) and (4.2) and which by definition are pure sinusoids.

For the periodic nonsinusoidal waveform shown in Figure 4.2, the simplified Fourier expression states:

$$\begin{aligned} v(t) = \; & V_0 + V_1 \sin(\omega t) + V_2 \sin(2\omega t) + V_3 \sin(3\omega t) + \ldots + V_n \sin(n\omega t) + \\ & V_{n+1} \sin((n+1)\omega t) + \ldots \end{aligned} \tag{4.3}$$

The Fourier expression is an infinite series. In this equation, V_0 represents the constant or the DC component of the waveform. $V_1, V_2, V_3, \ldots, V_n$ are the peak values of the successive terms of the expression. The terms are known as the harmonics of the periodic waveform. The fundamental (or first harmonic) frequency has a

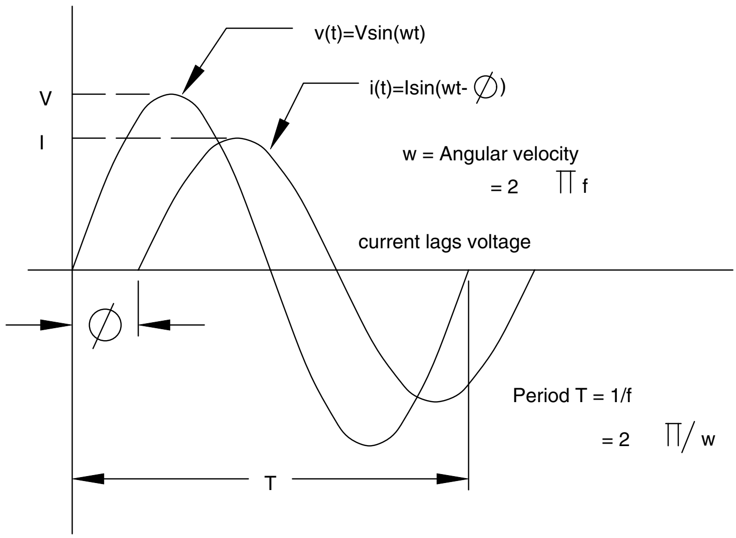


FIGURE 4.1 Sinusoidal voltage and current functions of time (t). Lagging functions are indicated by negative phase angle and leading functions by positive phase angle.

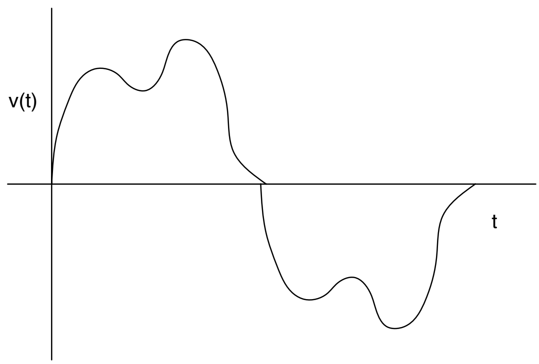


FIGURE 4.2 Nonsinusoidal voltage waveform Fourier series. The Fourier series allows expression of nonsinusoidal periodic waveforms in terms of sinusoidal harmonic frequency components.

frequency of f, the second harmonic has a frequency of $2 \times f$, the third harmonic has a frequency of $3 \times f$, and the nth harmonic has a frequency of $n \times f$. If the fundamental frequency is 60 Hz (as in the U.S.), the second harmonic frequency is 120 Hz, and the third harmonic frequency is 180 Hz.

The significance of harmonic frequencies can be seen in Figure 4.3. The second harmonic undergoes two complete cycles during one cycle of the fundamental frequency, and the third harmonic traverses three complete cycles during one cycle of the fundamental frequency. V_1, V_2, and V_3 are the peak values of the harmonic components that comprise the composite waveform, which also has a frequency of f.

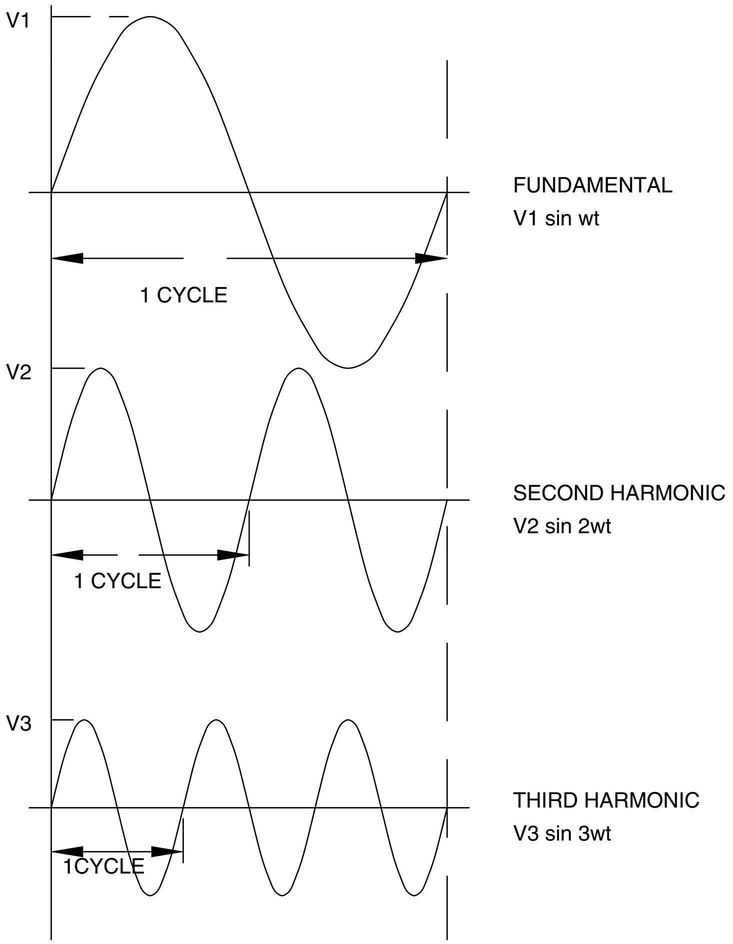


FIGURE 4.3 Fundamental, second, and third harmonics.

The ability to express a nonsinusoidal waveform as a sum of sinusoidal waves allows us to use the more common mathematical expressions and formulas to solve power system problems. In order to find the effect of a nonsinusoidal voltage or current on a piece of equipment, we only need to determine the effect of the individual harmonics and then vectorially sum the results to derive the net effect. Figure 4.4 illustrates how individual harmonics that are sinusoidal can be added to form a nonsinusoidal waveform.

The Fourier expression in Eq. (4.3) has been simplified to clarify the concept behind harmonic frequency components in a nonlinear periodic function. For the purist, the following more precise expression is offered. For a periodic voltage wave with fundamental frequency of $\omega = 2\pi f$,

$$v(t) = V_0 + \sum (a_k \cos k\omega t + b_k \sin k\omega t) \text{ (for } k = 1 \text{ to } \infty) \tag{4.4}$$

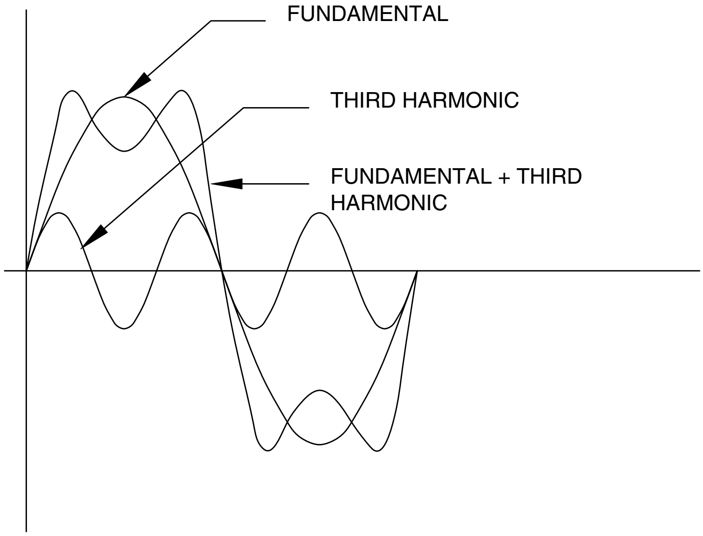


FIGURE 4.4 Creation of nonlinear waveform by adding the fundamental and third harmonic frequency waveforms.

where a_k and b_k are the coefficients of the individual harmonic terms or components. Under certain conditions, the cosine or sine terms can vanish, giving us a simpler expression. If the function is an even function, meaning $f(-t) = f(t)$, then the sine terms vanish from the expression. If the function is odd, with $f(-t) = -f(t)$, then the cosine terms disappear. For our analysis, we will use the simplified expression involving sine terms only. It should be noted that having both sine and cosine terms affects only the displacement angle of the harmonic components and the shape of the nonlinear wave and does not alter the principle behind application of the Fourier series.

The coefficients of the harmonic terms of a function $f(t)$ contained in Eq. (4.4) are determined by:

$$a_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(t).\cos kt.dt,\ (k = 1,2,3,\ \ldots,\ n) \tag{4.5}$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(t).\sin kt.dt,\ (k = 1,2,3,\ \ldots,\ n) \tag{4.6}$$

The coefficients represent the peak values of the individual harmonic frequency terms of the nonlinear periodic function represented by $f(t)$. It is not the intent of this book to explore the intricacies of the Fourier series. Several books in mathematics are available for the reader who wants to develop a deeper understanding of this very essential tool for solving power quality problems related to harmonics.

4.2 HARMONIC NUMBER (*h*)

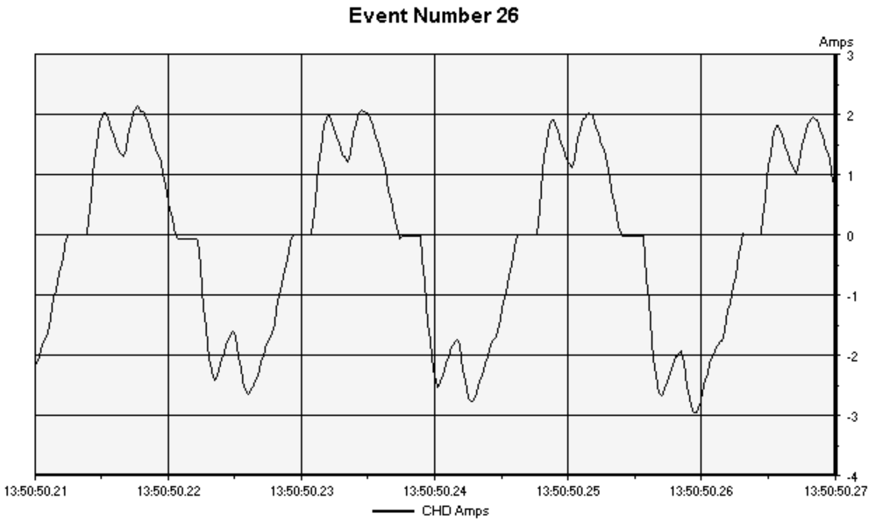
Harmonic number (h) refers to the individual frequency elements that comprise a composite waveform. For example, $h = 5$ refers to the fifth harmonic component with a frequency equal to five times the fundamental frequency. If the fundamental frequency is 60 Hz, then the fifth harmonic frequency is 5×60, or 300 Hz. The harmonic number 6 is a component with a frequency of 360 Hz. Dealing with harmonic numbers and not with harmonic frequencies is done for two reasons. The fundamental frequency varies among individual countries and applications. The fundamental frequency in the U.S. is 60 Hz, whereas in Europe and many Asian countries it is 50 Hz. Also, some applications use frequencies other than 50 or 60 Hz; for example, 400 Hz is a common frequency in the aerospace industry, while some AC systems for electric traction use 25 Hz as the frequency. The inverter part of an AC adjustable speed drive can operate at any frequency between zero and its full rated maximum frequency, and the fundamental frequency then becomes the frequency at which the motor is operating. The use of harmonic numbers allows us to simplify how we express harmonics. The second reason for using harmonic numbers is the simplification realized in performing mathematical operations involving harmonics.

4.3 ODD AND EVEN ORDER HARMONICS

As their names imply, odd harmonics have odd numbers (e.g., 3, 5, 7, 9, 11), and even harmonics have even numbers (e.g., 2, 4, 6, 8, 10). Harmonic number 1 is assigned to the fundamental frequency component of the periodic wave. Harmonic number 0 represents the constant or DC component of the waveform. The DC component is the net difference between the positive and negative halves of one complete waveform cycle. Figure 4.5 shows a periodic waveform with net DC content. The DC component of a waveform has undesirable effects, particularly on transformers, due to the phenomenon of core saturation. Saturation of the core is caused by operating the core at magnetic field levels above the knee of the magnetization curve. Transformers are designed to operate below the knee portion of the curve. When DC voltages or currents are applied to the transformer winding, large DC magnetic fields are set up in the transformer core. The sum of the AC and the DC magnetic fields can shift the transformer operation into regions past the knee of the saturation curve. Operation in the saturation region places large excitation power requirements on the power system. The transformer losses are substantially increased, causing excessive temperature rise. Core vibration becomes more pronounced as a result of operation in the saturation region.

We usually look at harmonics as integers, but some applications produce harmonic voltages and currents that are not integers. Electric arc furnaces are examples of loads that generate non-integer harmonics. Arc welders can also generate non-integer harmonics. In both cases, once the arc stabilizes, the non-integer harmonics mostly disappear, leaving only the integer harmonics.

The majority of nonlinear loads produce harmonics that are odd multiples of the fundamental frequency. Certain conditions need to exist for production of even



AV,BV,CV Impulse event at 10/21/98 13:50:50.14

	PrevRMS	MinRMS	MaxRMS	WorstIMP	Phase	HFHits
AV Volts	408.3	423.1	442.6	-167.0	89 deg	25
BV Volts	416.5	429.4	456.6	-149.0	116 deg	20
CV Volts	420.9	440.3	453.9	182.0	267 deg	37
DI Amps	1.529	1.530	1.712	0.000	0 deg	0

FIGURE 4.5 Current waveform with DC component (scale, 1 A = 200 A). This waveform has a net negative DC component as indicated by the larger area of the negative half compared to the positive half of each cycle.

harmonics. Uneven current draw between the positive and negative halves of one cycle of operation can generate even harmonics. The uneven operation may be due to the nature of the application or could indicate problems with the load circuitry. Transformer magnetizing currents contain appreciable levels of even harmonic components and so do arc furnaces during startup.

Subharmonics have frequencies below the fundamental frequency and are rare in power systems. When subharmonics are present, the underlying cause is resonance between the harmonic currents or voltages with the power system capacitance and inductance. Subharmonics may be generated when a system is highly inductive (such as an arc furnace during startup) or if the power system also contains large capacitor banks for power factor correction or filtering. Such conditions produce slow oscillations that are relatively undamped, resulting in voltage sags and light flicker.

4.4 HARMONIC PHASE ROTATION AND PHASE ANGLE RELATIONSHIP

So far we have treated harmonics as stand-alone entities working to produce waveform distortion in AC voltages and currents. This approach is valid if we are looking at single-phase voltages or currents; however, in a three-phase power system, the harmonics of one phase have a rotational and phase angle relationship with the

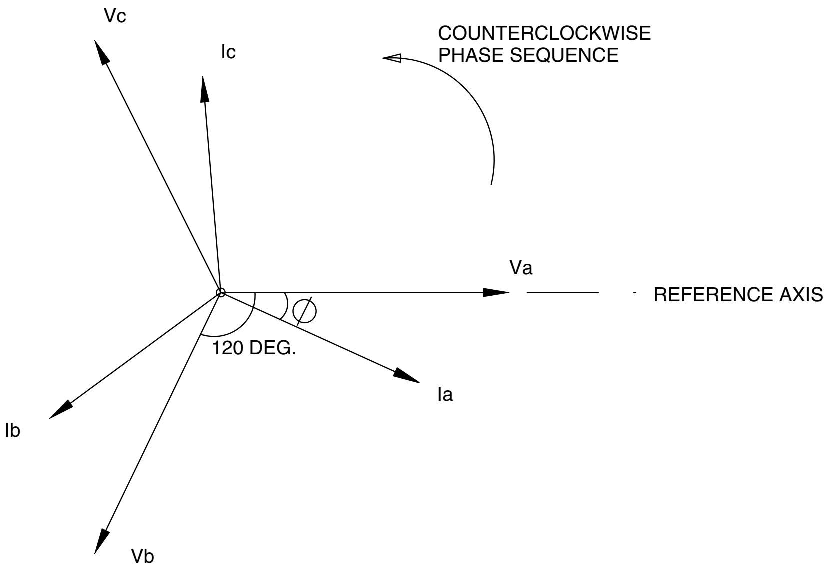


FIGURE 4.6 Balanced three-phase power system. Phase sequence refers to the order in which phasors move past a reference axis. The positive phase sequence is assigned a counterclockwise rotation.

harmonics of the other phases. In power system studies involving harmonics, this relationship is important.

In a balanced three-phase electrical system, the voltages and currents have a positional relationship as shown in Figure 4.6. The three voltages are 120° apart and so are the three currents. The normal phase rotation or sequence is a–b–c, which is counterclockwise and designated as the positive-phase sequence in this book. For harmonic analyses, these relationships are still applicable, but the fundamental components of voltages and currents are used as reference. All other harmonics use the fundamental frequency as the reference. The fundamental frequencies have a positive-phase sequence. The angle between the fundamental voltage and the fundamental current is the displacement power factor angle, as defined in Chapter 1.

So how do the harmonics fit into this space–time picture? For a clearer understanding, let us look only at the current harmonic phasors. We can further simplify the picture by limiting the discussion to odd harmonics only, which under normal and balanced conditions are the most prevalent. The following relationships are true for the fundamental frequency current components in a three-phase power system:

$$i_{a1} = I_{a1} \sin \omega t \tag{4.7}$$

$$i_{b1} = I_{b1} \sin (\omega t\text{-}120°) \tag{4.8}$$

$$i_{c1} = I_{c1} \sin (\omega t\text{-}240°) \tag{4.9}$$

The negative displacement angles indicate that the fundamental phasors i_{b1} and i_{c1} trail the i_{a1} phasor by the indicated angle. Figure 4.7a shows the fundamental current phasors.

The expressions for the third harmonic currents are:

$$i_{a3} = I_{a3} \sin 3\omega t \tag{4.10}$$

$$i_{b3} = I_{b3} \sin 3(\omega t\text{-}120°) = I_{b3} \sin (3\omega t\text{-}360°) = I_{b3} \sin 3\omega t \tag{4.11}$$

$$i_{c3} = I_{c3} \sin 3(\omega t\text{-}240°) = I_{c3} \sin (3\omega t\text{-}720°) = I_{c3} \sin 3\omega t \tag{4.12}$$

The expressions for the third harmonics show that they are in phase and have zero displacement angle between them. Figure 4.7b shows the third harmonic phasors. The third harmonic currents are known as zero sequence harmonics due to the zero displacement angle between the three phasors.

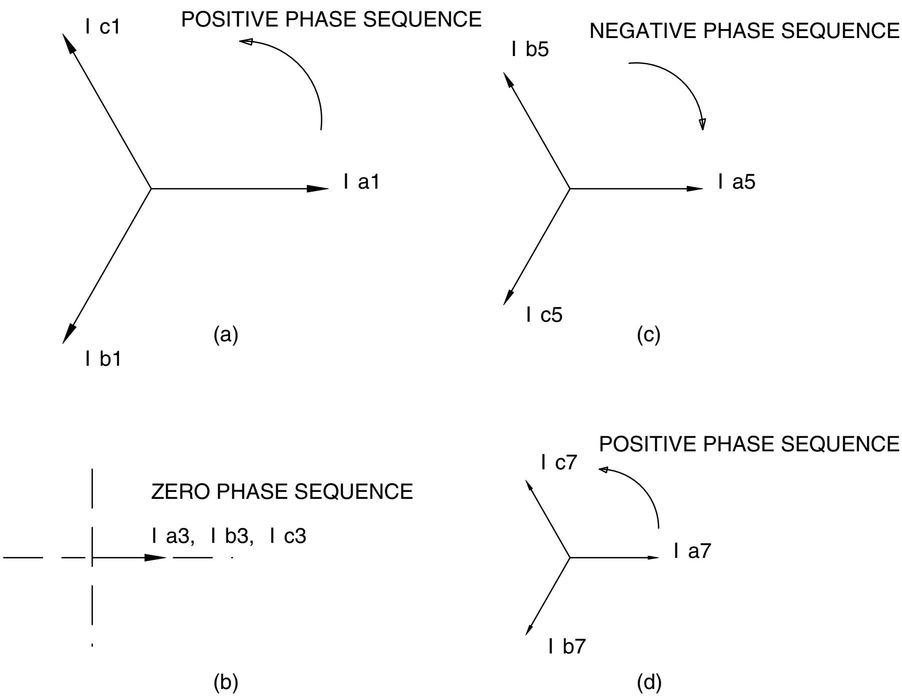


FIGURE 4.7 (a) Fundamental phasors. (b) Third harmonic phasors. (c) Fifth harmonic phasors. (d) Seventh harmonic phasors.

The expressions for the fifth harmonic currents are:

$$i_{a5} = I_{a5} \sin 5\omega t \tag{4.13}$$

$$i_{b5} = I_{b5} \sin 5(\omega t\text{-}120°) = I_{b5} \sin(5\omega t\text{-}600°) = I_{b5} \sin(5\omega t\text{-}240°) \tag{4.14}$$

$$i_{c5} = I_{c5} \sin 5(\omega t\text{-}240°) = I_{c5} \sin(5\omega t\text{-}1200°) = I_{c5} \sin(5\omega t\text{-}120°) \tag{4.15}$$

Figure 4.7c shows the fifth harmonic phasors. Note that the phase sequence of the fifth harmonic currents is clockwise and opposite to that of the fundamental. The fifth harmonics are negative sequence harmonics.

Similarly the expressions for the seventh harmonic currents are:

$$i_{a7} = I_{a7} \sin 7\omega t \tag{4.16}$$

$$i_{b7} = I_{b7} \sin 7(\omega t\text{-}120°) = I_{b7} \sin(7\omega t\text{-}840°) = I_{b7} \sin(7\omega t\text{-}120°) \tag{4.17}$$

$$i_{c7} = I_{c7} \sin 7(\omega t\text{-}240°) = I_{c7} \sin(7\omega t\text{-}1680°) = I_{c7} \sin(7\omega t\text{-}240°) \tag{4.18}$$

Figure 4.7d shows the seventh harmonic current phasors. The seventh harmonics have the same phase sequence as the fundamental and are positive sequence harmonics. So far, we have not included even harmonics in the discussion; doing so is left to the reader as an exercise. Table 4.1 categorizes the harmonics in terms of their respective sequence orders.

TABLE 4.1
Harmonic Order vs. Phase Sequence

Harmonic Order	Sequence
1, 4, 7, 10, 13, 16, 19	Positive
2, 5, 8, 11, 14, 17, 20	Negative
3, 6, 9, 12, 15, 18, 21	Zero

The expressions shown so far for harmonics have zero phase shifts with respect to the fundamental. It is not uncommon for the harmonics to have a phase-angle shift with respect to the fundamental. Figure 4.8 depicts a fifth harmonic current waveform with and without phase shift from the fundamental. Expressions for the fifth harmonics with a phase-shift angle of θ degrees are:

$$i_{a5} = I_{a5} \sin 5(\omega t\text{-}\theta) \tag{4.19}$$

$$i_{b5} = I_{b5} \sin 5(\omega t\text{-}120°\text{-}\theta) \tag{4.20}$$

$$i_{c5} = I_{c5} \sin 5(\omega t\text{-}240°\text{-}\theta) \tag{4.21}$$

While the phase-shift angle has the effect of altering the shape of the composite waveform, the phase sequence order of the harmonics is not affected.

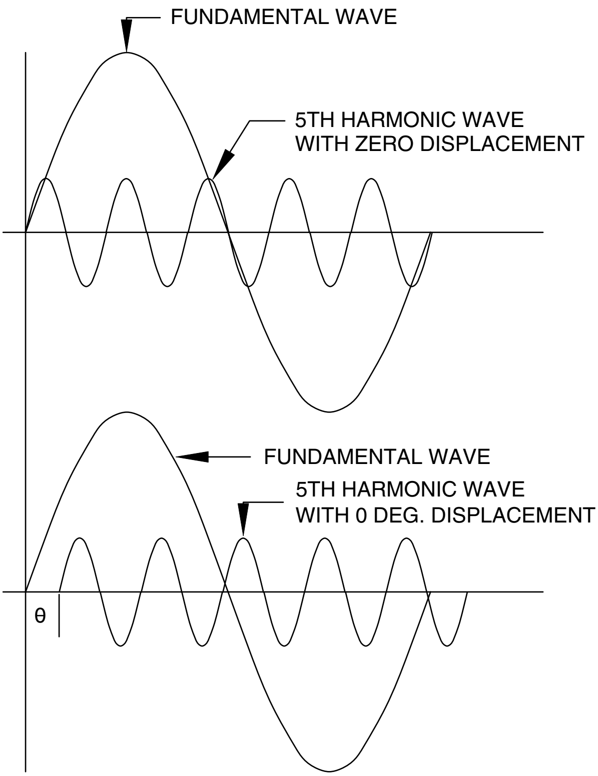


FIGURE 4.8 Nonsymmetry of the waveform with respect to a vertical reference plane introduced by a displacement of harmonics. Periodicity is still maintained.

4.5 CAUSES OF VOLTAGE AND CURRENT HARMONICS

A pure sinusoidal waveform with zero harmonic distortion is a hypothetical quantity and not a practical one. The voltage waveform, even at the point of generation, contains a small amount of distortion due to nonuniformity in the excitation magnetic field and discrete spatial distribution of coils around the generator stator slots. The distortion at the point of generation is usually very low, typically less than 1.0%. The generated voltage is transmitted many hundreds of miles, transformed to several levels, and ultimately distributed to the power user. The user equipment generates currents that are rich in harmonic frequency components, especially in large commercial or industrial installations. As harmonic currents travel to the power source, the current distortion results in additional voltage distortion due to impedance voltages associated with the various power distribution equipment, such as transmission and distribution lines, transformers, cables, buses, and so on. Figure 4.9 illustrates how current distortion is transformed into voltage distortion. Not all voltage distortion, however, is due to the flow of distorted current through the power system impedance. For instance, static uninterruptible power source (UPS) systems can

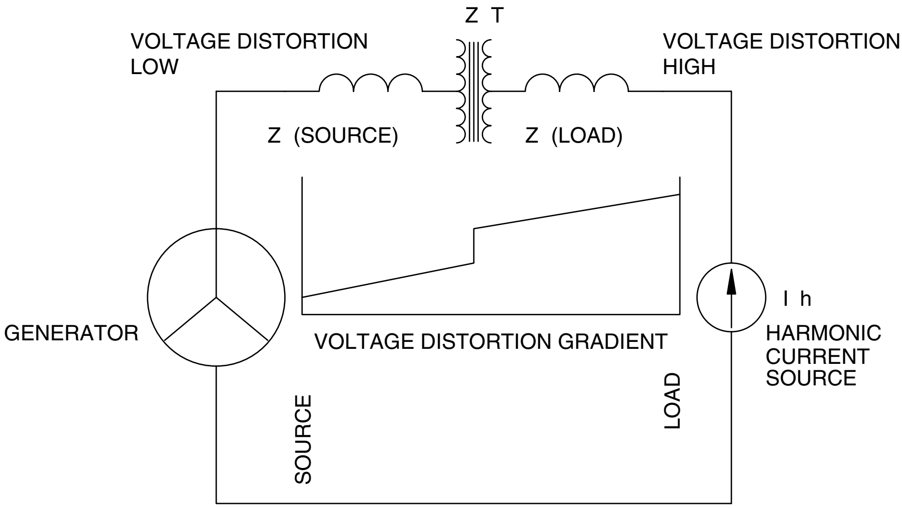


FIGURE 4.9 Voltage distortion due to current distortion. The gradient graph indicates how distortion changes from source to load.

generate appreciable voltage distortion due to the nature of their operation. Normal AC voltage is converted to DC and then reconverted to AC in the inverter section of the UPS. Unless waveform shaping circuitry is provided, the voltage waveforms generated in UPS units tend to be distorted.

As nonlinear loads are propagated into the power system, voltage distortions are introduced which become greater moving from the source to the load because of the circuit impedances. Current distortions for the most part are caused by loads. Even loads that are linear will generate nonlinear currents if the supply voltage waveform is significantly distorted. When several power users share a common power line, the voltage distortion produced by harmonic current injection of one user can affect the other users. This is why standards are being issued that will limit the amount of harmonic currents that individual power users can feed into the source (an issue that we will examine later in this chapter). The major causes of current distortion are nonlinear loads due to adjustable speed drives, fluorescent lighting, rectifier banks, computer and data-processing loads, arc furnaces, and so on. One can easily visualize an environment where a wide spectrum of harmonic frequencies are generated and transmitted to other loads or other power users, thereby producing undesirable results throughout the system.

4.6 INDIVIDUAL AND TOTAL HARMONIC DISTORTION

Individual harmonic distortion (IHD) is the ratio between the root mean square (RMS) value of the individual harmonic and the RMS value of the fundamental

$$IHD_n = I_n/I_1 \tag{4.22}$$

For example, assume that the RMS value of the third harmonic current in a nonlinear load is 20 A, the RMS value of the fifth harmonic current is 15 A, and the RMS value of the fundamental is 60 A. Then, the individual third harmonic distortion is:

$$IHD_3 = 20/60 = 0.333, \text{ or } 33.3\%$$

and the individual fifth harmonic distortion is:

$$IHD_5 = 15/60 = 0.25, \text{ or } 25.0\%$$

Under this definition, the value of IHD_1 is always 100%. This method of quantifying the harmonics is known as harmonic distortion based on the fundamental. This is the convention used by the Institute of Electrical and Electronic Engineers (IEEE) in the U.S. The European International Electrotechnical Commission (IEC) quantifies harmonics based on the total RMS value of the waveform. Using the same example shown above, the RMS value of the waveform is:

$$I_{\text{rms}} = \sqrt{(60^2 + 20^2 + 15^2)} = 65 \text{ A}$$

Based on the IEC convention,

$$IHD_1 = 60/65 = 0.923, \text{ or } 92.3\%$$

$$IHD_3 = 20/65 = 0.308, \text{ or } 30.8\%$$

$$IHD_5 = 15/65 = 0.231, \text{ or } 23.1\%$$

The examples illustrate that even though the magnitudes of the harmonic currents are the same, the distortion percentages are different because of a change in the definition. It should be pointed out that it really does not matter what convention is used as long as the same one is maintained throughout the harmonic analysis. In this book, the IEEE convention will be followed, and all harmonic distortion calculations will be based on the fundamental.

Total harmonic distortion (THD) is a term used to describe the net deviation of a nonlinear waveform from ideal sine waveform characteristics. Total harmonic distortion is the ratio between the RMS value of the harmonics and the RMS value of the fundamental. For example, if a nonlinear current has a fundamental component of I_1 and harmonic components of I_2, I_3, I_4, I_5, I_6, I_7, …, then the RMS value of the harmonics is:

$$I_{\text{H}} = \sqrt{(I_2^2 + I_3^2 + I_4^2 + I_5^2 + I_6^2 + I_7^2 + \ldots)} \tag{4.23}$$

$$THD = (I_{\text{H}}/I_1) \times 100\% \tag{4.24}$$

Example: Find the total harmonic distortion of a voltage waveform with the following harmonic frequency make up:

$$\text{Fundamental} = V_1 = 114 \text{ V}$$

$$\text{3rd harmonic} = V_3 = 4 \text{ V}$$

$$\text{5th harmonic} = V_5 = 2 \text{ V}$$

$$\text{7th harmonic} = V_7 = 1.5 \text{ V}$$

$$\text{9th harmonic} = V_9 = 1 \text{ V}$$

This problem can be solved in two ways:

$$\text{RMS value of the harmonics} = V_{\mathrm{H}} = \sqrt{(4^2 + 2^2 + 1.5^2 + 1^2)} = 4.82 \text{ V}$$

$$THD = (4.82/114) \times 100 \cong 4.23\%$$

or find the individual harmonic distortions:

$$IHD_3 = 4/114 = 3.51\%$$

$$IHD_5 = 2/114 = 1.75\%$$

$$IHD_7 = 1.5/114 = 1.32\%$$

$$IHD_9 = 1/114 = 0.88\%$$

By definition, $IHD_1 = 100\%$, so

$$THD = \sqrt{(IHD_3^2 + IHD_5^2 + IHD_7^2 + IHD_9^2)} \cong 4.23\%$$

The results are not altered by using either the magnitude of the RMS quantities or the individual harmonic distortion values.

The individual harmonic distortion indicates the contribution of each harmonic frequency to the distorted waveform, and the total harmonic distortion describes the net deviation due to all the harmonics. These are both important parameters. In order to solve harmonic problems, we require information on the composition of the individual distortions so that any treatment may be tailored to suit the problem. The total harmonic distortion, while conveying no information on the harmonic makeup, is used to describe the degree of pollution of the power system as far as harmonics are concerned. Defining the individual and total harmonic distortions will be helpful as we look at some typical nonlinear waveforms and their harmonic frequency characteristics.

4.7 HARMONIC SIGNATURES

Many of the loads installed in present-day power systems are harmonic current generators. Combined with the impedance of the electrical system, the loads also produce harmonic voltages. The nonlinear loads may therefore be viewed as both harmonic current generators and harmonic voltage generators. Prior to the 1970s, speed control of AC motors was primarily achieved using belts and pulleys. Now, adjustable speed drives (ASDs) perform speed control functions very efficiently. ASDs are generators of large harmonic currents. Fluorescent lights use less electrical energy for the same light output as incandescent lighting but produce substantial harmonic currents in the process. The explosion of personal computer use has resulted in harmonic current proliferation in commercial buildings. This section is devoted to describing, in no particular order, a few of the more common nonlinear loads that surround us in our everyday life.

4.7.1 Fluorescent Lighting

Figure 4.10 shows a current waveform at a distribution panel supplying exclusively fluorescent lights. The waveform is primarily comprised of the third and the fifth harmonic frequencies. The individual current harmonic distortion makeup is provided in Table 4.2. The waveform also contains slight traces of even harmonics, especially of the higher frequency order. The current waveform is flat topped due to initiation of arc within the gas tube, which causes the voltage across the tube and the current to become essentially unchanged for a portion of each half of a cycle.

4.7.2 Adjustable Speed Drives

While several technologies exist for creating a variable voltage and variable frequency power source for the speed control of AC motors, the pulse-width modulation (PWM) drive technology is currently the most widely used. Figures 4.11 and 4.12 show current graphs at the ASD input lines with a motor operating at 60 and 45 Hz, respectively. Tables 4.3 and 4.4 show the harmonic current distortion spectrum for the two respective frequencies. The characteristic double hump for each half cycle of the AC waveform is due to conduction of the input rectifier modules for a duration of two 60° periods for each half cycle. As the operating frequency is reduced, the humps become pronounced with a large increase in the total harmonic distortion. The THD of 74.2% for 45-Hz operation is excessive and can produce many deleterious effects, as will be shown in later sections of this chapter.

Figure 4.13 is the waveform of the voltage at the ASD input power lines. It was stated earlier that large current distortions can produce significant voltage distortions. In this particular case, the voltage THD is 8.3%, which is higher than levels typically found in most industrial installations. High levels of voltage THD also produce unwanted results. Table 4.5 provides the voltage harmonic distortion distribution.

Figure 4.14 is the current waveform of an ASD of smaller horsepower. This drive contains line side inductors which, along with the higher inductance of the motor, produce a current waveform with less distortion. Table 4.6 provides the harmonic frequency distribution for this ASD.

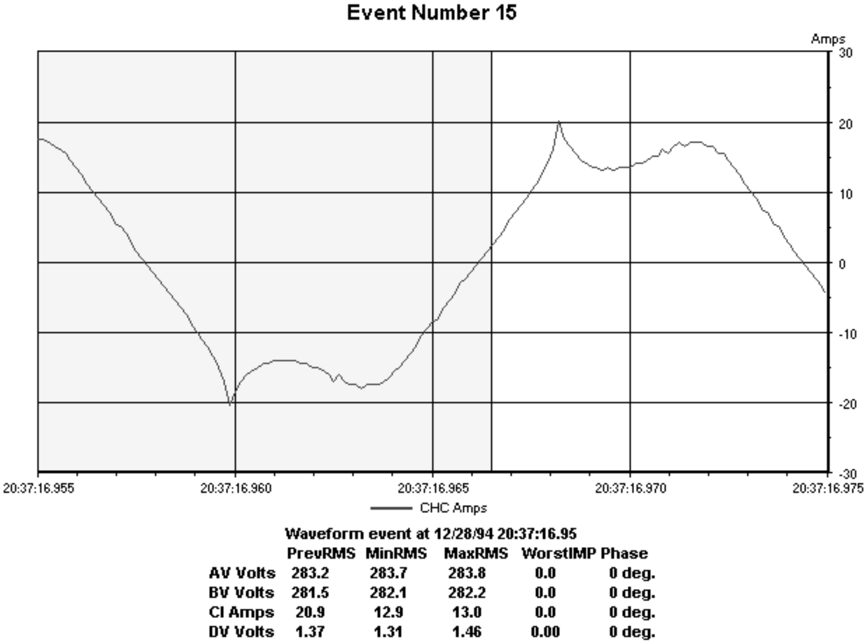


FIGURE 4.10 Nonlinear current drawn by fluorescent lighting.

TABLE 4.2
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion (IHD) for a Fluorescent Lighting Load

Harmonic Distortion Spectrum					
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	—	11	2.2	22	0.6
1	100	12	0.3	23	0.6
2	0.3	13	1.7	24	0.7
3	13.9	14	0.3	25	1.4
4	0.3	15	1.9	26	1.1
5	9	16	0.3	27	0.3
6	0.2	17	0.8	28	0.9
7	3.3	18	0.5	29	1.5
8	0	19	1.4	30	1
9	3.2	20	0.4	31	0.3
10	0.1	21	1.2		

Note: Total harmonic distortion = 18.0%.

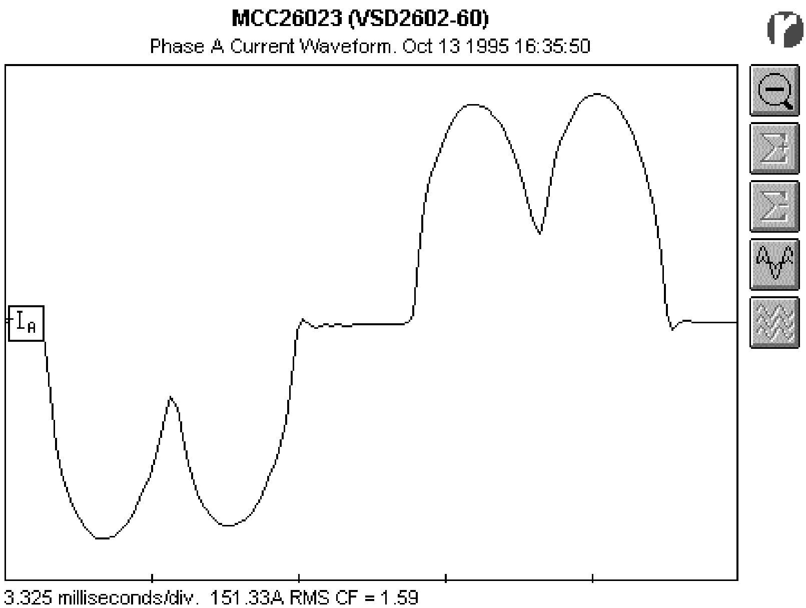


FIGURE 4.11 Adjustable speed drive input current with motor operating at 60 Hz.

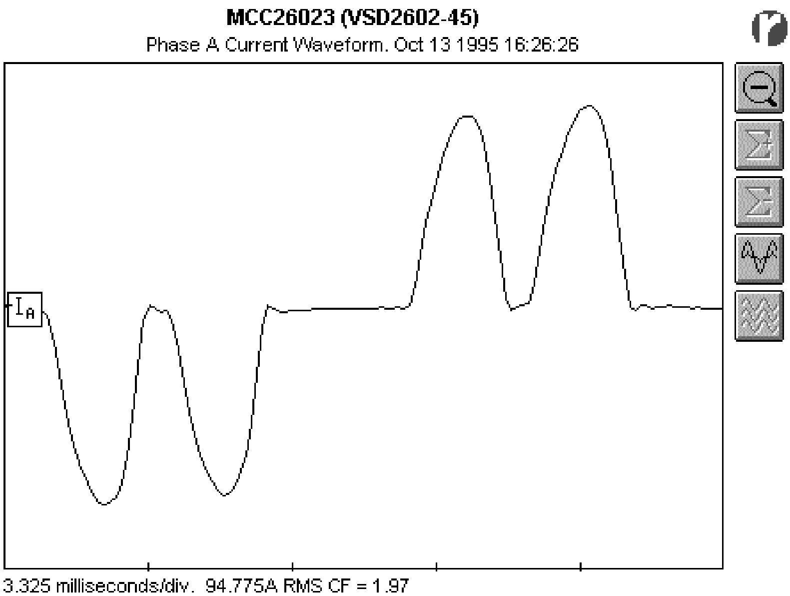


FIGURE 4.12 Adjustable speed drive input current with motor operating at 45 Hz.

TABLE 4.3
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion for an Adjustable Speed Drive Input Current with Motor Running at 60 Hz

Harmonic Distortion Spectrum					
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	0.15	11	9.99	22	0.39
1	100	12	0.03	23	2.95
2	4.12	13	0.19	24	0.02
3	0.78	14	0.48	25	0.66
4	1.79	15	0.07	26	0.15
5	35.01	16	0.52	27	0.05
6	0.215	17	4.85	28	0.22
7	2.62	18	0.03	29	1.79
8	1	19	0.67	30	0.03
9	0.06	20	0.31	31	0.64
10	0.73	21	0.04		

Note: Total harmonic distortion = 37.3%.

TABLE 4.4
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion for an Adjustable Speed Drive Input Current with Motor Running at 45 Hz

Harmonic Distortion Spectrum					
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	2.23	11	6.36	22	0.16
1	100	12	0.03	23	3.75
2	4.56	13	9.99	24	0.12
3	2.44	14	0.11	25	1.73
4	3.29	15	0.62	26	0.42
5	62.9	16	0.35	27	0.33
6	1.4	17	5.22	28	0.22
7	36.1	18	0.35	29	1.68
8	0.43	19	1.96	30	0.26
9	0.73	20	0.64	31	1.36
10	0.58	21	0.22		

Note: Total harmonic distortion = 74.2%.

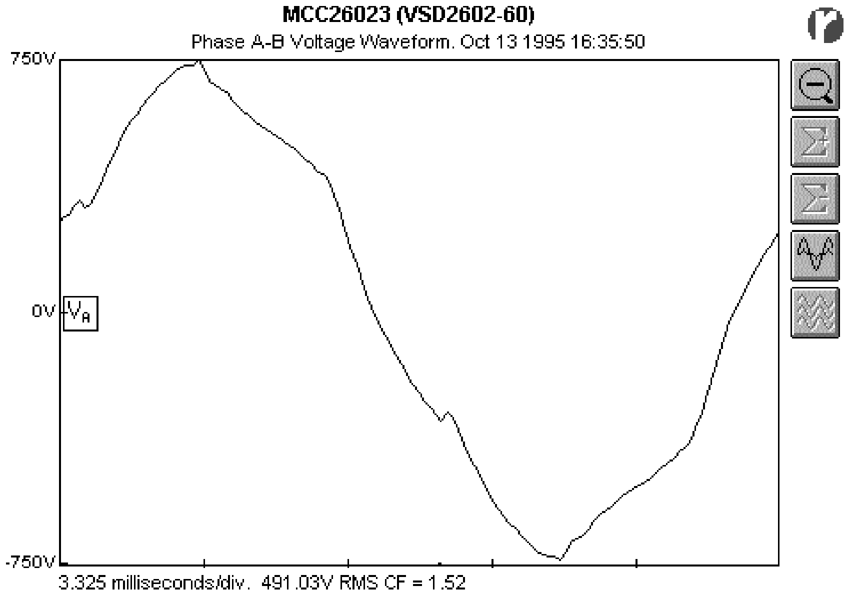


FIGURE 4.13 Adjustable speed drive input voltage with motor operating at 60 Hz.

TABLE 4.5
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion for an Adjustable Speed Drive Input Voltage with Motor Running at 60 Hz

Harmonic Distortion Spectrum					
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	0.02	11	1.87	22	0.07
1	100	12	0.02	23	0.46
2	0.12	13	0.92	24	0.04
3	0.09	14	0.07	25	0.36
4	0.11	15	0.01	26	0.06
5	7.82	16	0.04	27	0.03
6	0.01	17	0.61	28	0.07
7	1.42	18	0.06	29	0.4
8	0.06	19	0.36	30	0.02
9	0.04	20	0.06	31	0.34
10	0.03	21	0.12		

Note: Total harmonic distortion = 8.3%.

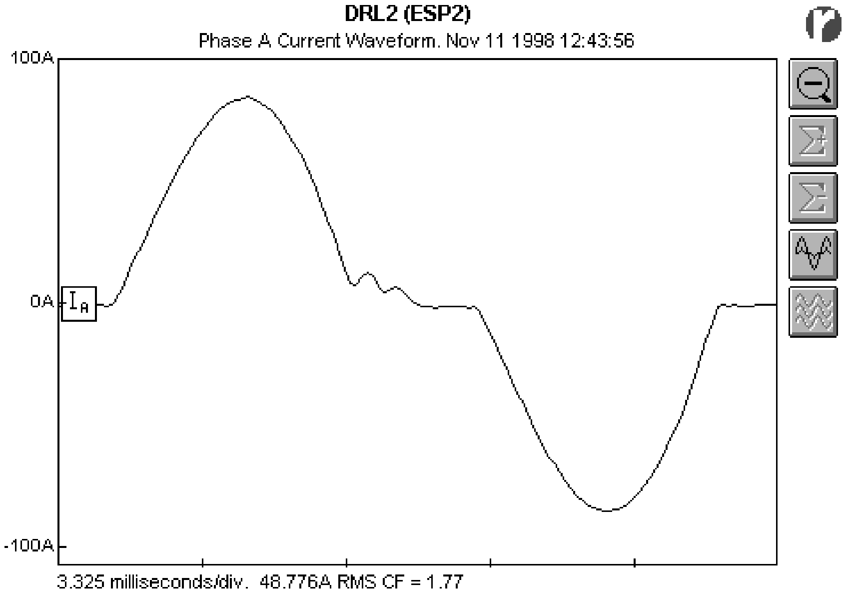


FIGURE 4.14 Adjustable speed drive input current for a smaller motor operating at 50 Hz (ASD with input line inductors).

TABLE 4.6
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion for an Adjustable Speed Drive Input Voltage with Line Inductor and Motor Running at 50 Hz

Harmonic Distortion Spectrum					
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	1.27	11	0.93	22	0.43
1	100	12	0.44	23	0.42
2	1.76	13	1.01	24	0.29
3	35.5	14	0.35	25	0.51
4	1.91	15	0.96	26	0.24
5	3.83	16	0.53	27	0.58
6	1.62	17	0.23	28	0.15
7	3.42	18	0.64	29	0.2
8	0.93	19	0.82	30	0.13
9	3.22	20	0.44	31	0.21
10	0.54	21	0.75		

Note: Total harmonic distortion = 36.3%.

4.7.3 Personal Computer and Monitor

Figures 4.15 and 4.16 show the nonlinear current characteristics of a personal computer and a computer monitor, respectively. Tables 4.7 and 4.8 provide the harmonic content of the currents for the two devices. The predominance of the third and fifth harmonics is evident. The current THD for both devices exceeds 100%, as the result of high levels of individual distortions introduced by the third and fifth harmonics. The total current drawn by a personal computer and its monitor is less than 2 A, but a typical high-rise building can contain several hundred computers and monitors. The net effect of this on the total current harmonic distortion of a facility is not difficult to visualize.

So far we have examined some of the more common harmonic current generators. The examples illustrate that a wide spectrum of harmonic currents is generated. Depending on the size of the power source and the harmonic current makeup, the composite harmonic picture will be different from facility to facility.

4.8 EFFECT OF HARMONICS ON POWER SYSTEM DEVICES

We are interested in the subject of harmonics because of the harmful effects they have on power system devices. What makes harmonics so insidious is that very often

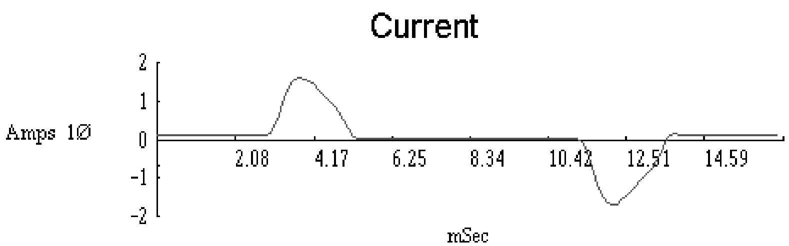


FIGURE 4.15 Nonlinear current drawn by single personal computer.

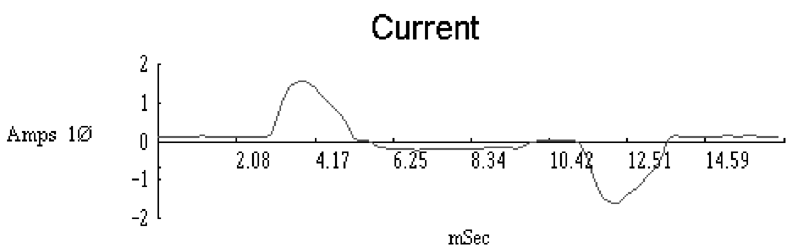


FIGURE 4.16 Nonlinear current drawn by single computer video monitor.

the effects of harmonics are not known until failure occurs. Insight into how harmonics can interact within a power system and how they can affect power system components is important for preventing failures. In this section, we will look at the effect of harmonics on some common power system devices.

TABLE 4.7
Harmonic Number *h*(*n*) vs. Individual Harmonic Distortion for a Personal Computer

		Harmonic Distortion Spectrum			
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	12.8	11	10.3	22	2.1
1	100	12	1.2	23	0
2	3.3	13	10.3	24	0
3	87.2	14	0	25	0
4	5.1	15	10.3	26	0
5	64.1	16	0	27	0
6	1.6	17	5.1	28	0
7	41.1	18	0	29	0
8	0	19	2.4	30	0
9	17.9	20	0	31	0
10	1.1	21	2.1		

Note: Total harmonic distortion = 118.3%.

TABLE 4.8
Harmonic Frequency *h*(*n*) vs. Individual Harmonic Distortion for Computer Monitor Current

		Harmonic Distortion Spectrum			
***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**	***h*(*n*)**	**IHD (%)**
0	0	11	10	22	0
1	100	12	2.5	23	5
2	5	13	10	24	0
3	90	14	2.5	25	0
4	5	15	10	26	0
5	62.5	16	0	27	0
6	5	17	2.5	28	0
7	32.5	18	0	29	0
8	0	19	0	30	0
9	12.5	20	0	31	0
10	2.5	21	5		

Note: Total harmonic distortion = 116.3%.

4.8.1 Transformers

Harmonics can affect transformers primarily in two ways. Voltage harmonics produce additional losses in the transformer core as the higher frequency harmonic voltages set up hysteresis loops, which superimpose on the fundamental loop. Each loop represents higher magnetization power requirements and higher core losses. A second and a more serious effect of harmonics is due to harmonic frequency currents in the transformer windings. The harmonic currents increase the net RMS current flowing in the transformer windings which results in additional I^2R losses. Winding eddy current losses are also increased. Winding eddy currents are circulating currents induced in the conductors by the leakage magnetic flux. Eddy current concentrations are higher at the ends of the windings due to the crowding effect of the leakage magnetic field at the coil extremities. The winding eddy current losses increase as the square of the harmonic current and the square of the frequency of the current. Thus, the eddy loss (*EC*) is proportional to $I_h^2 \times h^2$, where I_h is the RMS value of the harmonic current of order *h,* and *h* is the harmonic frequency order or number. Eddy currents due to harmonics can significantly increase the transformer winding temperature. Transformers that are required to supply large nonlinear loads must be derated to handle the harmonics. This derating factor is based on the percentage of the harmonic currents in the load and the rated winding eddy current losses.

One method by which transformers may be rated for suitability to handle harmonic loads is by *k* factor ratings. The *k* factor is equal to the sum of the square of the harmonic frequency currents (expressed as a ratio of the total RMS current) multiplied by the square of the harmonic frequency numbers:

$$k = I_1^2(1)^2 + I_2^2(2)^2 + I_3^2(3)^2 + I_4^2(4)^2 + \dots + I_n^2(n)^2 \qquad (4.25)$$

where

I_1 is the ratio between the fundamental current and the total RMS current.
I_2 is the ratio between the second harmonic current and the total RMS current.
I_3 is the ratio between the third harmonic current and the total RMS current.

Equation (4.25) can be rewritten as:

$$k = \Sigma\, I_n^2 h^2 (h = 1, 2, 3, \dots, n) \qquad (4.26)$$

Example: Determine the *k* rating of a transformer required to carry a load consisting of 500 A of fundamental, 200 A of third harmonics, 120 A of fifth harmonics, and 90 A of seventh harmonics:

$$\text{Total RMS current } (I) = \sqrt{(500^2 + 200^2 + 120^2 + 90^2)} = 559 \text{ A}$$

$$I_1 = 500/559 = 0.894$$

$$I_3 = 200/559 = 0.358$$

$$I_5 = 120/559 = 0.215$$

$$I_7 = 90/559 = 0.161$$

$$k = (0.894)^2 1^2 + (0.358)^2 3^2 + (0.215)^2 5^2 + (0.161)^2 7^2 = 4.378$$

The transformer specified should be capable of handling 559 A of total RMS current with a k factor of not less than 4.378. Typically, transformers are marked with k ratings of 4, 9, 13, 20, 30, 40, and 50, so a transformer with a k rating of 9 should be chosen. Such a transformer would have the capability to carry the full RMS load current and handle winding eddy current losses equal to k times the normal rated eddy current losses.

The k factor concept is derived from the ANSI/IEEE C57.110 standard, Recommended Practices for Establishing Transformer Capability When Supplying Non-Sinusoidal Load Currents, which provides the following expression for derating a transformer when supplying harmonic loads:

$$I\ \text{max.(pu)} = [P_{\text{LL-R(pu)}}/1 + (\Sigma f_h^2 h^2/\Sigma f_h^2) P_{\text{EC-R(pu)}}]^{1/2} \tag{4.27}$$

where

I max.(pu) = ratio of the maximum nonlinear current of a specified harmonic makeup that the transformer can handle to the transformer rated current.

$P_{\text{LL-R(pu)}}$ = load loss density under rated conditions (per unit of rated load I^2R loss density.

$P_{\text{EC-R(pu)}}$ = winding eddy current loss under rated conditions (per unit of rated I^2R loss).

f_h = harmonic current distribution factor for harmonic h (equal to harmonic h current divided by the fundamental frequency current for any given load level).

h = harmonic number or order.

As difficult as this formula might seem, the underlying principle is to account for the increased winding eddy current losses due to the harmonics. The following example might help clarify the IEEE expression for derating a transformer.

Example: A transformer with a full load current rating of 1000 A is subjected to a load with the following nonlinear characteristics. The transformer has a rated winding eddy current loss density of 10.0% (0.10 pu). Find the transformer derating factor.

Harmonic number (*h*)	**f_h (pu)**
1	1
3	0.35
5	0.17
7	0.09

Maximum load loss density, $P_{LL-R(pu)} = 1 + 0.1 = 1.1$

Maximum rated eddy current loss density, $P_{EC-R(pu)} = 0.1$

$\Sigma f_h^2 h^2 = 1^2 + (0.35)^2 3^2 + (0.17)^2 5^2 + (0.09)^2 7^2 = 3.22$

$\Sigma f_h^2 = 1^2 + 0.35^2 + 0.17^2 + 0.09^2 = 1.16$

I max.(pu) $= [1.1/1 + (3.22 \times 0.1/1.16)]^{1/2} = 0.928$

The transformer derating factor is 0.928; that is, the maximum nonlinear current of the specified harmonic makeup that the transformer can handle is 928 A.

The ANSI/IEEE derating method is very useful when it is necessary to calculate the allowable maximum currents when the harmonic makeup of the load is known. For example, the load harmonic conditions might change on an existing transformer depending on the characteristics of new or replacement equipment. In such cases, the transformer may require derating. Also, transformers that supply large third harmonic generating loads should have the neutrals oversized. This is because, as we saw earlier, the third harmonic currents of the three phases are in phase and therefore tend to add in the neutral circuit. In theory, the neutral current can be as high as 173% of the phase currents. Transformers for such applications should have a neutral bus that is twice as large as the phase bus.

4.8.2 AC Motors

Application of distorted voltage to a motor results in additional losses in the magnetic core of the motor. Hysteresis and eddy current losses in the core increase as higher frequency harmonic voltages are impressed on the motor windings. Hysteresis losses increase with frequency and eddy current losses increase as the square of the frequency. Also, harmonic currents produce additional I^2R losses in the motor windings which must be accounted for.

Another effect, and perhaps a more serious one, is torsional oscillations due to harmonics. Table 4.1 classified harmonics into one of three categories. Two of the more prominent harmonics found in a typical power system are the fifth and seventh harmonics. The fifth harmonic is a negative sequence harmonic, and the resulting magnetic field revolves in a direction opposite to that of the fundamental field at a speed five times the fundamental. The seventh harmonic is a positive sequence harmonic with a resulting magnetic field revolving in the same direction as the fundamental field at a speed seven times the fundamental. The net effect is a magnetic field that revolves at a relative speed of six times the speed of the rotor. This induces currents in the rotor bars at a frequency of six times the fundamental frequency. The resulting interaction between the magnetic fields and the rotor-induced currents produces torsional oscillations of the motor shaft. If the frequency of the oscillation coincides with the natural frequency of the motor rotating members, severe damage to the motor can result. Excessive vibration and noise in a motor operating in a harmonic environment should be investigated to prevent failures.

Motors intended for operation in a severe harmonic environment must be specially designed for the application. Motor manufacturers provide motors for operation with ASD units. If the harmonic levels become excessive, filters may be applied at the motor terminals to keep the harmonic currents from the motor windings. Large motors supplied from ASDs are usually provided with harmonic filters to prevent motor damage due to harmonics.

4.8.3 Capacitor Banks

Capacitor banks are commonly found in commercial and industrial power systems to correct for low power factor conditions. Capacitor banks are designed to operate at a maximum voltage of 110% of their rated voltages and at 135% of their rated kVARS. When large levels of voltage and current harmonics are present, the ratings are quite often exceeded, resulting in failures. Because the reactance of a capacitor bank is inversely proportional to frequency, harmonic currents can find their way into a capacitor bank. The capacitor bank acts as a sink, absorbing stray harmonic currents and causing overloads and subsequent failure of the bank.

A more serious condition with potential for substantial damage occurs due to a phenomenon called harmonic resonance. Resonance conditions are created when the inductive and capacitive reactances become equal at one of the harmonic frequencies. The two types of resonances are series and parallel. In general, series resonance produces voltage amplification and parallel resonance results in current multiplication. Resonance will not be analyzed in this book, but many textbooks on electrical circuit theory are available that can be consulted for further explanation. In a harmonic-rich environment, both series and parallel resonance may be present. If a high level of harmonic voltage or current corresponding to the resonance frequency exists in a power system, considerable damage to the capacitor bank as well as other power system devices can result. The following example might help to illustrate power system resonance due to capacitor banks.

Example: Figure 4.17 shows a 2000-kVA, 13.8-kV to 480/277-V transformer with a leakage reactance of 6.0% feeding a bus containing two 500-hp adjustable speed drives. A 750-kVAR Y-connected capacitor bank is installed on the 480-V bus for power factor correction. Perform an analysis to determine the conditions for resonance (consult Figure 4.18 for the transformer and capacitor connections and their respective voltages and currents):

$$\text{Transformer secondary current } (I) = 2000 \times 10^3/\sqrt{3 \times 480} = 2406 \text{ A}$$

$$\text{Transformer secondary volts} = (V) = 277$$

$$\text{Transformer reactance} = I \times X_L \times 100/V = 6.0$$

$$\text{Transformer leakage reactance } (X_L) = 0.06 \times 277/2406 = 0.0069\ \Omega$$

$$X_L = 2\pi f L, \text{ where } L = 0.0069/377 = 0.183 \times 10^{-4} \text{ H}$$

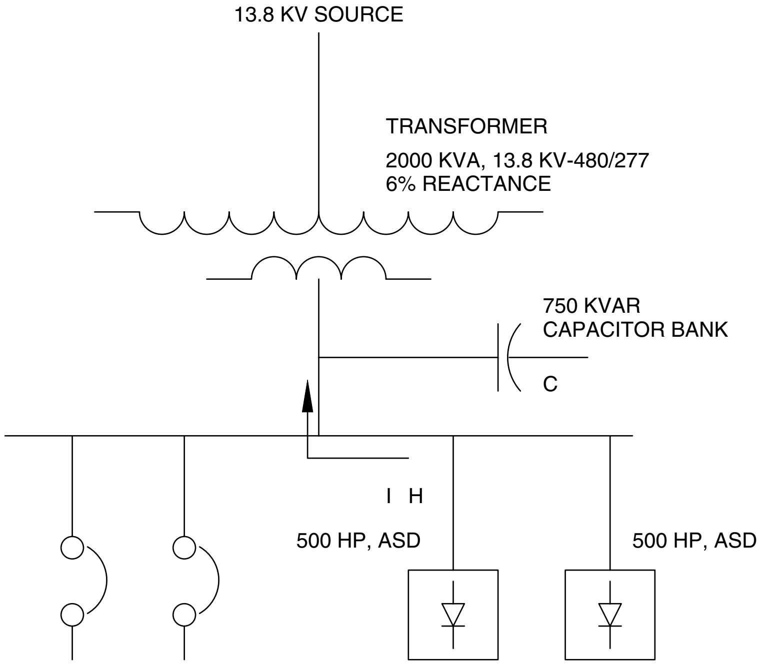


FIGURE 4.17 Schematic representation of an adjustable speed drive and a capacitor bank supplied from a 2000-kVA power transformer.

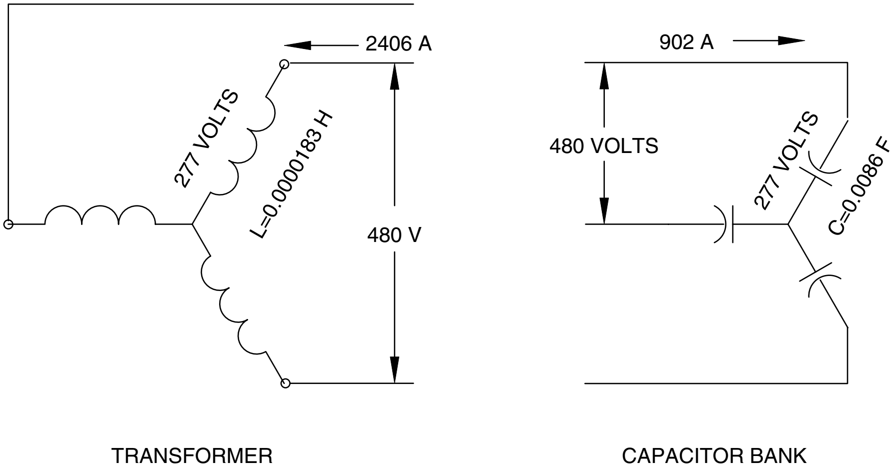


FIGURE 4.18 Transformer and capacitor bank configuration.

For the capacitor bank,

$$\sqrt{3} \times 480 \times I_C = 750 \times 10^3, \text{ where } I_C = 902 \text{ A}$$

$$\text{Capacitive reactance } (X_C) = V/I_C = 277/902 = 0.307\ \Omega$$

$$X_C = 1/2\pi fC, \text{ where } C = 1/(377 \times 0.307) = 86 \times 10^{-4} \text{ F}$$

For resonance, $X_L = X_C$; therefore,

$$2\pi f_R L = 1/2\pi f_R C$$

where f_R is the resonance frequency

$$f_R = 1/2\pi\sqrt{LC} \cong 401 \text{ Hz}$$

The resonance frequency is 401 Hz or the 6.7th (401/60) harmonic frequency. The resonance frequency is close to the seventh harmonic frequency, which is one of the more common harmonic frequency components found in power systems. This condition can have very serious effects.

The following expression presents a different way to find the harmonic resonance frequency:

$$\text{Resonance frequency order} = R_n = \sqrt{(MVA_{SC} \div MVAR_C)} \tag{4.28}$$

where MVA_{SC} is the available symmetrical fault *MVA* at the point of connection of the capacitor in the power system, and $MVAR_C$ is the rating of the capacitor bank in *MVAR*. In the above example, neglecting the source impedance, the available fault current = 2406 ÷ 0.06 ≅ 40,100 A.

$$\text{Available fault } MVA = \sqrt{3} \times 480 \times 40,100 \times 10^{-6} = 33.34$$

$$\text{Capacitor } MVAR = 0.75$$

Therefore, the resonance frequency number = $\sqrt{33.34 \div 0.75} = 6.67$, and the harmonic frequency = 6.67 × 60 = 400.2. This proves that similar results are obtained by using Eq. (4.28). The expression in Eq. (4.28) is derived as follows: The available three-phase fault current at the common bus is given by $I_{SC} = V \div X$, where V is the phase voltage in kilovolts and X is the total reactance of the power system at the bus. I_{SC} is in units of kiloamperes.

$$I_{SC} = V \div 2\pi f_1 L, \text{ where } f_1 \text{ is the fundamental frequency}$$

$$\text{Short circuit } MVA = MVA_{SC} = 3 \times V \times I_{SC} = 3V^2 \div 2\pi f_1 L$$

From this,

$$L = 3V^2 \div 2\pi f_1(MVA_{SC})$$

At resonance,

$$X_{LR} = 2\pi f_R L = 3V^2 f_R \div f_1(MVA_{SC})$$

Because $f_R \div f_1$ = resonance frequency order, R_n, then

$$X_{LR} = 3V^2 R_n \div (MVA_{SC})$$

For the capacitor bank, $I_C = V \div X_C$, and capacitor reactive power $MVAR_C = 3 \times V \times I_C = 3V^2(2\pi f_1 C)$. We can derive an expression for the capacitive reactance at resonance $X_{CR} = 3V^2 \div R_n(MVAR_C)$. Equating X_{LR} and X_{CR}, the harmonic order at resonance is the expression given by Eq. (4.28).

The capacitor bank and the transformer form a parallel resonant circuit with the seventh harmonic current from the ASDs acting as the harmonic source. This condition is represented in Figure 4.19. Two adjustable speed drives typically draw a current of 550 A each, for a total load of 1100 A. If the seventh harmonic current is 5.0% of the fundamental (which is typical in drive applications), the seventh harmonic current seen by the parallel resonant circuit is 55 A = I_7.

If the resistance of the transformer and the associated cable, bus, etc. is 1.0%, then $R \cong 0.0012\ \Omega$.

The quality factor, Q, of an electrical system is a measure of the energy stored in the inductance and the capacitance of the system. The current amplification factor (CAF) of a parallel resonance circuit is approximately equal to the Q of the circuit:

$$Q = 2\pi(\text{maximum energy stored})/\ \text{energy dissipated per cycle}$$

$$Q = (2\pi)(1/2)LI_m^2 \div (I^2R)/f,\ \text{where } I_m = \sqrt{2}I$$

$$Q = X/R$$

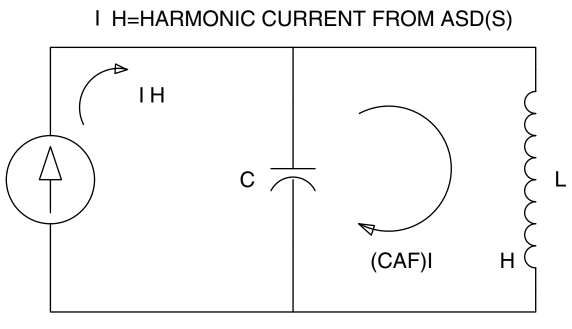


FIGURE 4.19 Parallel resonance circuit formed by transformer inductance and capacitor bank capacitance at harmonic frequency f_H.

For the seventh harmonic frequency, $CAF = X_7/R = 7 \times 0.0069/0.0012 = 40.25$. Therefore, current $I_R = 40.25 \times 55 = 2214$ A. The net current through the capacitor bank $= \sqrt{(I_C^2 + I_R^2)} = 2390$ A. It is easy to see that the capacitor bank is severely overloaded. If the capacitor protective device does not operate to isolate the bank, the capacitor bank will be damaged.

In the above example, by changing the capacitor bank to a 500-kVAR unit, the resonance frequency is increased to 490 Hz, or the 8.2 harmonic. This frequency is potentially less troublesome. (The reader is encouraged to work out the calculations.) In addition, the transformer and the capacitor bank may also form a series resonance circuit as viewed from the power source. This condition can cause a large voltage rise on the 480-V bus with unwanted results. Prior to installing a capacitor bank, it is important to perform a harmonic analysis to ensure that resonance frequencies do not coincide with any of the characteristic harmonic frequencies of the power system.

4.8.4 CABLES

Current flowing in a cable produces I^2R losses. When the load current contains harmonic content, additional losses are introduced. To compound the problem, the effective resistance of the cable increases with frequency because of the phenomenon known as skin effect. Skin effect is due to unequal flux linkage across the cross section of the conductor which causes AC currents to flow only on the outer periphery of the conductor. This has the effect of increasing the resistance of the conductor for AC currents. The higher the frequency of the current, the greater the tendency of the current to crowd at the outer periphery of the conductor and the greater the effective resistance for that frequency.

The capacity of a cable to carry nonlinear loads may be determined as follows. The skin effect factor is calculated first. The skin effect factor depends on the skin depth, which is an indicator of the penetration of the current in a conductor. Skin depth (δ) is inversely proportional to the square root of the frequency:

$$\delta = S \div \sqrt{f}$$

where S is a proportionality constant based on the physical characteristics of the cable and its magnetic permeability and f is the frequency of the current.

If R_{dc} is the DC resistance of the cable, then the AC resistance at frequency f, $(R_f) = K \times R_{dc}$. The value of K is determined from Table 4.9 according to the value of X, which is calculated as:

$$X = 0.0636 \sqrt{f\mu \div R_{dc}} \tag{4.29}$$

where 0.0636 is a constant for copper conductors, f is the frequency, μ is the magnetic permeability of the conductor material, and R_{dc} is the DC resistance per mile of the conductor. The magnetic permeability of a nonmagnetic material such as copper is approximately equal to 1.0. Tables or graphs containing values of X and K are available from cable manufacturers.

TABLE 4.9
Cable Skin Effect Factor

X	K	X	K	X	K
0	1	1.4	1.01969	2.7	1.22753
0.1	1	1.5	1.02558	2.8	1.2662
0.2	1	1.6	1.03323	2.9	1.28644
0.3	1.00004	1.7	1.04205	3.0	1.31809
0.5	1.00032	1.8	1.0524	3.1	1.35102
0.6	1.00067	1.9	1.0644	3.1	1.38504
0.7	1.00124	2.0	1.07816	3.3	1.41999
0.8	1.00212	2.1	1.09375	3.4	1.4577
0.9	1.0034	2.1	1.11126	3.5	1.49202
1.0	1.00519	2.3	1.13069	3.6	1.52879
1.1	1.00758	2.4	1.15207	3.7	1.56587
1.2	1.01071	2.5	1.17538	3.8	1.60312
1.3	1.0147	2.6	1.20056	3.9	1.64051

Example: Find the 60-Hz and 420-Hz resistance of a 4/0 copper cable with a DC resistance of 0.276 Ω per mile. Using Eq. (4.29),

$$X_{60} = 0.0636\ \sqrt{(60 \times 1 \div 0.276)}\ = 0.938$$

From Table 4.2, $K \cong 1.004$, and $R_{60} = 1.004 \times 0.276 = 0.277\ \Omega$ per mile. Also,

$$X_{420} = 0.0636\ \sqrt{(420 \times 1 \div 0.276)}\ = 2.48$$

From Table 4.2, $K \cong 1.154$, and $R_{420} = 1.154 \times 0.276 = 0.319\ \Omega$ per mile.

The ratio of the resistance of the cable at a given frequency to its resistance at 60 Hz is defined as the skin effect ratio, E. According to this definition,

$E_2 =$ resistance at second harmonic frequency ÷ resistance at the fundamental frequency $= R_{120} \div R_{60}$

$E_3 =$ resistance at third harmonic frequency ÷ resistance at the fundamental frequency $= R_{180} \div R_{60}$

Also, remember that the general form expression for the individual harmonic distortions states that I_n is equal to the RMS value of the nth harmonic current divided by the RMS value of the fundamental current, thus an expression for the current rating factor for cables can be formulated. The current rating factor (q) is the equivalent fundamental frequency current at which the cable should be rated for carrying nonlinear loads containing harmonic frequency components:

$$q = I_1^2 E_1 + I_2^2 E_2 + I_3^2 E_3 + \ldots + I_n^2 E_n \tag{4.30}$$

where I_1, I_2, I_3 are the ratios of the harmonic frequency currents to the fundamental current, and E_1, E_2, E_3 are the skin effect ratios.

Example: Determine the current rating factor for a 300-kcmil copper conductor required to carry a nonlinear load with the following harmonic frequency content:

Fundamental = 250 A

3rd harmonic = 25 A

5th harmonic = 60 A

7th harmonic = 45 A

11th harmonic = 20 A

The DC resistance of 300-kcmil cable = 0.17 Ω per mile. Using Eq. (4.29),

$$X_{60} = 0.0636\ \sqrt{(60 \times 1 \div 0.17)}\ = 1.195,\ K \cong 1.0106$$

$$X_{180} = 0.0636\ \sqrt{(180 \times 1 \div 0.17)}\ = 2.069,\ K \cong 1.089$$

$$X_{300} = 0.0636\ \sqrt{(300 \times 1 \div 0.17)}\ = 2.672,\ K \cong 1.220$$

$$X_{420} = 0.0636\ \sqrt{(420 \times 1 \div 0.17)}\ = 3.161,\ K \cong 1.372$$

$$X_{660} = 0{,}0636\ \sqrt{(660 \times 1 \div 0.17)}\ = 3.963,\ K \cong 1.664$$

$$R_{60} = 1.0106 \times 0.17 = 0.1718\ \Omega/\text{mile}$$

$$R_{180} = 1.089 \times 0.17 = 0.1851\ \Omega/\text{mile}$$

$$R_{300} = 1.220 \times 0.17 = 0.2074\ \Omega/\text{mile}$$

$$R_{420} = 1.372 \times 0.17 = 0.2332\ \Omega/\text{mile}$$

$$R_{660} = 1.664 \times 0.17 = 0.2829\ \Omega/\text{mile}$$

Skin effect ratios are:

$$E_1 = 1,\ E_3 = 1.077,\ E_5 = 1.207,\ E_7 = 1.357,\ E_{11} = 1.647$$

The individual harmonic distortion factors are:

$$I_1 = 1.0,\ I_3 = 25/250 = 0.1,\ I_5 = 60/250 = 0.24,\ I_7 = 0.18,\ I_{11} = 20/250 = 0.08$$

The current rating factor from Eq. (4.30) is given by:

$$q = 1 + (0.1)^2(1.077) + (0.24)^2(1.207) + (0.18)^2(1.357) + (0.08)^2(1.647) = 1.135$$

The cable should be capable of handling a 60-Hz equivalent current of $1.135 \times 250 \cong 284$ A.

4.8.5 Busways

Most commercial multistory installations contain busways that serve as the primary source of electrical power to various floors. Busways that incorporate sandwiched busbars are susceptible to nonlinear loading, especially if the neutral bus carries large levels of triplen harmonic currents (third, ninth, etc.). Under the worst possible conditions, the neutral bus may be forced to carry a current equal to 173% of the phase currents. In cases where substantial neutral currents are expected, the busways must be suitably derated. Table 4.10 indicates the amount of nonlinear loads that may be allowed to flow in the phase busbars for different neutral currents. The data are shown for busways with neutral busbars that are 100 and 200% in size.

TABLE 4.10
Bus Duct Derating Factor for Harmonic Loading

$I_N/I_{\varnothing H}$	$I_{\varnothing H}/I_{\varnothing}$	
	100% N	**200% N**
0	1.000	1.000
0.25	0.99	0.995
0.50	0.961	0.98
0.75	0.918	0.956
1.00	0.866	0.926
1.25	0.811	0.891
1.50	0.756	0.853
1.75	0.703	0.814
2.00	0.655	0.775

Note: I_N is the neutral current, $I_{\varnothing H}$ is the harmonic current component in each phase, and $I_{\varnothing}$ is the total phase current. N = size of neutral bus bar in relation to phase bus bar.

4.8.6 Protective Devices

Harmonic currents influence the operation of protective devices. Fuses and motor thermal overload devices are prone to nuisance operation when subjected to nonlinear currents. This factor should be given due consideration when sizing protective devices for use in a harmonic environment. Electromechanical relays are also affected by harmonics. Depending on the design, an electromechanical relay may operate faster or slower than the expected times for operation at the fundamental frequency alone. Such factors should be carefully considered prior to placing the relays in service.

4.9 GUIDELINES FOR HARMONIC VOLTAGE AND CURRENT LIMITATION

So far we have discussed the adverse effects of harmonics on power system operation. It is important, therefore, that attempts be made to limit the harmonic distortion that a facility might produce. There are two reasons for this. First, the lower the harmonic currents produced in an electrical system, the better the equipment within the confinement of the system will perform. Also, lower harmonic currents produce less of an impact on other power users sharing the same power lines of the harmonic generating power system. The IEEE 519 standard provides guidelines for harmonic current limits at the point of common coupling (PCC) between the facility and the utility. The rationale behind the use of the PCC as the reference location is simple. It is a given fact that within a particular power use environment, harmonic currents will be generated and propagated. Harmonic current injection at the PCC determines how one facility might affect other power users and the utility that supplies the power. Table 4.11 (per IEEE 519) lists harmonic current limits based on the size of the power user. As the ratio between the maximum available short circuit current at the PCC and the maximum demand load current increases, the percentage of the harmonic currents that are allowed also increases. This means that larger power users are allowed to inject into the system only a minimal amount of harmonic current (as a percentage of the fundamental current). Such a scheme tends to equalize the amounts of harmonic currents that large and small users of power are allowed to inject into the power system at the PCC.

IEEE 519 also provides guidelines for maximum voltage distortion at the PCC (see Table 4.12). Limiting the voltage distortion at the PCC is the concern of the utility. It can be expected that as long as a facility's harmonic current contribution is within the IEEE 519 limits the voltage distortion at the PCC will also be within the specified limits.

TABLE 4.11
Harmonic Current Limits for General Distribution Systems (120–69,000 V)

I_{SC}/I_L	$h < 11$	$11 \le h < 17$	$17 \le h < 23$	$23 \le h < 35$	$35 \le h$	THD
<20	4.0	2.0	1.5	0.6	0.3	5.0
20–50	7.0	3.5	2.5	1.0	0.5	8.0
50–100	10.0	4.5	4.0	1.5	0.7	12.0
100–1000	12.0	5.5	5.0	2.0	1.0	15.0
>1000	15.0	7.0	6.0	2.5	1.4	20.0

Note: I_{SC} = maximum short-circuit current at PCC; I_L = maximum fundamental frequency demand load current at PCC (average current of the maximum demand for the preceding 12 months); h = individual harmonic order; THD = total harmonic distortion. based on the maximum demand load current. The table applies to odd harmonics; even harmonics are limited to 25% of the odd harmonic limits shown above.

TABLE 4.12
Voltage Harmonic Distortion Limits

Bus Voltage at PCC	Individual Voltage Distortion (%)	Total Voltage Distortion THD (%)
69 kV and below	3.0	5.0
69.001 kV through 161 kV	1.5	2.5
161.001 kV and above	1.0	1.5

Note: PCC = point of common coupling; THD = total harmonic distortion.

When the IEEE 519 harmonic limits are used as guidelines within a facility, the PCC is the common junction between the harmonic generating loads and other electrical equipment in the power system. It is expected that applying IEEE guidelines renders power system operation more reliable. In the future, more and more utilities might require facilities to limit their harmonic current injection to levels stipulated by IEEE 519. The following section contains information on how harmonic mitigation might be achieved.

4.10 HARMONIC CURRENT MITIGATION

4.10.1 EQUIPMENT DESIGN

The use of electronic power devices is steadily increasing. It is estimated that more than 70% of the loading of a facility by year 2010 will be due to nonlinear loads, thus demand is increasing for product manufacturers to produce devices that generate lower distortion. The importance of equipment design in minimizing harmonic current production has taken on greater importance, as reflected by technological improvements in fluorescent lamp ballasts, adjustable speed drives, battery chargers, and uninterruptible power source (UPS) units. Computers and similar data-processing devices contain switching mode power supplies that generate a substantial amount of harmonic currents, as seen earlier. Designing power supplies for electronic equipment adds considerably to the cost of the units and can also make the equipment heavier. At this time, when computer prices are extremely competitive, attempts to engineer power supplies that draw low harmonic currents are not a priority.

Adjustable speed drive (ASD) technology is evolving steadily, with greater emphasis being placed on a reduction in harmonic currents. Older generation ASDs using current source inverter (CSI) and voltage source inverter (VSI) technologies produced considerable harmonic frequency currents. The significant harmonic frequency currents generated in power conversion equipment can be stated as:

$$n = kq \pm 1$$

where n is the significant harmonic frequency, k is any positive integer (1, 2, 3, etc.), and q is the pulse number of the power conversion equipment which is the number of power pulses that are in one complete sequence of power conversion. For example, a three-phase full wave bridge rectifier has six power pulses and therefore has a pulse number of 6. With six-pulse-power conversion equipment, the following significant harmonics may be generated:

For $k = 1$, $n = (1 \times 6) \pm 1 = $ 5th and 7th harmonics.
For $k = 2$, $n = (2 \times 6) \pm 1 = $ 11th and 13th harmonics.

With six-pulse-power conversion equipment, harmonics below the 5th harmonic are insignificant. Also, as the harmonic number increases, the individual harmonic distortions become lower due to increasing impedance presented to higher frequency components by the power system inductive reactance. So, typically, for six-pulse-power conversion equipment, the 5th harmonic current would be the highest, the 7th would be lower than the 5th, the 11th would be lower than the 7th, and so on, as shown below:

$$I_{13} < I_{11} < I_7 < I_5$$

We can deduce that, when using 12-pulse-power conversion equipment, harmonics below the 11th harmonic can be made insignificant. The total harmonic distortion is also considerably reduced. Twelve-pulse-power conversion equipment costs more than six-pulse-power equipment. Where harmonic currents are the primary concern, 24-pulse-power conversion equipment may be considered.

4.10.2 Harmonic Current Cancellation

Transformer connections employing phase shift are sometimes used to effect cancellation of harmonic currents in a power system. Triplen harmonic (3rd, 9th, 15th, etc.) currents are a set of currents that can be effectively trapped using a special transformer configuration called the zigzag connection. In power systems, triplen harmonics add in the neutral circuit, as these currents are in phase. Using a zigzag connection, the triplens can be effectively kept away from the source. Figure 4.20 illustrates how this is accomplished.

The transformer phase-shifting principle is also used to achieve cancellation of the 5th and the 7th harmonic currents. Using a Δ–Δ and a Δ–Y transformer to supply harmonic producing loads in parallel as shown in Figure 4.21, the 5th and the 7th harmonics are canceled at the point of common connection. This is due to the 30° phase shift between the two transformer connections. As the result of this, the source does not see any significant amount of the 5th and 7th harmonics. If the nonlinear loads supplied by the two transformers are identical, then maximum harmonic current cancellation takes place; otherwise, some 5th and 7th harmonic currents would still be present. Other phase-shifting methods may be used to cancel higher harmonics if they are found to be a problem. Some transformer manufacturers offer multiple phase-shifting connections in a single package which saves cost and space compared to using individual transformers.

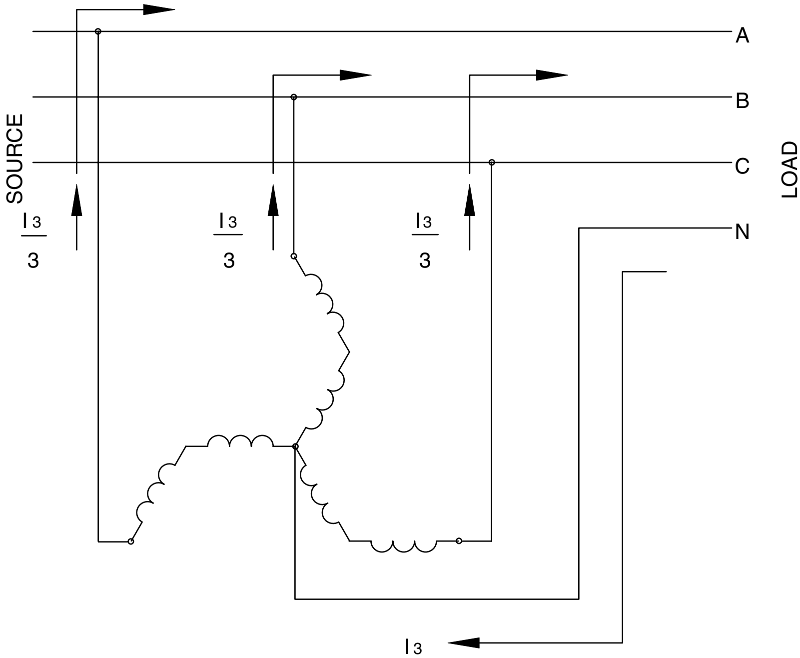


FIGURE 4.20 Zig-zag transformer application as third harmonic filter.

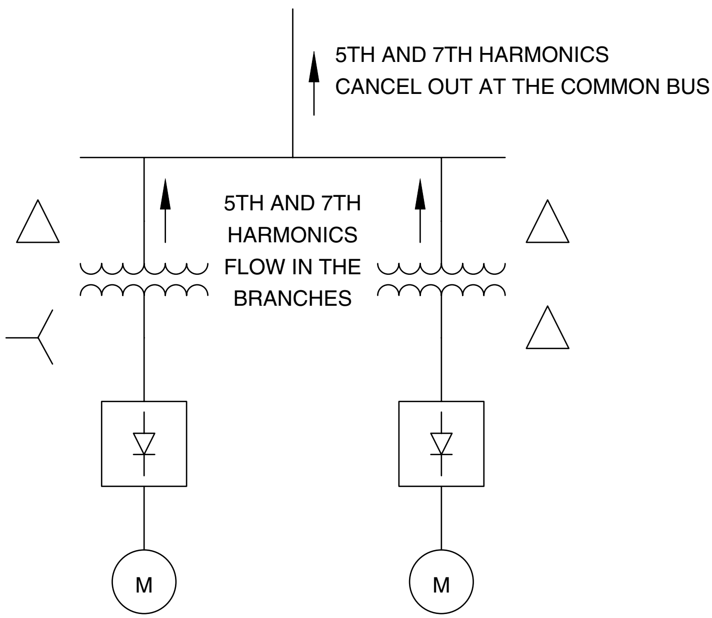


FIGURE 4.21 Cancellation of fifth and seventh harmonic currents by using 30° phase-shifted transformer connections.

4.10.3 Harmonic Filters

Nonlinear loads produce harmonic currents that can travel to other locations in the power system and eventually back to the source. As we saw earlier, harmonic currents can produce a variety of effects that are harmful to the power system. Harmonic currents are a result of the characteristics of particular loads. As long as we choose to employ those loads, we must deal with the reality that harmonic currents will exist to a degree dependent upon the loads. One means of ensuring that harmonic currents produced by a nonlinear current source will not unduly interfere with the rest of the power system is to filter out the harmonics. Application of harmonic filters helps to accomplish this.

Harmonic filters are broadly classified into passive and active filters. Passive filters, as the name implies, use passive components such as resistors, inductors, and capacitors. A combination of passive components is tuned to the harmonic frequency that is to be filtered. Figure 4.22 is a typical series-tuned filter. Here the values of the inductor and the capacitor are chosen to present a low impedance to the harmonic frequency that is to be filtered out. Due to the lower impedance of the filter in comparison to the impedance of the source, the harmonic frequency current will circulate between the load and the filter. This keeps the harmonic current of the desired frequency away from the source and other loads in the power system. If other harmonic frequencies are to be filtered out, additional tuned filters are applied in parallel. Applications such as arc furnaces require multiple harmonic filters, as they generate large quantities of harmonic currents at several frequencies.

Applying harmonic filters requires careful consideration. Series-tuned filters appear to be of low impedance to harmonic currents but they also form a parallel resonance circuit with the source impedance. In some instances, a situation can be created that is worse than the condition being corrected. It is imperative that computer simulations of the entire power system be performed prior to applying harmonic filters. As a first step in the computer simulation, the power system is modeled to indicate the locations of the harmonic sources, then hypothetical harmonic filters are placed in the model and the response of the power system to the filter is examined. If unacceptable results are obtained, the location and values of the filter parameters are changed until the results are satisfactory. When applying harmonic filters, the units are almost never tuned to the exact harmonic frequency. For example, the 5th harmonic frequency may be designed for resonance at the 4.7th harmonic frequency.

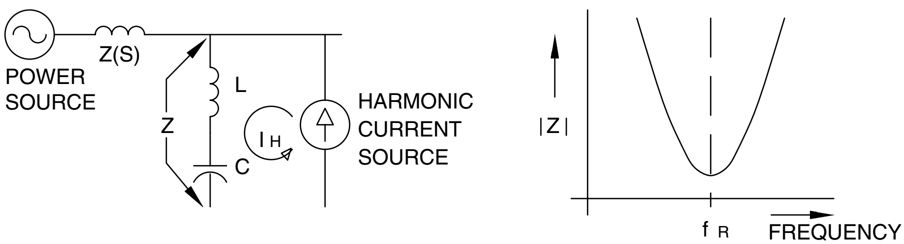


FIGURE 4.22 Series-tuned filter and filter frequency response.

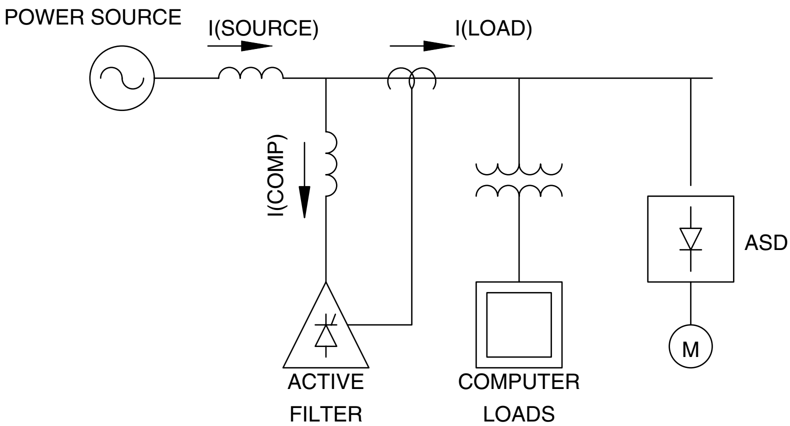


FIGURE 4.23 Active filter to cancel harmonic currents.

By not creating a resonance circuit at precisely the 5th harmonic frequency, we can minimize the possibility of the filter resonating with other loads or the source, thus forming a parallel resonance circuit at the 5th harmonic. The 4.7th harmonic filter would still be effective in filtering out the 5th harmonic currents. This is evident from the series-tuned frequency vs. impedance curve shown in Figure 4.22.

Sometimes, tuned filters are configured to provide power factor correction for a facility as well as harmonic current filtering. In such cases the filter would be designed to carry the resonant harmonic frequency current and also the normal frequency current at the fundamental frequency. In either case, a power system harmonic study is paramount to ensure that no ill effects would be produced by the application of the power factor correction/filter circuit.

Active filters use active conditioning to compensate for harmonic currents in a power system. Figure 4.23 shows an active filter applied in a harmonic environment. The filter samples the distorted current and, using power electronic switching devices, draws a current from the source of such magnitude, frequency composition, and phase shift to cancel the harmonics in the load. The result is that the current drawn from the source is free of harmonics. An advantage of active filters over passive filters is that the active filters can respond to changing load and harmonic conditions, whereas passive filters are fixed in their harmonic response. As we saw earlier, application of passive filters requires careful analysis. Active filters have no serious ill effects associated with them. However, active filters are expensive and not suited for application in small facilities.

4.11 CONCLUSIONS

The term *harmonics* is becoming very common in power systems, small, medium, or large. As the use of power electronic devices grows, so will the need to understand the effects of harmonics and the application of mitigation methods. Fortunately, harmonics in a strict sense are not transient phenomena. Their presence can be easily measured and identified. In some cases, harmonics can be lived with indefinitely, but in other cases they should be minimized or eliminated. Either of these approaches requires a clear understanding of the theory behind harmonics.