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# 6 Power Factor

## 6.1 INTRODUCTION

Power factor is included in the discussion of power quality for several reasons. Power factor is a power quality issue in that low power factor can sometimes cause equipment to fail. In many instances, the cost of low power factor can be high; utilities penalize facilities that have low power factor because they find it difficult to meet the resulting demands for electrical energy. The study of power quality is about optimizing the performance of the power system at the lowest possible operating cost. Power factor is definitely an issue that qualifies on both counts.

## 6.2 ACTIVE AND REACTIVE POWER

Several different definitions and expressions can be applied to the term power factor, most of which are probably correct. Apparent power ( $S$ ) in an electrical system can be defined as being equal to voltage times current:

$$S = V \times I(1\emptyset)$$

$$S = \sqrt{3} \times V \times I(3\emptyset)$$

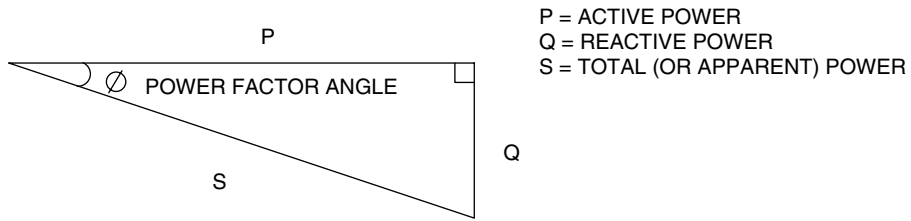
where  $V$  = phase-to-phase voltage (V) and  $I$  = line current (VA).

Power factor ( $PF$ ) may be viewed as the percentage of the total apparent power that is converted to real or useful power. Thus, active power ( $P$ ) can be defined by:

$$P = V \times I \times PF - 1\emptyset$$

$$P = \sqrt{3} \times V \times I \times PF - 3\emptyset$$

In an electrical system, if the power factor is 0.80, 80% of the apparent power is converted into useful work. Apparent power is what the transformer that serves a home or business has to carry in order for that home or business to function. Active power is the portion of the apparent power that performs useful work and supplies losses in the electrical equipment that are associated with doing the work. Higher power factor leads to more optimum use of electrical current in a facility. Can a power factor reach 100%? In theory it can, but in practice it cannot without some form of power factor correction device. The reason why it can approach 100% power factor but not quite reach it is because all electrical circuits have inductance and capacitance, which introduce reactive power requirements. The reactive power is that



**FIGURE 6.1** Power triangle and relationship among active, reactive, and apparent power.

portion of the apparent power that prevents it from obtaining a power factor of 100% and is the power that an AC electrical system requires in order to perform useful work in the system. Reactive power sets up a magnetic field in the motor so that a torque is produced. It is also the power that sets up a magnetic field in a transformer core allowing transfer of power from the primary to the secondary windings.

All reactive power requirements are not necessary in every situation. Any electrical circuit or device when subjected to an electrical potential develops a magnetic field that represents the inductance of the circuit or the device. As current flows in the circuit, the inductance produces a voltage that tends to oppose the current. This effect, known as Lenz's law, produces a voltage drop in the circuit that represents a loss in the circuit. At any rate, inductance in AC circuits is present whether it is needed or not. In an electrical circuit, the apparent and reactive powers are represented by the power triangle shown in [Figure 6.1](#). The following relationships apply:

$$S = \sqrt{P^2 + Q^2} \quad (6.1)$$

$$P = S \cos \emptyset \quad (6.2)$$

$$Q = S \sin \emptyset \quad (6.3)$$

$$Q/P = \tan \emptyset \quad (6.4)$$

where  $S$  = apparent power,  $P$  = active power,  $Q$  = reactive power, and  $\emptyset$  is the power factor angle. In [Figure 6.2](#),  $V$  is the voltage applied to a circuit and  $I$  is the current in the circuit. In an inductive circuit, the current lags the voltage by angle  $\emptyset$ , as shown in the figure, and  $\emptyset$  is called the power factor angle.

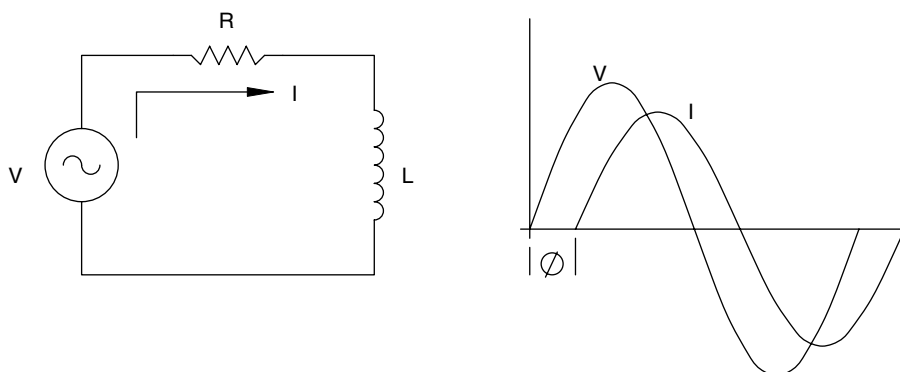
If  $X_L$  is the inductive reactance given by:

$$X_L = 2\pi fL$$

then total impedance ( $Z$ ) is given by:

$$Z = R + jX_L$$

where  $j$  is the imaginary operator =  $\sqrt{-1}$



**FIGURE 6.2** Voltage, current, and power factor angle in a resistive/inductive circuit.

The power factor angle is calculated from the expression:

$$\tan \varnothing = (X_L/R) \text{ or } \varnothing = \tan^{-1}(X_L/R) \quad (6.5)$$

*Example:* What is the power factor of a resistive/inductive circuit characterized by  $R = 2 \, \Omega$ ,  $L = 2.0 \, \text{mH}$ ,  $f = 60 \, \text{Hz}$ ?

$$X_L = 2\pi fL = 2 \times \pi \times 60 \times 2 \times 10^{-3} = 0.754 \, \Omega$$

$$\tan \varnothing = X_L/R = 0.754/2 = 0.377$$

$$\varnothing = 20.66^\circ$$

$$\text{Power factor} = PF = \cos(20.66) = 0.936$$

*Example:* What is the power factor of a resistance/capacitance circuit when  $R = 10 \, \Omega$ ,  $C = 100 \, \mu\text{F}$ , and frequency ( $f$ ) = 60 Hz? Here,

$$X_C = 1/2\pi fC = 1/2 \times \pi \times 60 \times 100 \times 10^{-6} = 26.54 \, \Omega$$

$$\tan \varnothing = (-X_C/R) = -2.654$$

$$\varnothing = -69.35^\circ$$

$$\text{Power factor} = PF = \cos \varnothing = 0.353$$

The negative power factor angle indicates that the current leads the voltage by  $69.35^\circ$ .

Let's now consider an inductive circuit where application of voltage  $V$  produces current  $I$  as shown in Figure 6.2 and the phasor diagram for a single-phase circuit is as shown. The current is divided into active and reactive components,  $I_p$  and  $I_Q$ :

$$I_p = I \times \cos\phi$$

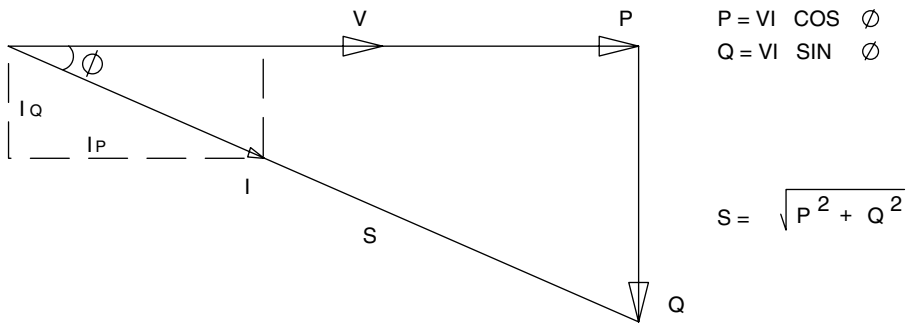
$$I_Q = I \times \sin\phi$$

$$\text{Active power} = P = V \times \text{active current} = V \times I \times \cos\phi$$

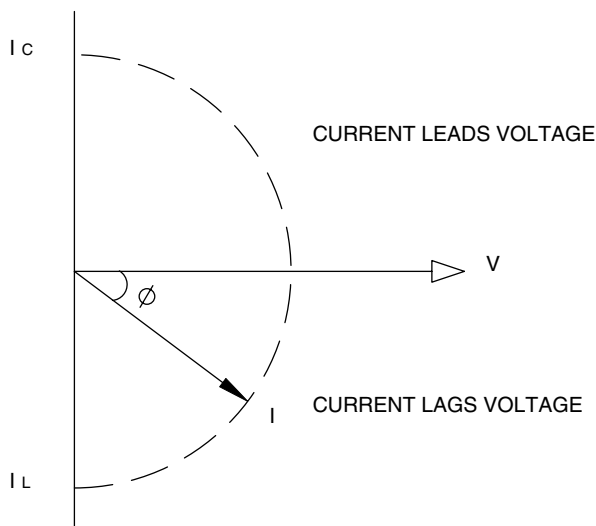
$$\text{Reactive power} = Q = V \times \text{reactive current} = V \times I \times \sin\phi$$

$$\text{Total or apparent power} = S = \sqrt{(P^2 + Q^2)} = \sqrt{(V^2 I^2 \cos^2\phi + V^2 I^2 \sin^2\phi)} = V \times I$$

Voltage, current, and power phasors are as shown in Figure 6.3. Depending on the reactive power component, the current phasor can swing, as shown in Figure 6.4. The  $\pm 90^\circ$  current phasor displacement is the theoretical limit for purely inductive and capacitive loads with zero resistance, a condition that does not really exist in practice.



**FIGURE 6.3** Relationship among voltage, current, and power phasors.



**FIGURE 6.4** Theoretical limits of current.

### 6.3 DISPLACEMENT AND TRUE POWER FACTOR

The terms *displacement* and *true power factor*, are widely mentioned in power factor studies. Displacement power factor is the cosine of the angle between the fundamental voltage and current waveforms. The fundamental waveforms are by definition pure sinusoids. But, if the waveform distortion is due to harmonics (which is very often the case), the power factor angles are different than what would be for the fundamental waves alone. The presence of harmonics introduces additional phase shift between the voltage and the current. True power factor is calculated as the ratio between the total active power used in a circuit (including harmonics) and the total apparent power (including harmonics) supplied from the source:

$$\text{True power factor} = \text{total active power} / \text{total apparent power}$$

Utility penalties are based on the true power factor of a facility.

### 6.4 POWER FACTOR IMPROVEMENT

Two ways to improve the power factor and minimize the apparent power drawn from the power source are:

- Reduce the lagging reactive current demand of the loads
- Compensate for the lagging reactive current by supplying leading reactive current to the power system

The second method is the topic of interest in this chapter. Lagging reactive current represent the inductance of the power system and power system components. As observed earlier, lagging reactive current demand may not be totally eliminated but may be reduced by using power system devices or components designed to operate with low reactive current requirements. Practically no devices in a typical power system require leading reactive current to function; therefore, in order to produce leading currents certain devices must be inserted in a power system. These devices are referred to as *power factor correction equipment*.

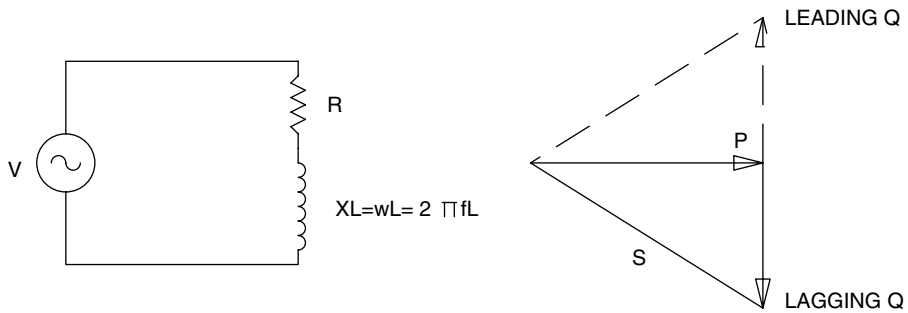
### 6.5 POWER FACTOR CORRECTION

In simple terms, power factor correction means reduction of lagging reactive power ( $Q$ ) or lagging reactive current ( $I_Q$ ). Consider [Figure 6.5](#). The source  $V$  supplies the resistive/inductive load with impedance ( $Z$ ):

$$Z = R + j\omega L$$

$$I = V/Z = V/(R + j\omega L)$$

$$\text{Apparent power} = S = V \times I = V^2/(R + j\omega L)$$



**FIGURE 6.5** Lagging and leading reactive power representation.

Multiplying the numerator and the denominator by  $(R - j\omega L)$ ,

$$S = V^2(R - j\omega L)/(R^2 + \omega^2 L^2)$$

Separating the terms,

$$S = V^2 R/(R^2 + \omega^2 L^2) - jV^2 \omega L/(R^2 + \omega^2 L^2)$$

$$S = P - jQ \quad (6.6)$$

The  $-Q$  indicates that the reactive power is lagging. By supplying a leading reactive power equal to  $Q$ , we can correct the power factor to unity.

From Eq. (6.4),  $Q/P = \tan \phi$ . From Eq. (6.5),  $Q/P = \omega L/R = \tan \phi$  and  $\phi = \tan^{-1}(\omega L/R)$ , thus:

$$\text{Power factor} = \cos \phi = \cos(\tan^{-1} \omega L/R) \quad (6.7)$$

*Example:* In the circuit shown in Figure 6.5,  $V = 480$  V,  $R = 1 \Omega$ , and  $L = 1$  mH; therefore,

$$X_L = \omega L = 2\pi f L = 2\pi \times 60 \times .001 = 0.377 \Omega$$

From Eq. (6.6),

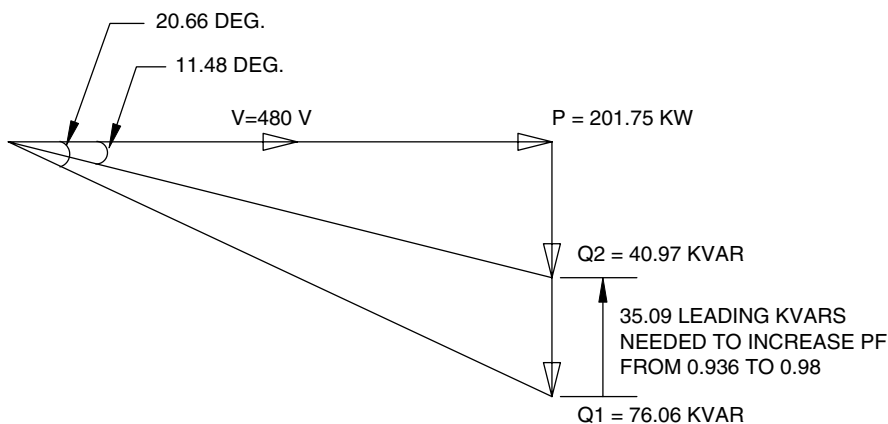
$$\text{Active power} = P = V^2 R/(R^2 + \omega^2 L^2) = 201.75 \text{ kW}$$

$$\text{Reactive power} = Q = V^2 \omega L/(R^2 + \omega^2 L^2) = 76.06 \text{ kVAR}$$

$$\text{Power factor angle} = \phi = \tan^{-1}(Q/P) = \tan^{-1}(0.377) = 20.66^\circ$$

$$\text{Power factor} = PF = \cos \phi = 0.936$$

The leading reactive power necessary to correct the power factor to 1.0 is 76.06 kVAR.



**FIGURE 6.6** Power factor correction triangle.

In the same example, what is the leading kVAR required to correct the power factor to 0.98? At 0.98 power factor lag, the lagging kVAR permitted can be calculated from the following:

$$\text{Power factor angle at } 0.98 = 11.48^\circ$$

$$\tan(11.48^\circ) = Q/201.75 = 0.203$$

$$Q = 0.203 \times 201.75 = 40.97 \text{ kVAR}$$

The leading kVAR required in order to correct the power factor to 0.98 = 76.06 – 40.97 = 35.09 (see Figure 6.6).

In a typical power system, power factor calculations, values of resistance, and inductance data are not really available. What is available is total active and reactive power. From this, the kVAR necessary to correct the power factor from a given value to another desired value can be calculated. Figure 6.7 shows the general power factor correction triangles. To solve this triangle, three pieces of information are needed: existing power factor ( $\cos\theta_1$ ), corrected power factor ( $\cos\theta_2$ ), and any one of the following: active power ( $P$ ), reactive power ( $Q$ ), or apparent power ( $S$ ).

- Given  $P$ ,  $\cos\theta_1$ , and  $\cos\theta_2$ :  
From the above,  $Q_1 = P \tan\theta_1$  and  $Q_2 = P \tan\theta_2$ . The reactive power required to correct the power factor from  $\cos\theta_1$  to  $\cos\theta_2$  is:

$$\Delta Q = P(\tan\theta_1 - \tan\theta_2)$$

- Given  $S_1$ ,  $\cos\theta_1$ , and  $\cos\theta_2$ :  
From the above,  $Q_1 = S_1 \sin\theta_1$ ,  $P = S_1 \cos\theta_1$ , and  $Q_2 = P \tan\theta_2$ . The leading reactive power necessary is:

$$\Delta Q = Q_1 - Q_2$$

- Given  $Q_1$ ,  $\cos\phi_1$ , and  $\cos\phi_2$ :

From the above,  $P = Q_1/\tan\phi_1$  and  $Q_2 = P\tan\phi_2$ . The leading reactive power necessary is:

$$\Delta Q = Q_1 - Q_2$$

*Example:* A 5-MVA transformer is loaded to 4.5 MVA at a power factor of 0.82 lag. Calculate the leading kVAR necessary to correct the power factor to 0.95 lag. If the transformer has a rated conductor loss equal to 1.0% of the transformer rating, calculate the energy saved assuming 24-hour operation at the operating load. [Figure 6.8](#) contains the power triangle of the given load and power factor conditions:

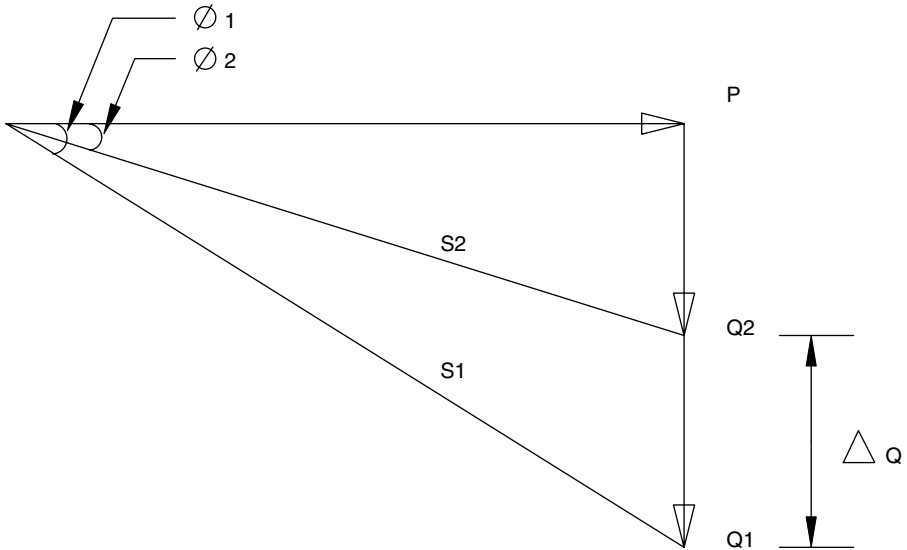
$$\text{Existing power factor angle} = \phi_1 = \cos^{-1}(0.82) = 34.9^\circ$$

$$\text{Corrected power factor angle} = \phi_2 = \cos^{-1}(0.95) = 18.2^\circ$$

$$Q_1 = S_1 \sin\phi_1 = 4.5 \times 0.572 = 2.574 \text{ MVAR}$$

$$P = S_1 \cos\phi_1 = 4.5 \times 0.82 = 3.69 \text{ MW}$$

$$Q_2 = P \tan\phi_2 = 3.69 \times 0.329 = 1.214 \text{ MVAR}$$



**FIGURE 6.7** General power factor correction triangle.



The leading MVAR necessary to improve the power factor from 0.82 to 0.95 =  $Q_1 - Q_2 = 1.362$ . For a transformer load with improved power factor  $S_2$ :

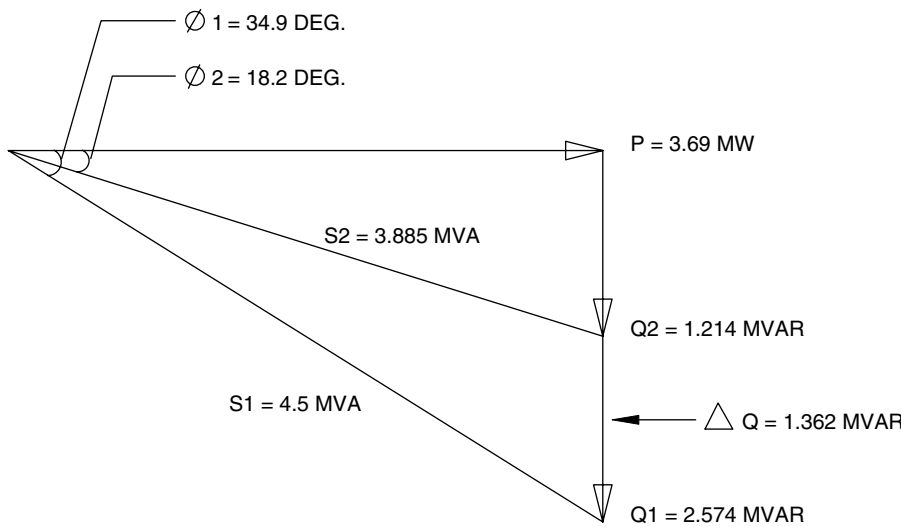
$$S_2 = \sqrt{(P^2 + Q_2^2)} = 3.885 \text{ MVA}$$

The change in transformer conductor loss =  $1.0 [(4.5/5)^2 - (3.885/5)^2] = 0.206$  p.u. of rated losses, thus the total energy saved =  $0.206 \times 50 \times 24 = 247.2$  kWhr/day. At a cost of \$0.05/kWhr, the energy saved per year =  $247.2 \times 365 \times 0.05 = \$4511.40$ .

### 6.6 POWER FACTOR PENALTY

Typically, electrical utilities charge a penalty for power factors below 0.95. The method of calculating the penalty depends on the utility. In some cases, the formula is simple, but in other cases the formula for the power factor penalty can be much more complex. Let's assume that one utility charges a rate of 0.20¢/kVAR–hr for all the reactive energy used if the power factor falls below 0.95. No kVar–hr charges are levied if the power factor is above 0.95.

In the example above, at 0.82 power factor the total kVar–hr of reactive power used per month =  $2574 \times 24 \times 30$ . The total power factor penalty incurred each month =  $2574 \times 24 \times 30 \times 0.20 \times 0.01 = \$3707$ . The cost of having a low power factor per year is \$44,484. The cost of purchasing and installing power factor correction equipment in this specific case would be about \$75,000. It is not difficult to see the cost savings involved by correcting the power factor to prevent utility penalties.



**FIGURE 6.8** Power factor triangle for Section 6.4 example.

Another utility calculates the penalty using a different formula. First, kW demand is increased by a factor equal to the 0.95 divided by actual power factor. The difference between this and the actual demand is charged at a rate of \$3.50/kW. In the example, the calculated demand due to low power factor =  $3690 \times 0.95/0.82 = \$4275$ , thus the penalty kW =  $4275 - 3690 = \$585$ , and the penalty each month =  $585 \times \$3.50 = \$2047$ . In this example, the maximum demand is assumed to be equal to the average demand calculated for the period. The actual demand is typically higher than the average demand. The penalty for having a poor power factor will be correspondingly higher. In the future, as the demand for electrical power continues to grow, the penalty for poor power factors is expected to get worse.

## 6.7 OTHER ADVANTAGES OF POWER FACTOR CORRECTION

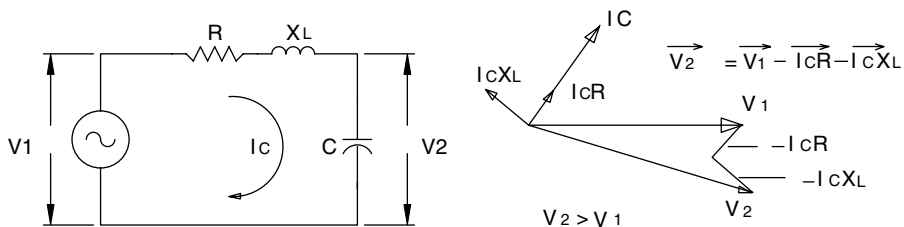
Correcting low power factor has other benefits besides avoiding penalties imposed by the utilities. Other advantages of improving the power factor include:

- Reduced heating in equipment
- Increased equipment life
- Reduction in energy losses and operating costs
- Freeing up available energy
- Reduction of voltage drop in the electrical system

In Figure 6.9, the total apparent power saved due to power factor correction =  $4500 - 3885 = 615$  kVA, which will be available to supply other plant loads or help minimize capital costs in case of future plant expansion. As current drawn from the source is lowered, the voltage drop in the power system is also reduced. This is important in large industrial facilities or high-rise commercial buildings, which are typically prone to excessive voltage sags.

## 6.8 VOLTAGE RISE DUE TO CAPACITANCE

When large power factor correction capacitors are present in an electrical system, the flow of capacitive current through the power system impedance can actually

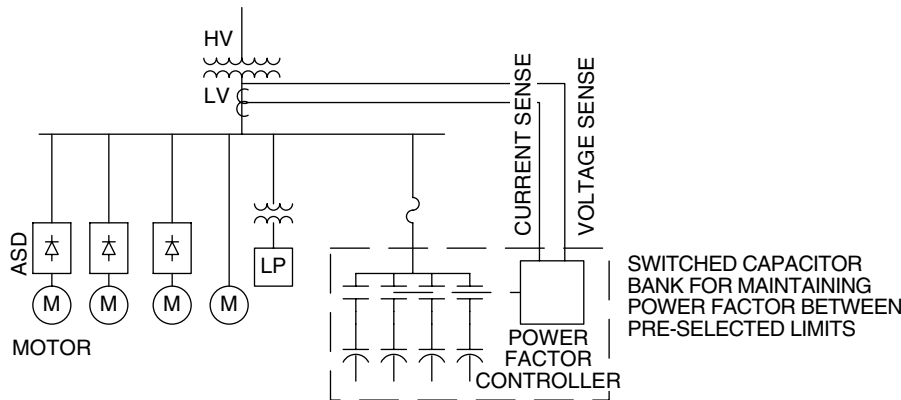


**FIGURE 6.9** Schematic and phasor diagram showing voltage rise due to capacitive current flowing through line impedance.

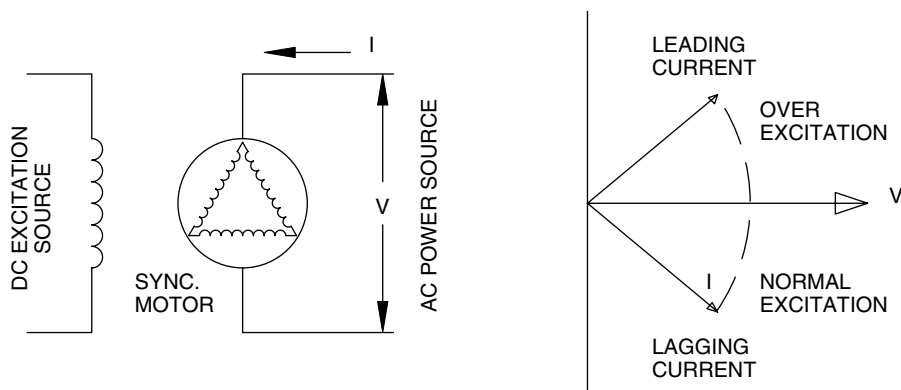
produce a voltage rise, as shown in [Figure 6.9](#). In some instances, utilities will actually switch on large capacitor banks to effect a voltage rise on the power system at the end of long transmission lines. Depending on the voltage levels and the reactive power demand of the loads, the capacitors may be switched in or out in discreet steps. Voltage rise in the power system is one reason why the utilities do not permit large levels of uncompensated leading kVARs to be drawn from the power lines. During the process of selecting capacitor banks for power factor correction, the utilities should be consulted to determine the level of leading kVARs that can be drawn. This is not a concern when the plant or the facility is heavily loaded, because the leading kVARs would be essentially canceled by the lagging reactive power demand of the plant. But, during light load periods, the leading reactive power is not fully compensated and therefore might be objectionable to the utility. For applications where large swings in reactive power requirements are expected, a switched capacitor bank might be worth the investment. Such a unit contains a power factor controller that senses and regulates the power factor by switching blocks of capacitors in and out. Such equipment is more expensive. [Figure 6.10](#) depicts a switched capacitor bank configured to maintain a power factor between two preset limits for various combinations of plant loading conditions.

### 6.9 APPLICATION OF SYNCHRONOUS CONDENSERS

It was observed in Chapter 4 that capacitor banks must be selected and applied based on power system harmonic studies. This is necessary to eliminate conditions that can actually amplify the harmonics and create conditions that can render the situation considerably worse. One means of providing leading reactive power is by the use of synchronous motors. Synchronous motors applied for power factor control are called synchronous condensers. A synchronous motor normally draws lagging currents, but when its field is overexcited, the motor draws leading reactive currents ([Figure 6.11](#)). By adjusting the field currents, the synchronous motor can be made



**FIGURE 6.10** Schematic of a switched capacitor bank for power factor control between preselected limits for varying plant load conditions.



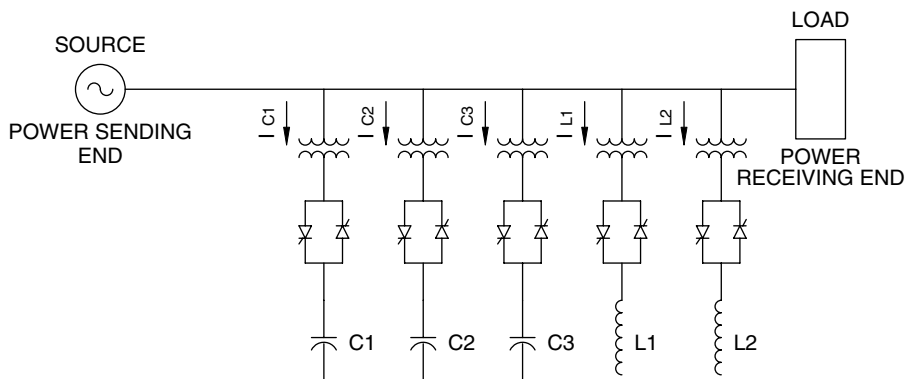
**FIGURE 6.11** Synchronous condenser for power factor correction.

to operate in the lagging, unity, or leading power factor region. Facilities that contain large AC motors are best suited for the application. Replacing an AC induction motor with a synchronous motor operating in the leading power factor region is an effective means of power factor control. Synchronous motors are more expensive than conventional induction motors due to their construction complexities and associated control equipment. Some facilities and utilities use unloaded synchronous motors strictly for leading reactive power generation. The advantage of using a synchronous condenser is the lack of harmonic resonance problems sometimes found with the use of passive capacitor banks.

## 6.10 STATIC VAR COMPENSATORS

Static VAR compensators (SVCs) use static power control devices such as SCRs or IGBTs and switch a bank of capacitors and inductors to generate reactive currents of the required makeup. Reactive power is needed for several reasons. As we saw earlier, leading reactive power is needed to improve the power factor and also to raise the voltage at the end of long power lines. Lagging reactive power is sometimes necessary at the end of long transmission lines to compensate for the voltage rise experienced due to capacitive charging currents of the lines. Uncompensated, such power lines can experience a voltage rise beyond what is acceptable. The reactors installed for such purposes are called line charge compensators.

Static VAR compensators perform both functions as needed. [Figure 6.12](#) contains a typical arrangement of an SVC. By controlling the voltage to the capacitors and inductors, accurate reactive current control is obtained. One drawback of using SVCs is the generation of a considerable amount of harmonic currents that may have to be filtered. The cost of SVCs is also high, so they will not be economical for small power users.



**FIGURE 6.12** Static VAR compensator draws optimum amount of leading and lagging currents to maintain required voltage and power factor levels.

## 6.11 CONCLUSIONS

Good power factor is not necessarily critical for most equipment to function in a normal manner. Having low power factor does not cause a piece of machinery to shut down, but high power factor is important for the overall health of the power system. Operating in a high power factor environment ensures that the power system is functioning efficiently. It also makes economic sense. Electrical power generation, transmission, and distribution equipment have maximum rated currents that the machines can safely handle. If these levels are exceeded, the equipment operates inefficiently and suffers a loss of life expectancy. This is why it is important not to exceed the rated currents for power system equipment. It is also equally important that the available energy production capacity be put to optimum use. Such an approach helps to provide an uninterrupted supply of electrical energy to industries, hospitals, commercial institutions, and our homes. As the demand for electrical energy continues to grow and the resources for producing the energy become less and less available, the idea of not using more than what we need takes on more relevance.