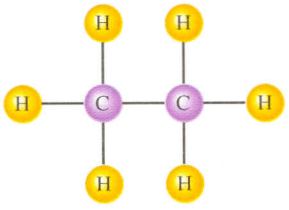
APPENDIX: GRAPH THEORY

DID YOU KNOW

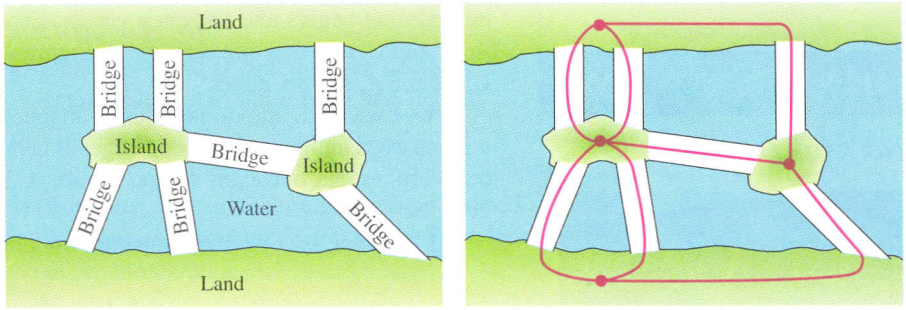
Making Connections



Ethane

A chemist sketches the relationship among atoms in a molecule. An engineer draws a schematic diagram of an electrical network. A genealogist charts a family tree. A sociologist maps the power structure of a large corporation. What do all of these drawings have in common? Each one is a system of points (atoms, electrical components, people, segments) connected by line segments. The German mathematician Denes Konig, who made the first systematic study of such patterns, named them "graphs."

In the eighteenth-century Prussian town of Königsberg (now the city of Kaliningrad, Russia near the Baltic Sea) seven bridges crossed the Prigel River (Fig. A.1a). Individuals in the area tried to determine whether it was possible to walk a path that would cross each of the seven bridges exactly once. They found that they ended up either not crossing one of the bridges, or crossing one of the bridges more than once. The problem was brought to the attention of the Swiss mathematician Leonhard Euler (pronounced "oiler," 1707–1783). His study of this problem, now known as the Königsberg bridge problem, laid the groundwork for a modern branch of mathematics called ***graph theory***, a topic in a more general area called ***topology***. To solve the problem, Euler drew figures called ***graphs*** *or* ***networks*** like that shown in red in Fig. A.1(b). Each dot represented a plot of land and each line a bridge or path. Can you find a path that crosses each bridge exactly once?



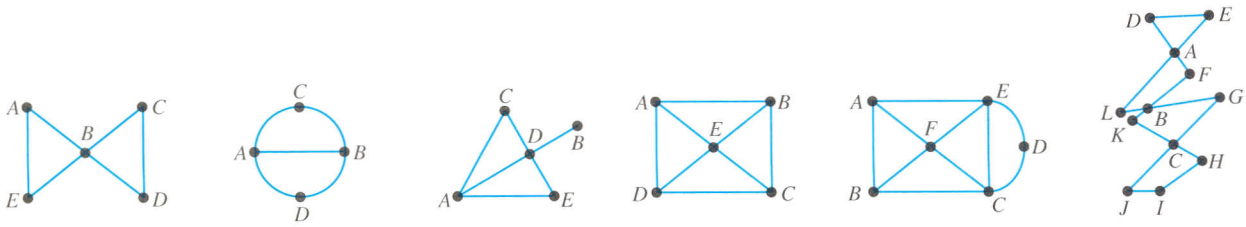
(a) (b)

Figure A.1

Before we determine whether there is a path that will cross each bridge exactly once, let's consider Example 1.

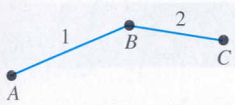
EXAMPLE 1

In Fig. A.2 start at any point and try to trace each figure without retracing a line and without removing your pencil from the paper. If you succeed, indicate your starting point and ending point.



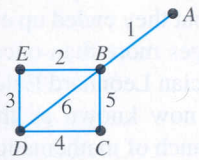
(a) (b) (c) (d) (e) (f)

Figure A.2



Odd vertices: A and C
Even vertex: B

(a)



Odd vertices: A and D
Even vertices: B, C, and E

(b)

Figure A.3

SOLUTION:

a) The figure can be traced if you start at any point. You will end at the point at which you started.

b) The figure can be traced, but only if you start at point A or point B. If you start at point A, you will end at point B, and vice versa.

c) The figure can be traced, but only if you start at point A or point B. If you start at point A, you will end at point B, and vice versa.

d) The figure cannot be traced without retracing a line.

e) The figure can be traced, but only if you start at point A or point B. If you start at point A, you will end at point B, and vice versa.

f) The figure can be traced if you start at any point. You will end at the point at which you started. ▲

In order for you to be able to answer the Königsberg bridge problem and understand why we were able to trace all but one of the figures in Example 1 without retracing a line, we must introduce some new terms and concepts.

A ***vertex*** is any designated point. An ***edge*** (or an ***arc***) is any line, either straight or curved, that begins and ends at a vertex. Figure A.3(a) has three designated vertices, A, B, and C, and two edges, 1 and 2. Figure A.3(b) has five designated vertices, A, B, C, D, and E, and six edges.

A vertex with an odd number of attached edges is called an ***odd vertex***. A vetex with an even number of attached edges is called an ***even vertex***. Figure A.3(a) has two odd vertices and one even vertex. Figure A.3(b) has two odd vertices and three even vertices.

A ***network*** is any continuous (not broken) system of edges and vertices.

A network is said to be **traversable** if it can be traced without removing the pencil from the paper and without tracing an edge more than once.

After completing Example 1, do you have an intuitive feeling as to when a figure is traversable? Think about odd and even vertices. Leonhard Euler discovered an important scientific principle concealed in the Königsberg bridge problem. He presented his simple and ingenious solution of that problem to the Russian Academy at St. Petersburg in 1735. Euler developed the following rules of traversability in solving the problem.

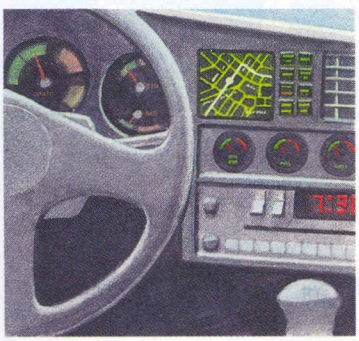
Rules of Traversability

1. A network with no odd (all even) vertices is traversable; you may start from any vertex, and you will end where you began.
2. A network with exactly two odd vertices is traversable; you must start at either of the odd vertices and you will finish at the other.
3. A network with more than two odd vertices is not traversable.

The network in the Königsberg bridge problem (Fig. A.1b) has four odd vertices, so it cannot be traversed. Therefore crossing each bridge only once is impossible. Note that it is impossible for a network to contain an odd number of odd vertices. (If you don't believe this statement, try to construct such a network.)

DID YOU KNOW

Networks



Today, graph (network) theory is a flourishing field, useful for dealing with a variety of everyday problems. It is used by the airline industry to arrange the complex network of flights connecting cities and by sports officials planning tournaments and playoff schedules. City planners use it to work out new roads and traffic patterns. Tests are underway in traffic-clogged cities such as Los Angeles to link cars, via an on-board navigational computer, to the city's central traffic-control computer. In this way motorists can be rerouted to less congested roadways.

Now go back to Example 1 and determine which figures are traversable, using these rules. Note that Figs. A.2(a) and A.2(f) are traversable from any point because they contain only even vertices. Figures A.2(b), (c), and (e) have exactly two odd vertices and can be traversed but only by starting at one of the odd vertices, either point *A* or point *B*. Figure A.2(d) contains more than two odd vertices and therefore cannot be traversed.

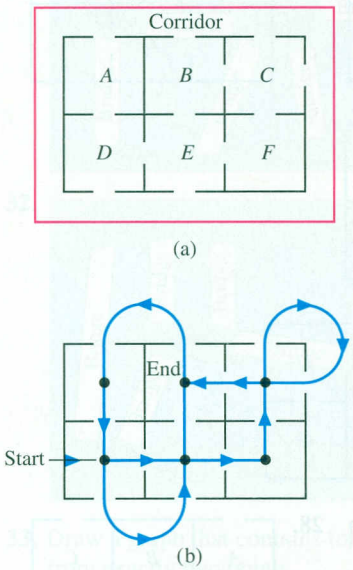


Figure A.4

EXAMPLE 2

The floor plan of a six-gallery art museum is shown in Fig. A.4(a). The openings represent doors, and the letters represent galleries.

a) Determine the galleries that contain an odd number of doors; an even number of doors.

b) Each gallery can be represented as an odd or even vertex. Use this information to determine whether it is possible to walk through each gallery by using each door only once.

c) Determine a path to walk through each gallery by using each door only once.

SOLUTION:

a) Galleries *B* and *D* contain three doors each. Galleries *A* and *F* contain two doors each. Galleries *C* and *E* have four doors.

b) There are only two odd vertices, *B* and *D*, so the figure is traversable, and you can walk through the museum by using each door only once.

c) You must start in either Gallery *B* or *D* (see Fig. A.4b). If you start in *B*, you will end in *D*, and vice versa.. When you leave Gallery *B*, you can leave by any of the three doors. ▲

The floor plan in Example 2 can be reduced to a map, where the rooms are the vertices and the doors are the paths. Construct a map of that floor plan now.

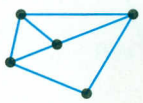
APPENDIX EXERCISES

Concept/Writing Exercises

1. What is a vertex?
2. What is an edge (or an arc)?
3. Explain how to determine whether a vertex is odd or even.
4. In your own words, explain the rules for determining whether a graph is traversable.

Practice the Skills

In Exercises 5–8, determine the number of vertices and the number of edges.

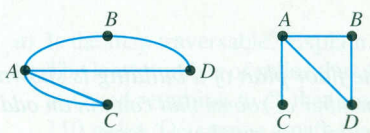
5. 

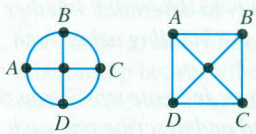
6.

7.

8.

In Exercises 9 and 10, explain why these two figures represent the same graph.

9. 

10. 

In Exercises 11 and 12, list the vertices that are odd and the vertices that are even.

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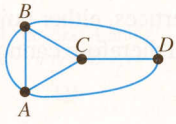

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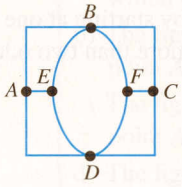

Problem Solving

In Exercises 13–20, determine whether the network is traversable. If it is, state the points from which you may start and end.

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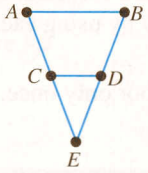

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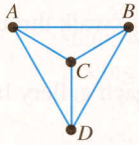

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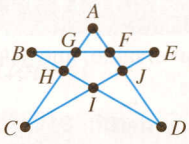

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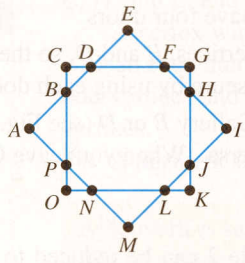

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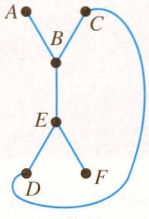

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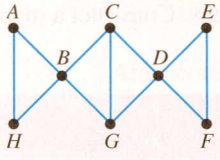

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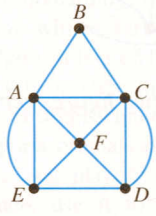

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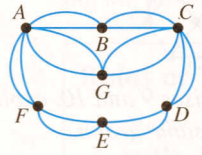
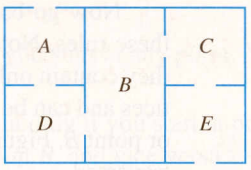


In Exercises 21–28, the floor plan of a building is shown.

- **a)** *Determine the number of rooms that contain an odd number of doors; an even number of doors.*
- **b)** *Use the rules of traversability to determine whether it is possible to walk through the building using each door only once.*
- **c)** *If the answer to part (b) is yes, indicate where you can start and where you will end and describe one such path (for example, A to D to B to . . . etc.).*

21.



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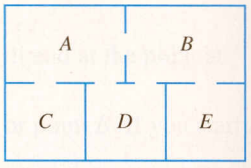

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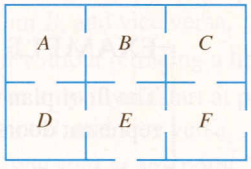

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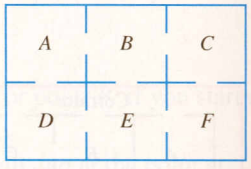

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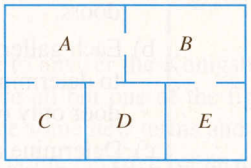

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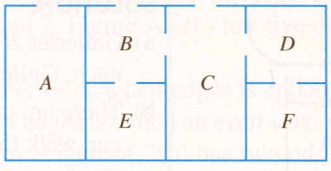

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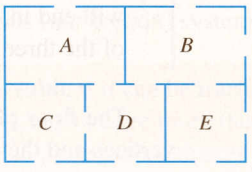

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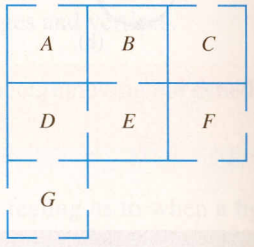
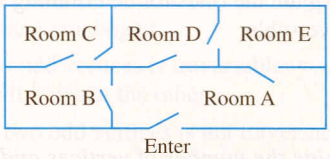


In Exercises 29 and 30, the floor plan for a suite of rooms is shown. Add an exit door in one of the rooms so that the security guard can enter through the door marked enter, pass through each door only once locking it behind him, and then exit by the door you added. Explain why there is only one possible room in which the door may be placed.

29.



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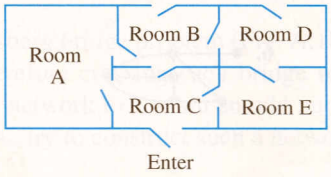

In Exercises 31 and 32, is it possible to cross each bridge exactly once? If it is possible, indicate where the person can start and where the person will finish. Explain your answer.

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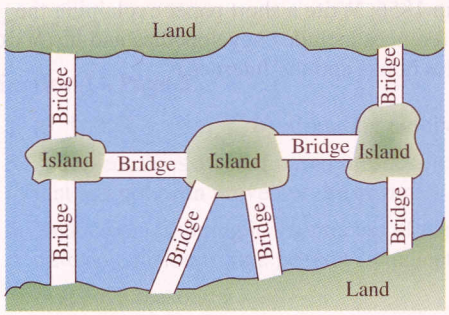

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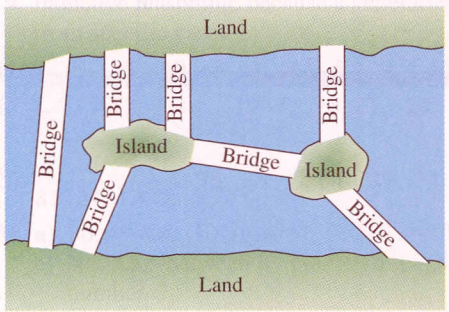
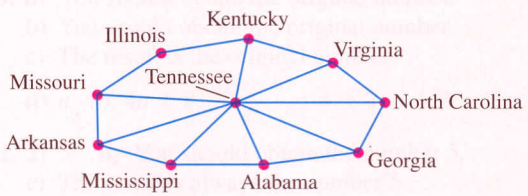


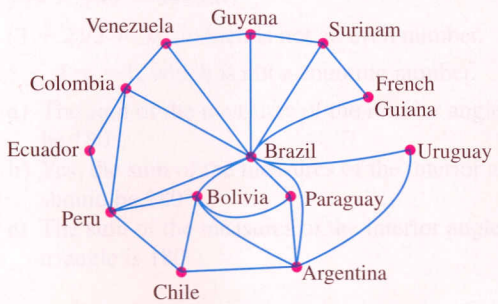
33. Draw a graph that contains four vertices and is traversable from exactly two points.

34. Draw a graph that contains five vertices that is traversable from exactly two points.

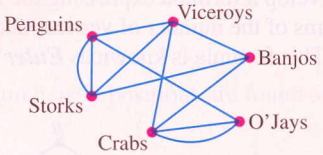
35. In the figure, lines connecting two states indicate that the two states share a common border. Which of these states share a common border with (a) Tennessee? (b) Missouri?



36. In the figure, a line connecting two countries indicates that they share a common border. Which of these countries share a common border with (a) Brazil? (b) Bolivia?



37. The remaining games in a soccer league are illustrated in the graph.



a) How many games do the Viceroys have remaining?
b) How many games do the Penguins have remaining?
c) How many games still need to be played?

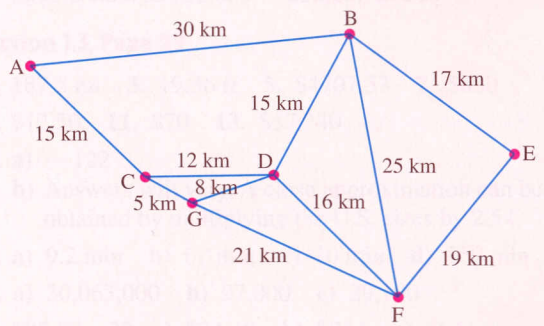
38. Dawn, Jessica, Pam, Bill, Ed, and Scott go to a dance. Dawn dances with Bill and Scott. Jessica dances with all the boys and Pam dances only with Scott. Draw a graph that displays this information.

39. France has common borders with Belgium, Germany, Switzerland, Italy, and Spain. Belgium has common borders with the Netherlands, France, and Germany. Germany has common borders with Belgium, Poland, the Czech Republic, Austria, Switzerland, and France. Switzerland has common borders with France, Italy, Austria, and Germany. Draw a graph that displays this information.

40. Can you draw a graph that contains an odd number of odd vertices? If you answer yes draw such a graph.

Challenge problems/Group Activities

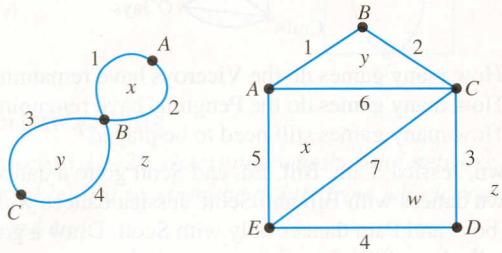
41. Gretchen's Delivery service, located in town C, makes deliveries every day to each town shown on the map. The map also shows the highways connecting the towns and the distances between towns. The delivery service's expenses are lowest when the driver's route is the shortest.



a) Is the map traversable? Explain.
b) The least number of miles that can be driven, starting at C and returning to C, that covers every city is 110 miles. Determine a path that covers all the cities and whose distance is 110 miles.

42. Networks (a) and (b) on page AA-6 illustrate a relationship between the number of edges, regions, and vertices. Network (a) has three vertices (A, B, and C), four edges (1, 2, 3, and 4), and three regions (x, y, and z). Network (b) has five vertices (A, B, C, D, and E), seven

edges (1, 2, 3, 4, 5, 6, 7), and four regions (w, x, y, and z). Using these networks and others that you can make up yourself, develop a formula expressing the number of edges in terms of the number of vertices and the number of regions. This formula is known as ***Euler's formula for networks***.



(a) (b)

Internet/Research Activity

43. In 1856 William Rowan Hamilton (1805–1865) introduced a problem similar to the Königsberg bridge problem. Hamilton turned his problem into a game that was marketed in 1856. Write a short paper explaining the game. (References include encyclopedias and history of mathematics books and the Internet.)

ANSWERS

Chapter 1

Section 1.1, Page 5

1. a) 1, 2, 3, 4, 5, . . . **b)** Counting numbers

3. A conjecture is a belief based on specific observations that has not been proven or disproven.

5. Deductive reasoning is the process of reasoning to a specific conclusion from a general statement.

7. Inductive reasoning

9. Inductive reasoning, because a generalization was made from specific cases.

11. 1 5 10 10 5 1 **13.** $5 \times 9 = 45$

15. **17.** **19.** 15, 18, 21 **21.** $-1, 1, -1$

23. $\frac{1}{81}, \frac{1}{243}, \frac{1}{729}$ **25.** 36, 49, 64 **27.** 34, 55, 89 **29.** Y

31. a) 36, 49, 64 **b)** square 6, 7, 8, 9 and 10
c) No, 72 is between 8^2 and 9^2, so it is not a square number.

33. Blue: 1, 5, 7, 10, 12 Purple: 2, 4, 6, 9, 11 Yellow: 3, 8

35. a) ≈58 million **b)** ≈45 million
c) We are using specific cases to make a prediction.

37.

39. a) You should obtain the original number.
b) You should obtain the original number.
c) The result is the original number.
d) $n, 4n, 4n + 8, \frac{4n + 8}{4} = n + 2, n + 2 - 2 = n$

41. a) 5 **b)** You should obtain the number 5.
c) The result is always the number 5.
d) $n, n + 1, \frac{n + (n + 1) + 9}{2} = \frac{2n + 10}{2} = n + 5,$
$n + 5 - n = 5$

43. $999 \times 999 = 998{,}001$

45. $(3 + 2)/2 = 5/2$, which is not an even number.

47. $1 - 2 = -1$, which is not a counting number.

49. a) The sum of the measures of the interior angles should be 180°.
b) Yes, the sum of the measures of the interior angles should be 180°.
c) The sum of the measures of the interior angles of a triangle is 180°.

51. 129, the numbers in positions are found as follows:
$a \quad b$
$c \quad a + b + c$

53. Counterexample **54.** (c)

Section 1.2, Page 14

Answers in this section will vary depending on how you round your numbers. All answers are approximate.

1. 1170 **3.** 1,200,000,000 **5.** 8000 **7.** 100

9. 364,000,000 **11.** 1,200,000,000 **13.** $20

15. 8500 mi **17.** $85.70 **19.** $9000 **21.** 45 lb

23. 2000 mi **25.** $840 **27.** $6 **29.** $41 **31.** ≈ 375 mi

33. a) 32.5 million **b)** 558 counties
c) Answers will vary.

35. a) 4 million **b)** 98 million **c)** 64 million
d) 275 million

37. a) 83% **b)** 20% **c)** 91,771 square miles
d) No, since we are not given the area of each state.

39. 25 **41.** ≈ 90 berries **43.** 150° **45.** 10%

47. 9 square units **49.** 150 feet

51.–59. Answers will vary. **60.** 118 ridges

61. There are 336 dimples on a regulation golf ball.

62. b) 11.6 days

63. Answers will vary. The U.S. government categorized the middle class as $32,000 − $50,000 in 2001.

Section 1.3, Page 29

1. 187.5 mi **3.** 19.36 ft **5.** $4707.53 **7.** $800

9. $12.50 **11.** $70 **13.** $57,240

15. a) ≈ 122
b) Answers will vary. A close approximation can be obtained by multiplying the U.S. sizes by 2.54.

17. a) 9.2 min **b)** 62 min **c)** 40 min **d)** 150 min

19. a) 30,063,000 **b)** 97,000 **c)** 29,100

21. $82.08 **23. a)** $74.40 **b)** $264 **c)** $64

25. $34,600 **27.** 13,906 violations

29. a) Answers will vary. **b)** ≈ 267.65 million
c) ≈ 1241.2 million or 1.2412 billion

31. $990, less than initial investment

33. a) 48 rolls **b)** $198 if she purchases four 10 packs and two 4 packs

35. a) Water/milk: 3 cups; salt: $\frac{3}{8}$ tsp; Cream of Wheat: 9 tbsp (or $\frac{9}{16}$ cup)

b) Water/milk: $2\frac{7}{8}$ cups; salt: $\frac{3}{8}$ tsp; Cream of Wheat: $\frac{5}{8}$ cup (or 10 tbsp)

c) Water/milk: $2\frac{3}{4}$ cups; salt: $\frac{3}{8}$ tsp; Cream of Wheat: $\frac{9}{16}$ cup (or 9 tbsp)

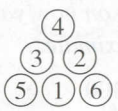
d) Differences exist in water/milk because the amount for 4 servings is not twice that for 2 servings. Differences also exist in Cream of Wheat because $\frac{1}{2}$ cup is not twice 3 tbsp.

37. 144 square inches **39.** The area is 4 times as large.

41. 1 and 9 **43.** at -1 **45.** 10 birds and 12 lizards

47. a) 30 **b)** 140

49.



51.

8	6	16
18	10	2
4	14	12

53. The sum of the four corners is 4 times the number in the center.

55. Multiply the center number by 9. **57.** 6 ways

59.

	7	
3	1	4
5	8	6
	2	

Other answers are possible, but 1 and 8 must appear in the center.

61.

1	2	3	4	5
2	3	4	5	1
3	4	5	1	2
4	5	1	2	3
5	1	2	3	4

Other answers are possible.

63. Mary is the skier. **65.** 714 square units

66. 3 ostriches

Review Exercises, Page 35

1. 23, 28, 33 **2.** 25, 36, 49 **3.** $-48, 96, -192$

4. 25, 32, 40 **5.** 15, 9, 2 **6.** $\frac{3}{8}, \frac{3}{16}, \frac{3}{32}$

7. **8.** **9.** (c)

10. a) The original number and the final number are the same.

b) The original number and the final number are the same.

c) The final number is the same as the original number.

d) $n, 2n, 2n + 10, \frac{2n + 10}{2} = n + 5, n + 5 - 5 = n$

11. This process will always result in an answer of 3.

12. $1^2 + 2^2 = 5$

The answers to Exercises 13-25 will vary, depending upon how you round the numbers. All answers are approximate.

13. 420,000,000 **14.** 2150 **15.** 200

16. Answers will vary. **17.** \$88 **18.** \$12

19. 3 mph **20.** \$14.00 **21.** 2 mi **22.** 70% **23.** 5%

24. 13 square units **25.** Length $\approx$ 22 ft; height $\approx$ 8 ft

26. \$7.50 **27.** \$1.16

28. Berkman's is cheaper by \$20.00.

29. \$32,996 **30.** \$16.20 **31.** \$311 **32.** 7.05 mg

33. \$744.80 **34.** 6 hr 45 min **35.** July 26, 11:00 A.M.

36. a) 6.45 cm^2

b) 16.39 cm^3

c) 1 cm $\approx$ 0.39 in.

37. 201

38.

21	7	8	18
10	16	15	13
14	12	11	17
9	19	20	6

39.

23	25	15
13	21	29
27	17	19

40. 59 min 59 sec **41.** 6

42.
\$25 Room
\$ 3 Men
\$ 2 Clerk
\$30

43. 140 lb

44. Yes; 3 quarters and 4 dimes, or 1 half dollar, 1 quarter and 4 dimes, or 1 quarter and 9 dimes. Other answers are possible.

45. 216 cm^3

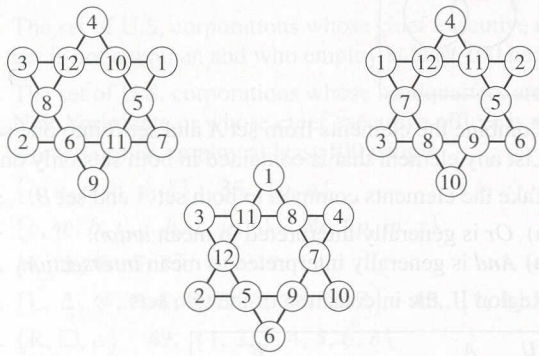
46. Place six coins in each pan with one coin off to the side. If it balances, the heavier coin is the one on the side. If the pan does not balance, take the six coins on the heavier side and split them into two groups of three. Select the three heavier coins and weigh two coins. If the pan balances, it is the third coin. If the pan does not balance, you can identify the heavier coin.

47. 125,250 **48.** 16 blue **49.** 90

50. The fifth figure will be an octagon with sides of equal length. Inside the octagon will be a seven sided figure with each side of equal length. The figure will have one antenna.

51. 61

52. Some possible answers are shown. Others are possible.



53. a) 2 **b)** 6 **c)** 24 **d)** 120
e) $n(n-1)(n-2)\ldots 1$, (or $n!$), where n = the number of people in line

Chapter Test, Page 38

1. 18, 21, 24

2. $\frac{1}{81}, \frac{1}{243}, \frac{1}{729}$

3. a) The result is the original number plus 1.
b) The result is the original number plus 1.
c) The result will always be the original number plus 1.
d) n, $5n$, $5n+10$, $\frac{5n+10}{5} = n+2$,
$n+2-1 = n+1$

The answers for Exercises 4–6 are approximate.

4. 6000 **5.** 33,000,000 **6.** 7 square units

7. a) ≈ 23.03
b) He is in the at risk range.

8. 159.25 therms **9.** 32 cans **10.** $7\frac{1}{2}$ min

11. $\approx$ 39.5 in. by 29.6 in. (The actual dimensions are 100.5 cm by 76.5 cm)

12. $49.00

13.

40	15	20
5	25	45
30	35	10

14. Less time if she had driven at 45 mph for the entire trip

15. $2 \cdot 6 \cdot 8 \cdot 9 \cdot 13$; 11 does not divide 11,232.

16. 243 jelly beans

17. a) $11.97
b) $11.81
c) Save 16 cents by using the 25% off coupon.

18. 8

Chapter 2

Section 2.1, Page 46

1. A set is a collection of objects.

3. Description, roster form, and set-builder notation; the set of counting numbers less than 7, $\{1, 2, 3, 4, 5, 6\}$, and $\{x \mid x \in N \text{ and } x < 7\}$

5. An infinite set is a set that is not finite.

7. Two sets are equivalent if they contain the same number of elements.

9. The empty set is a set that contains no elements.

11. Two sets that can be placed in a one-to-one correspondence have the same cardinal number and are equivalent.

13. Not well defined **15.** Well defined **17.** Well defined

19. Infinite **21.** Infinite **23.** Infinite

25. {Atlantic, Pacific, Arctic, Indian}

27. $\{11, 12, 13, 14, \ldots, 177\}$ **29.** $B = \{2, 4, 6, 8, \ldots\}$

31. $\{\ \}$ or $\emptyset$ **33.** $E = \{6, 7, 8, 9, \ldots, 71\}$

35. {Sony DSC-S50, Sony DSC-S70, Sony Mavica FD-90}

37. {Sony Mavica FD-73, Olympus D-360L, Sony DSC-S50, Kodak DC215, H-P Photo Smart C315}

39. {2002, 2003, 2004, 2005, 2006, 2007, 2008}

41. {2002, 2005, 2006, 2007, 2008}

43. $B = \{x \mid x \in N \text{ and } 3 < x < 11\}$ or
$B = \{x \mid x \in N \text{ and } 4 \le x \le 10\}$

45. $C = \{x \mid x \in N \text{ and } x \text{ is a multiple of } 3\}$

47. $E = \{x \mid x \in N \text{ and } x \text{ is odd}\}$

49. $C = \{x \mid x \text{ is February}\}$

51. Set A is the set of natural numbers less than or equal to 7.

53. Set V is the set of vowels in the English alphabet.

55. Set C is the set of companies that make calculators.

57. Set B is the set of members of the Beatles.

59. {St. Louis} **61.** $\{\ \}$ or $\emptyset$

63. {1999, 2000, 2001, 2002} **65.** {1999, 2001, 2002}

67. False; $\{b\}$ is a set, and not an element of the set.

69. False; h is not an element of the set.

71. False; 3 is an element of the set.

73. True **75.** 4 **77.** 0 **79.** Both **81.** Neither

83. Equivalent

85. a) Set A is the set of natural numbers greater than 2. Set B is the set of all numbers greater than 2.
b) Set A contains only natural numbers. Set B contains other types of numbers, including fractions and decimal numbers.
c) $A = \{3, 4, 5, 6, \ldots\}$

d) No; set B cannot be written in roster form since we cannot list all the elements in set B.

87. Cardinal **89.** Ordinal **91.** Answers will vary.

93. Answers will vary.

95.

	2		7		2			
7	H	6	6	D	1	1	C	6
	8		4		3			
	8		4		3			
4	F	7	7		5	5	E	4
	3		2		8			
	3		2		8			
6	A	5	5	G	3	3	B	7
	1		6		5			

Section 2.2 Page 54

1. Set A is a subset of set B, symbolized $A \subseteq B$, if and only if all the elements of set A are also elements of set B.

3. If $A \subseteq B$, then every element of set A is an element of set B. If $A \subset B$, then every element of set A is an element of set B and set $A \neq$ set B.

5. The number of proper subsets is determined by the formula $2^n - 1$, where n is the number of elements in the set.

7. False; gold is an element of the set, not a subset.

9. True **11.** True

13. False; the set $\{\emptyset\}$ contains the element $\emptyset$.

15. True **17.** False; the set $\{0\}$ contains the element 0.

19. False **21.** True

23. False; no set is a proper subset of itself.

25. $B \subseteq A, B \subset A$ **27.** $B \subseteq A, B \subset A$ **29.** $B \subseteq A, B \subset A$

31. $A = B, A \subseteq B, B \subseteq A$ **33.** $\{\ \}$

35. $\{\ \}, \{\text{pen}\}, \{\text{pencil}\}, \{\text{pen, pencil}\}$

37. a) $\{\ \}, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}$

b) $\{a, b, c, d\}$

39. False **41.** True **43.** True **45.** True **47.** True

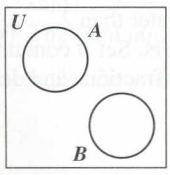
49. True **51.** 2^4 or 16 **53.** 2^6 or 64 **55.** $E = F$

57. a) Yes **b)** No **c)** Yes **59.** 1 **60.** Yes **61.** Yes

62. No

Section 2.3, Page 62

1.



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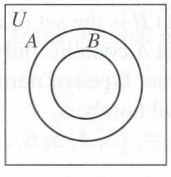
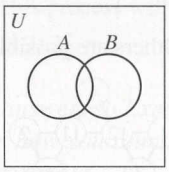


5.



7. Combine the elements from set A and set B into one set. List any element that is contained in both sets only once.

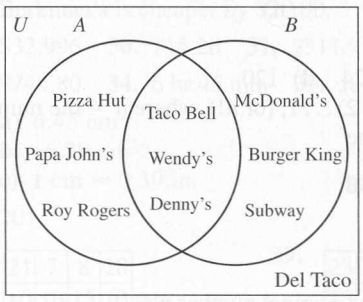
9. Take the elements common to both set A and set B.

11. a) *Or* is generally interpreted to mean *union*.

b) *And* is generally interpreted to mean *intersection*.

13. Region II, the intersection of the two sets.

15.



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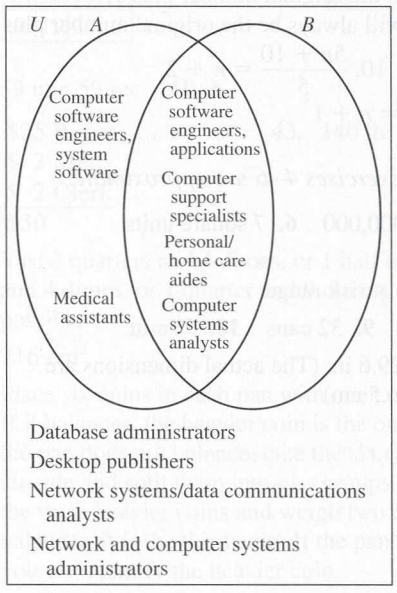


19. The set of U.S. colleges and universities that are not in the state of North Dakota

21. The set of insurance companies in the United States that do not offer life insurance

23. The set of insurance companies in the United States that offer life insurance or car insurance

25. The set of insurance companies in the United States that offer life insurance and do not offer car insurance

27. The set of U.S. corporations whose headquarters are in New York State and whose chief executive officer is a woman

29. The set of U.S. corporations whose chief executive officer is not a woman and who employ at least 100 people

31. The set of U.S. corporations whose headquarters are in New York State or whose chief executive officer is a woman or that employ at least 100 people

33. $\{b, c, t, w, a, h\}$ **35.** $\{a, h\}$

37. $\{c, w, b, t, a, h, f, g, r\}$ **39.** $\{p, m, z\}$

41. $\{L, \Delta, @, \$, *\}$

43. $\{L, \Delta, @, *, \$, R, \square, \alpha, \infty, \Sigma, Z\}$ **45.** $\{*, \$\}$

47. $\{R, \square, \alpha\}$ **49.** $\{1, 2, 3, 4, 5, 6, 8\}$

51. $\{1, 5, 7, 8\}$ **53.** $\{7\}$ **55.** $\{\ \}$ **57.** $\{7\}$

59. $\{a, e, h, i, j, k\}$ **61.** $\{a, f, i\}$

63. $\{b, c, d, e, g, h, j, k\}$ **65.** $\{a, c, d, e, f, g, h, i, j, k\}$

67. $\{a, b, c, d, e, f, g, h, i, j, k\}$, or U **69.** $\{\ \}$

71. $\{2, 4, 6, 8\}$, or B **73.** $\{7, 9\}$

75. $\{1, 3, 5, 6, 7, 8, 9\}$ **77.** $\{1, 2, 3, 4, 5, 7, 9\}$

79. $\{2, 4, 6, 8\}$, or B **81.** $\{1, 2, 3, 4, 5\}$, or C

83. A set and its complement will always be disjoint. For example, if $U = \{1, 2, 3\}$, $A = \{1, 2\}$, and $A' = \{3\}$, then $A \cap A' = \{\ \}$.

85. 49

87. a) $8 = 4 + 6 - 2$ **b)** and **c)** Answers will vary.

89. $\{1, 2, 3, 4, \ldots\}$, or A **91.** $\{4, 8, 12, 16, \ldots\}$, or B

93. $\{2, 4, 6, 8, \ldots\}$, or C **95.** $\{2, 6, 10, 14, 18, \ldots\}$

97. $\{2, 6, 10, 14, 18, \ldots\}$ **99.** U **101.** A **103.** U

105. U **107.** $B \subseteq A$ **109.** A and B are disjoint sets.

111. $A \subseteq B$ **113.** $\{e, f, h\}$ **115.** $\{d, j, k\}$ **117.** $\{13\}$

119. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$

121. $\{2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15\}$

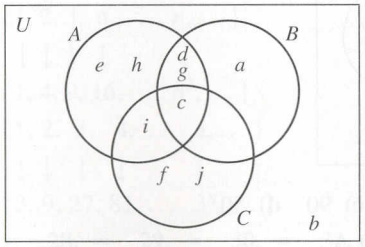
123. Complement

Section 2.4, Page 71

1. 8 **3.** II, IV, VI **5.** 8

7. a) Yes **b)** No, one specific case cannot be used as proof. **c)** No

9.



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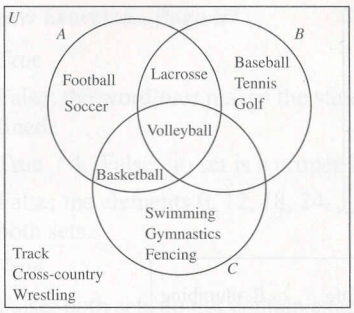

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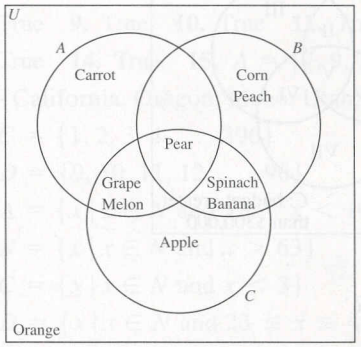

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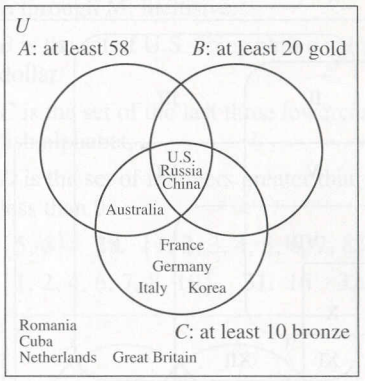


17. V **19.** VIII **21.** VI **23.** IV **25.** II **27.** VII

29. VI **31.** III **33.** III **35.** V **37.** II **39.** VII

41. I **43.** VIII **45.** VI **47.** $\{1, 2, 3, 4, 5, 6\}$

49. $\{3, 4, 5, 7, 8, 9, 12\}$ **51.** $\{3, 4, 5\}$

53. $\{1, 2, 3, 6, 9, 10, 11, 12\}$

55. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 12\}$ **57.** $\{9, 11, 12\}$

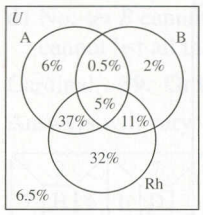
59. $\{7, 8, 9, 10, 11, 12\}$ **61.** Yes **63.** No **65.** No

67. Yes **69.** No **71.** Yes **73.** Yes **75.** Yes **77.** No

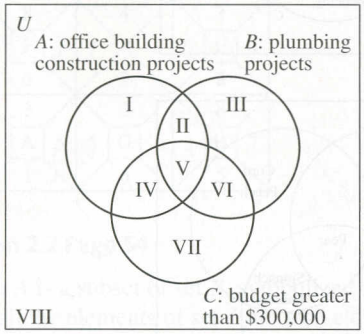
79. $(A \cup B)'$ **81.** $(A \cup B) \cap C'$

83. a) Both equal $\{6, 7\}$. **b)** Answers will vary. **c)** Both are represented by the regions IV, V, VI.

85.

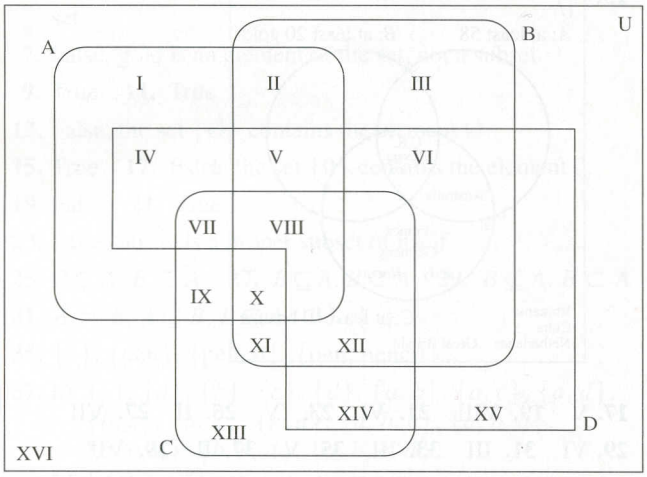


87. a)



b) V; $A \cap B \cap C$
c) VI; $A' \cap B \cap C$
d) I; $A \cap B' \cap C'$

89. a)

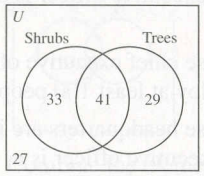


b)

Region	*Set*	*Region*	*Set*
I	$A \cap B' \cap C' \cap D'$	IX	$A \cap B' \cap C \cap D'$
II	$A \cap B \cap C' \cap D'$	X	$A \cap B \cap C \cap D'$
III	$A' \cap B \cap C' \cap D'$	XI	$A' \cap B \cap C \cap D'$
IV	$A \cap B' \cap C' \cap D$	XII	$A' \cap B \cap C \cap D$
V	$A \cap B \cap C' \cap D$	XIII	$A' \cap B' \cap C \cap D'$
VI	$A' \cap B \cap C' \cap D$	XIV	$A' \cap B' \cap C \cap D$
VII	$A \cap B' \cap C \cap D$	XV	$A' \cap B' \cap C' \cap D$
VIII	$A \cap B \cap C \cap D$	XVI	$A' \cap B' \cap C' \cap D'$

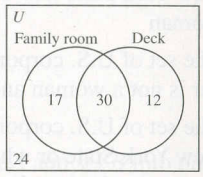
Section 2.5, Page 80

1.



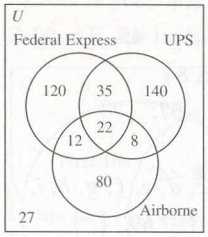
a) 33
b) 29
c) 27

3.



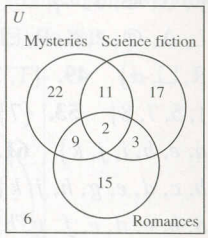
a) 17
b) 12
c) 59

5.



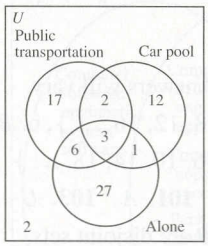
a) 27
b) 80
c) 340
d) 55
e) 337

7.



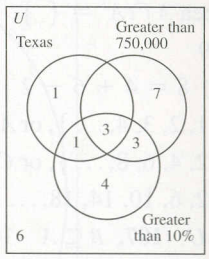
a) 22
b) 11
c) 64
d) 50
e) 23

9.



a) 17
b) 27
c) 2
d) 31
e) 2

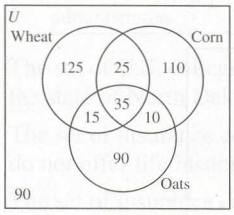
11.



a) 10
b) 15
c) 0
d) 6

13. In a Venn diagram, regions II, IV, and V contain a total of 37 cars driven by women. This total is greater than the 35 cars driven by women, as given in the exercise.

15.



a) 410 **b)** 35 **c)** 90 **d)** 50

16. a) 10 **b)** 10 **c)** 6

Section 2.6, Page 86

1. An infinite set is a set that can be placed in a one-to-one correspondence with a proper subset of itself.

3. $\{7, 8, 9, 10, 11, \ldots, n + 6, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{8, 9, 10, 11, 12, \ldots, n + 7, \ldots\}$

5. $\{3, 5, 7, 9, 11, \ldots, 2n + 1, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{5, 7, 9, 11, 13, \ldots, 2n + 3, \ldots\}$

7. $\{4, 7, 10, 13, \ldots, 3n + 1, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{7, 10, 13, 16, \ldots, 3n + 4, \ldots\}$

9. $\{6, 11, 16, 21, 26, \ldots, 5n + 1, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{11, 16, 21, 26, 31, \ldots, 5n + 6, \ldots\}$

11. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots, \frac{1}{2n}, \ldots\right\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n + 2}, \ldots\right\}$

13. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{6, 12, 18, 24, \ldots, 6n, \ldots\}$

15. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{4, 6, 8, 10, \ldots, 2n + 2, \ldots\}$

17. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{2, 5, 8, 11, \ldots, 3n - 1, \ldots\}$

19. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{5, 8, 11, 14, \ldots, 3n + 2, \ldots\}$

21. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n + 2}, \ldots\right\}$

23. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{1, 4, 9, 16, \ldots, n^2, \ldots\}$

25. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \quad \downarrow$
$\{3, 9, 27, 81, \ldots, 3^n, \ldots\}$

27. = **28.** = **29.** = **30.** = **31.** =

Review Exercises, Page 87

1. True

2. False; the word *best* makes the statement not well defined.

3. True **4.** False; no set is a proper subset of itself.

5. False; the elements 6, 12, 18, 24, ... are members of both sets.

6. True

7. False; both sets do not contain exactly the same elements.

8. True **9.** True **10.** True **11.** True **12.** True

13. True **14.** True **15.** $A = \{7, 9, 11, 13, 15\}$

16. {California, Oregon, Idaho, Utah, Arizona}

17. $C = \{1, 2, 3, 4, \ldots, 296\}$

18. $D = \{9, 10, 11, 12, \ldots, 96\}$

19. $A = \{x \mid x \in N \text{ and } 52 < x < 100\}$

20. $B = \{x \mid x \in N \text{ and } x > 63\}$

21. $C = \{x \mid x \in N \text{ and } x < 3\}$

22. $D = \{x \mid x \in N \text{ and } 23 \leq x \leq 41\}$

23. A is the set of capital letters in the English alphabet from E through M, inclusive.

24. B is the set of U.S. coins with a value of less than a dollar.

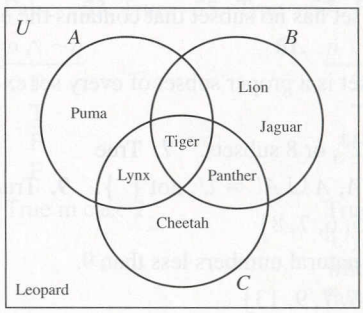
25. C is the set of the last three lowercase letters in the English alphabet.

26. D is the set of numbers greater than or equal to 3 and less than 9.

27. $\{5, 6\}$ **28.** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **29.** $\{9, 10\}$

30. $\{1, 2, 4, 6, 7, 8, 10\}$ **31.** 16 **32.** 15

33.



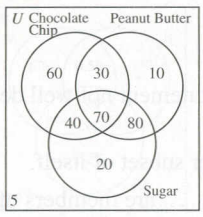
34. {b, e, g, k, c, d, f, a} **35.** {b, d}

36. {b, e, g, k, c, d, f, a, i} **37.** {f} **38.** {d, f, a}

39. {e, g, f, d, a, i} **40.** True **41.** True **42.** II

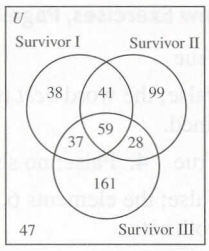
43. V **44.** VIII **45.** IV **46.** IV **47.** VII **48.** $450

49.



a) 315
b) 10
c) 30
d) 110

50.



a) 38
b) 298
c) 28
d) 236
e) 106

51. $\{2, 4, 6, 8, \ldots, 2n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{4, 6, 8, 10, \ldots, 2n + 2, \ldots\}$

52. $\{3, 5, 7, 9, \ldots, 2n + 1, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{5, 7, 9, 11, \ldots, 2n + 3, \ldots\}$

53. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{5, 8, 11, 14, \ldots, 3n + 2, \ldots\}$

54. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{4, 9, 14, 19, \ldots, 5n - 1, \ldots\}$

Chapter Test, Page 90

1. True

2. False; the sets do not contain exactly the same elements.

3. True

4. False; the second set has no subset that contains the element 7.

5. False; the empty set is a proper subset of every set except itself.

6. False; the set has 2^3, or 8 subsets. **7.** True

8. False; for any set A, $A \cup A' = U$, not $\{\ \}$. **9.** True

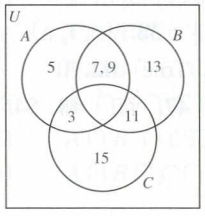
10. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

11. Set A is the set of natural numbers less than 9.

12. $\{7, 9\}$ **13.** $\{3, 5, 7, 9, 13\}$

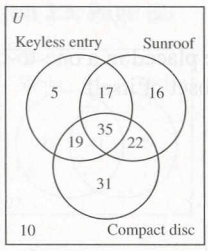
14. $\{3, 5, 7, 9\}$, or A **15.** 2

16.



17. Equal

18.



a) 52 **b)** 10 **c)** 93 **d)** 17 **e)** 38 **f)** 31

19. $\{7, 8, 9, 10, \ldots, n + 6, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{8, 9, 10, 11, \ldots, n + 7, \ldots\}$

20. $\{1, 2, 3, 4, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{1, 3, 5, 7, \ldots, 2n - 1, \ldots\}$

Chapter 3

Section 3.1, Page 103

1. a) A simple statement is a statement that conveys only one idea.
b) Compound statements are statements consisting of two or more simple statements.

3. a) Some are **b)** All are
c) Some are not **d)** None are

5. a) $\rightarrow$ **b)** $\vee$ **c)** $\wedge$ **d)** $\sim$ **e)** $\leftrightarrow$

7. When two simple statements are on the same side of the comma, they are placed together in parentheses when translated into symbolic form.

9. Compound; conjunction, $\wedge$

11. Compound; biconditional, $\leftrightarrow$

13. Compound; disjunction, $\vee$

15. Simple statement **17.** Compound; negation, $\sim$

19. Compound; conjunction, $\wedge$

21. Compound; negation, $\sim$

23. No picnic tables are portable.

25. Some chickens do not fly. **27.** All turtles have claws.

29. Some bicycles have three wheels.

31. All pine trees produce pinecones.

33. No pedestrians are in the crosswalk.

35. $\sim p$ **37.** $\sim q \vee \sim p$ **39.** $\sim p \rightarrow \sim q$ **41.** $\sim q \rightarrow \sim p$

43. $\sim p \wedge \sim q$ **45.** $\sim(q \rightarrow \sim p)$

47. Firemen do not work hard.

49. Firemen wear red suspenders or firemen work hard.

51. Firemen do not work hard if and only if firemen do not wear red suspenders.

53. It is false that firemen wear red suspenders or firemen work hard.

55. Firemen do not work hard and firemen do not wear red suspenders.

57. $(p \vee \sim q) \rightarrow r$ **59.** $(p \wedge q) \vee r$ **61.** $p \rightarrow (q \vee \sim r)$

63. $(r \leftrightarrow q) \wedge p$ **65.** $q \rightarrow (p \leftrightarrow r)$

67. The water is 70° or the sun is shining, and we do not go swimming.

69. The water is not 70°, and the sun is shining or we go swimming.

71. If we do not go swimming, then the sun is shining and the water is 70°.

73. If the sun is shining then we go swimming, and the water is 70°.

75. The sun is shining if and only if the water is 70°, and we go swimming.

77. Not permissible, you cannot have both soup and salad. The *or* used on menus is the exclusive *or*.

79. Not permissible, you cannot have both potatoes and pasta. The *or* used on menus is the exclusive *or*.

81. a) $(\sim p) \rightarrow q$ **b)** Conditional

83. a) $(\sim q) \wedge (\sim r)$ **b)** Conjunction

85. a) $(p \vee q) \rightarrow r$ **b)** Conditional

87. a) $r \rightarrow (p \vee q)$ **b)** Conditional

89. a) $(\sim p) \leftrightarrow (\sim q \rightarrow r)$ **b)** Biconditional

91. a) $(r \wedge \sim q) \rightarrow (q \wedge \sim p)$ **b)** Conditional

93. a) $\sim[(p \wedge q) \leftrightarrow (p \vee r)]$ **b)** Negation

95. a) $r \wedge \sim c$ **b)** Conjunction

97. a) $\sim(b \rightarrow \sim p)$ **b)** Negation

99. a) $(f \vee v) \rightarrow h$ **b)** Conditional

101. a) $c \leftrightarrow (\sim f \vee p)$ **b)** Biconditional

103. a) $(c \leftrightarrow w) \vee s$ **b)** Disjunction

105. $[(\sim q) \rightarrow (r \vee p)] \leftrightarrow [(\sim r) \wedge q]$; Biconditional

107. a) The conjunction and disjunction have the same dominance.

b) Answers will vary. **c)** Answers will vary.

Section 3.2, Page 115

1. a) 4 **b)**

p	q
T	T
T	F
F	T
F	F

3. a)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

b) Only when both p and q are false

5. T T **7.** F T F **9.** F F T F **11.** T F T T **13.** T F T T T **15.** T F T T T T **17.** F F T T F F **19.** T T T T F

21.

$p \wedge q$
T
F
F
F

23.

$p \wedge \sim q$
F
T
F
F

25.

$\sim(p \wedge q)$
F
T
T
T

27.

$p \vee (q \vee r)$
T
T
T
T
T
T
T
F

29.

$p \wedge (q \vee \sim q)$
T
T
F
F

31. a) False **b)** True **33. a)** False **b)** False

35. a) True **b)** True **37. a)** False **b)** True

39. a) True **b)** True **41. a)** True **b)** True

43. True **45.** True **47.** False **49.** False

51. False **53.** True **55.** True **57.** False

59.

$p \wedge \sim q$
F
T
F
F

True in case 2

61.

$p \vee \sim q$
T
T
F
T

True in cases 1, 2, and 4, when p is true, or when p and q are both false.

63.

$(r \vee q) \wedge p$
T
T
T
F
F
F
F
F

True in cases 1, 2, and 3

65.

$q \vee (p \wedge \sim r)$
T
T
F
T
T
T
F
F

True in cases 1, 2, 4, 5, and 6. True except when p, q, r have truth values TFT, FFT, or FFF.

67. a) Mr. Duncan and Mrs. Tuttle qualify.
b) Mrs. Rusinek does not qualify, since their combined income is less than \$46,000.

69. a) Wing Park qualifies; the other four do not.
b) Gina Vela is returning on April 2. Kara Sharo is returning on a Monday. Christos Supernaw is not staying over on a Saturday. Alex Chang is returning on a Monday.

71. T, T, T, T, F, T, F, T

73. Yes

Section 3.3, Page 125

1. a)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

b) The conditional is false only when the antecedent is true and the consequent is false.

3. a) Substitute the truth values for the simple statements. Then evaluate the compound statement, using the assigned truth values.
b) True

5. A self-contradiction is a compound statement that is never true.

7. T, F, T, T **9.** F, F, T, F **11.** F, T, T, F **13.** T, T, F, T **15.** F, T, T, T **17.** T, F, T, F, T, F, F, F **19.** F, T, F, F, F, F, F, T **21.** F, F, T, F, T, T, T, F

23. T, T, T, T, T, F, F, F **25.** T, T, T, F, T, T, F, T

27.

$p \rightarrow (q \wedge r)$
T
F
F
F
T
T
T
T

29.

$(p \leftrightarrow \sim q) \vee r$
T
F
T
T
T
T
T
F

31.

$(\sim p \rightarrow q) \vee r$
T
T
T
T
T
T
T
F

33. Neither **35.** Self-contradiction **37.** Tautology

39. Not an implication **41.** Implication **43.** Implication

45. True **47.** True **49.** False **51.** True **53.** True

55. True **57.** True **59.** False **61.** True **63.** True

65. True **67.** False **69.** False **71.** True **73.** True

75. True

77. No, the statement only states what will occur if your sister gets straight A's. If your sister does not get straight A's, your parents may still get her a computer.

79. T, F, F, T, T, F, F, F

81. It is a tautology. The statement may be expressed as $(p \rightarrow q) \vee (\sim p \rightarrow q)$, where p: It is a head and q: I win. This statement is a tautology.

83.

Tiger	Boots	Sam	Sue
Blue	Yellow	Red	Green
Nine Lives	Whiskas	Friskies	Meow Mix

84. Katie was born last. Katie and Mary are saying the same thing.

Section 3.4, Page 139

1. Statements that have exactly the same truth values

3. The two statements are equivalent.

5. a) $q \rightarrow p$ **b)** $\sim p \rightarrow \sim q$ **c)** $\sim q \rightarrow \sim p$ **7.** $\sim p \vee q$

9. Equivalent **11.** Not equivalent **13.** Equivalent

15. Equivalent **17.** Equivalent **19.** Equivalent

21. Equivalent **23.** Equivalent **25.** Not equivalent

27. Not equivalent **29.** Equivalent

31. The Mississippi River does not run through Ohio and the Ohio River does not run through Mississippi.

33. It is false that the snowmobile was an Arctic Cat or a Ski-Do.

35. It is false that the hotel has a weight room and the conference center has an auditorium.

37. If we go to Cozumel, then it is false that we will not go snorkeling and we will go to Senior Frogs.

39. You do not drink a glass of orange juice or you will get a full day supply of folic acid.

41. If Bob the Tomato didn't visit the nursing home then he did not visit the Cub Scout meeting.

43. The plumbers meet in Kansas City and the Rainmakers will not provide the entertainment.

45. It is cloudy if and only if the front is coming through.

47. If the chemistry teacher teaches mathematics then there is a shortage of mathematics teachers and if there is a shortage of mathematics teachers then the chemistry teacher teaches mathematics.

49. *Converse:* If I finish the book in one week, then the book is interesting.
Inverse: If the book is not interesting, then I will not finish the book in one week.
Contrapositive: If I do not finish the book in one week, then the book is not interesting.

51. *Converse:* If you can watch television, then you finish your homework.
Inverse: If you did not finish your homework, then you cannot watch television.
Contrapositive: If you cannot watch television, then you did not finish your homework.

53. *Converse:* If I scream, then that annoying paper clip shows up on my computer screen.
Inverse: If that annoying paper clip does not show up on my computer screen, then I will not scream.
Contrapositive: If I do not scream, then that annoying paper clip does not show up on my screen.

55. *Converse:* If we go down to the marina and take out the sailboat, then the sun is shining.
Inverse: If the sun is not shining, then we will not go down to the marina or we will not take out the sailboat.
Contrapositive: If we do not go down to the marina or we will not take out the sailboat, then the sun is not shining.

57. If a natural number is divisible by 10, then the natural number is divisible by 5. True.

59. If a natural number is not divisible by 6, then the natural number is not divisible by 3. False.

61. If two lines are not parallel, then the two lines intersect in at least one point. True.

63. If the polygon is a quadrilateral, then the sum of the interior angles of the polygon measures 360°. True.

65. **b)** and **c)** are equivalent. **67.** **a)** and **c)** are equivalent.

69. **b)** and **c)** are equivalent. **71.** **b)** and **c)** are equivalent.

73. None are equivalent. **75.** None are equivalent.

77. **a)** and **c)** are equivalent. **79.** **a)** and **b)** are equivalent.

81. True. If $p \rightarrow q$ is false, it must be of the form T $\rightarrow$ F. Therefore, the converse must be of the form F $\rightarrow$ T, which is true.

83. False. A conditional statement and its contrapositive always have the same truth values.

85. Answers will vary. **87.** Answers will vary.

89. **a)** Conditional **b)** Biconditional **c)** Inverse **d)** Converse **e)** Contrapositive

Section 3.5, Page 150

1. The conclusion necessarily follows from the given set of premises.

3. Yes, if the conclusion necessarily follows from the set of premises, the argument is valid, even if the conclusion is false.

5. Yes, if the conclusion necessarily follows from the set of premises, the argument is valid, even if the premises themselves are false.

7. a)
$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ \therefore\ q \end{array}$$

b) Answers will vary.

9. a)
$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore\ p \rightarrow r \end{array}$$

b) Answers will vary.

11. a)
$$\begin{array}{l} p \rightarrow q \\ \underline{\sim p} \\ \therefore\ \sim q \end{array}$$

b) Answers will vary.

13. Valid **15.** Valid **17.** Invalid **19.** Invalid

21. Invalid **23.** Valid **25.** Valid **27.** Invalid

29. Invalid **31.** Valid

33. a) $p \to q$
$\underline{\sim p}$
$\therefore \sim q$
b) Invalid

35. a) $p \to q$
$\underline{p}$
$\therefore q$
b) Valid

37. a) $p \to q$
$\underline{\sim q}$
$\therefore \sim p$
b) Valid

39. a) $p \to q$
$\underline{q}$
$\therefore p$
b) Invalid

41. a) $p \vee q$
$\underline{\sim p}$
$\therefore q$
b) Valid

43. a) $p \to q$
$\underline{q \to r}$
$\therefore p \to r$
b) Valid

45. a) $f \to w$
$\underline{w \wedge \sim t}$
$\therefore w \to \sim f$
b) Invalid

47. a) $s \wedge g$
$\underline{g \to c}$
$\therefore s \to c$
b) Valid

49. a) $h \to b$
$\underline{\sim p \to \sim b}$
$\therefore h \to p$
b) Valid

51. a) $p \to q$
$\underline{\sim q}$
$\therefore \sim p$
b) Valid

53. a) $t \vee \sim p$
$\underline{p}$
$\therefore t$
b) Valid

55. a) $t \wedge g$
$\underline{\sim t \vee \sim g}$
$\therefore \sim t$
b) Valid

57. a) $c \wedge \sim h$
$\underline{h \to c}$
$\therefore h$
b) Invalid

59. a) $f \to d$
$\underline{d \to \sim s}$
$\therefore f \to s$
b) Invalid

61. Therefore, your face will break out.

63. Therefore, a tick is an arachnid.

65. Therefore, you did not close the deal.

67. Therefore, if you do not pay off your credit card bills, then the bank makes money.

69. No. The conditional statement will always be true, and therefore it will be a tautology, and a valid argument.

Section 3.6, Page 158

1. It is an invalid argument.

3. The conclusion necessarily follows from the premises.

5. Yes, if the conclusion necessarily follows from the premises, the argument is valid.

7. Valid **9.** Valid **11.** Invalid **13.** Valid

15. Invalid **17.** Invalid **19.** Invalid **21.** Valid

23. Invalid **25.** Invalid **27.** Valid **29.** Invalid

Review Exercises, Page 161

1. No rock bands play ballads.

2. All bananas are ripe.

3. Some chickens have lips.

4. Some panthers are not endangered.

5. Some pens do not use ink.

6. Some rabbits wear glasses.

7. The coffee is Maxwell House or the coffee is hot.

8. The coffee is not hot and the coffee is strong.

9. If the coffee is hot, then the coffee is strong and the coffee is not Maxwell House.

10. The coffee is Maxwell House if and only if the coffee is not strong.

11. The coffee is not Maxwell House, if and only if the coffee is strong and the coffee is not hot.

12. The coffee is Maxwell House or the coffee is not hot, and the coffee is not strong.

13. $r \wedge q$ **14.** $p \to r$ **15.** $(r \to q) \vee \sim p$

16. $(q \leftrightarrow p) \wedge \sim r$ **17.** $(r \wedge q) \vee \sim p$ **18.** $\sim(r \wedge q)$

19. **20.** **21.** **22.** **23.** **24.**

F	T	T	T	F	F
F	F	T	F	T	T
T	F	T	T	F	T
F	F	T	T	F	T
		T	F	T	T
		F	F	T	T
		F	F	T	T
		T	F	T	T

25. False **26.** True **27.** False **28.** True **29.** True

30. True **31.** True **32.** True **33.** False **34.** False

35. Not equivalent **36.** Equivalent **37.** Equivalent

38. Not equivalent

39. It is false that Johnny Cash is not in the Rock and Roll Hall of Fame or India Arie did not record *Acoustic Soul*.

40. If her foot did not fall asleep then she injured her ankle.

41. Altec Lansing does not produce only speakers and Harman Kardon does not produce only stereo receivers.

42. It is false that Travis Tritt won an Academy Award or Randy Jackson does commercials for Milk Bone Dog Biscuits.

43. The temperature is above 32° or we will go ice fishing at O'Leary's Lake.

44. a) If you enjoy life, then you hear a beautiful songbird today.
b) If you do not hear a beautiful songbird today, then you do not enjoy life.
c) If you do not enjoy life, then you do not hear a beautiful songbird today.

45. a) If the quilt has a uniform design, then you followed the correct pattern.
b) If you did not follow the correct pattern, then the quilt does not have a uniform design.

c) If the quilt does not have a uniform design, then you did not follow the correct pattern.

46. a) If Maureen Gerald is helping at the school, then she is not in attendance.
b) If Maureen Gerald is in attendance, then she is not helping at the school.
c) If Maureen Gerald is not helping at the school, then she is in attendance.

47. a) If we will not buy a desk at Miller's Furniture, then the desk is made by Winner's Only and the desk is in the Rose catalog.
b) If the desk is not made by Winner's Only or the desk is not in the Rose catalog, then we will buy a desk at Miller's Furniture.
c) If we will buy a desk at Miller's Furniture, then the desk is not made by Winner's Only or the desk is not in the Rose catalog.

48. a) If I let you attend the prom, then you will get straight A's on your report card.
b) If you do not get straight A's on your report card, then I will not let you attend the prom.
c) If I will not let you attend the prom, then you did not get straight A's on your report card.

49. a), b), and **c)** are equivalent. **50.** None are equivalent.
51. a) and **c)** are equivalent. **52.** None are equivalent.
53. Invalid **54.** Valid **55.** Invalid **56.** Valid
57. Invalid **58.** Invalid

Chapter Test, Page 163

1. $(p \wedge r) \vee \sim q$ **2.** $(r \rightarrow q) \vee \sim p$ **3.** $\sim(r \leftrightarrow \sim q)$

4. Ann is the secretary, if and only if Dick is the vice president and Elaine is the president.

5. It is false that if Ann is the secretary, then Elaine is not the president.

6. F T F F F F F F

7. T T T T F T T F

8. True **9.** True **10.** True **11.** True **12.** Equivalent

13. a) and **b)** are equivalent. **14. a)** and **b)** are equivalent.

15. $s \rightarrow f$
$f \rightarrow p$
$\therefore s \rightarrow p$
Valid

16. Invalid

17. Some leopards are not spotted.

18. No jacks-in-the-box are electronic.

19. *Converse:* If today is Saturday, then the garbage truck comes.
Inverse: If the garbage truck does not come, then today is not Saturday.
Contrapositive: If today is not Saturday, then the garbage truck does not come.

20. Yes

Chapter 4

Section 4.1, Page 173

1. A number is a quantity, and it answers the question "How many?" A numeral is a symbol used to represent the number.

3. A system of numeration consists of a set of numerals and a scheme or rule for combining the numerals to represent numbers.

5. The Hindu–Arabic numeration system

7. In a multiplicative system, there are numerals for each number less than the base and for powers of the base. Each numeral less than the base is multiplied by a numeral for the power of the base, and these products are added to obtain the number.

9. 142 **11.** 2423 **13.** 334,214

15. 999999∩∩∩||||| **17.** 𝄞𝄞∩∩∩∩|||||

19. ⋈𝄐𝄐𝄐𝄐𝄐𝄞𝄞𝄞99999999∩∩∩∩|||||

21. 19 **23.** 547 **25.** 1492 **27.** 2946 **29.** 12,666

31. 9464 **33.** LIX **35.** CXXXIV **37.** MMV

39. $\overline{\text{IV}}$DCCXCIII **41.** $\overline{\text{IX}}$CMXCIX **43.** $\overline{\text{XX}}$DCXLIV

45. 74 **47.** 4081 **49.** 8550 **51.** 4003

53. 五十三 **55.** 三百七十八 **57.** 四千二百六十 **59.** 七千零五十六

61. 341 **63.** 22,505 **65.** 9607 **67.** $\nu\theta$ **69.** $\psi\kappa\zeta$

71. $\pi'\beta'\psi\delta$

73. Advantage: You can write some numbers more compactly.
Disadvantage: There are more numerals to memorize.

75. Advantage: You can write some numbers more compactly.
Disadvantage: There are more numerals to memorize.

77. 1936, 𝄞999999999∩∩∩|||||, α′ ⊓ λ ζ, 一千九百三十六

79. 422, 9999∩∩II, CDXXII, 四百二十二

81. ⊓′Q′θ′ ⊓Qθ **83.** Turn the book upside down.

84. MM **85.** 1888, MDCCCLXXXVIII

Section 4.2, Page 180

1. A base 10 place-value system **3.** Four tens, four hundreds

5. A symbol for zero and for each counting number less than the base are required.

7. Write each digit times its corresponding positional value.

9. a) There may be confusion because numbers could be interpreted in different ways. For example, 𒁹 could be interpreted to be either 1 or 60.

b) 𒁹𒁹 𒌋𒁹𒁹𒁹 for both numbers.

11. 1, 20, 18 × 20, 18 × $(20)^2$, 18 × $(20)^3$

13. (6 × 10) + (3 × 1)

15. (3 × 100) + (5 × 10) + (9 × 1)

17. (8 × 100) + (9 × 10) + (7 × 1)

19. (4 × 1000) + (3 × 100) + (8 × 10) + (7 × 1)

21. (1 × 10,000) + (6 × 1000) + (4 × 100) + (0 × 10) + (2 × 1)

23. (3 × 100,000) + (4 × 10,000) + (6 × 1000) + (8 × 100) + (6 × 10) + (1 × 1)

25. 42 **27.** 784 **29.** 4868 **31.** 𒁹 𒌋𒌋𒌋𒁹𒁹𒁹

33. 𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹 **35.** 𒁹 𒁹 𒌋𒌋𒁹𒁹𒁹𒁹𒁹

37. 92 **39.** 4321 **41.** 4000 **43.** ⁝⁝ **45.** ⁝⁝⁝⁝ ⁝⁝

47. ⁝ ⊖ ⁝⁝⁝

49. Advantages: In general, they are more compact; large and small numbers can be written more easily; there are fewer symbols to memorize.

Disadvantage: If many of the symbols in the numeral represent zero, the place-value system may be less compact.

51. 33, ⁝⁝⁝

53. (△ × ⊡^{2}) + (□ × ⊡) + (⬡ × 1)

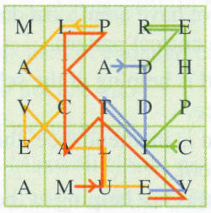
55. a) No largest number

b) 𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹

57. 𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹

59. ⁝⁝⁝ ⁝⁝⁝⁝ ⁝⁝⁝

61.



Section 4.3, Page 187

1. Answers will vary. **3.** 5 **5.** 22 **7.** 11 **9.** 100

11. 373 **13.** 1367 **15.** 867 **17.** 83 **19.** 6597

21. 1000_2 **23.** 10111_2 **25.** 2535_6 **27.** 1239_{12}

29. 1021_8 **31.** $17TE_{12}$ **33.** 1111110011_2 **35.** 4403_8

37. 1845 **39.** 447,415 **41.** $23D_{16}$ **43.** 1566_{16}

45. 11111010101_2 **47.** 31010_5 **49.** $11E1_{12}$

51. Incorrect; cannot have a 5 in base 5.

53. Written correctly

55. Incorrect; cannot have an 8 in base 7. **57.** 13

59. 73 **61.** ⊖⊙$_5$ **63.** ⊙⊙⊙$_5$ **65.** 7

67. 36 **69.** ●●$_4$ **71.** ●●●$_4$

73. a) Answers will vary. **b)** 10213_5 **c)** 1373_8

75. Answers will vary. **77.** $b = 6$ **78.** $d = 4$

79. a) 876 **b)** ●●●●$_4$

Section 4.4, Page 197

1. a) 1, b, b^2, b^3, b^4 **b)** 1, 6, 6^2, 6^3, 6^4

3. No, cannot have a 6 in base 5. **5.** Answers will vary.

7. 134_5 **9.** 3201_4 **11.** $9E5_{12}$ **13.** 2200_3

15. 24001_7 **17.** 10100_2 **19.** 213_4 **21.** 400_5

23. 644_{12} **25.** 11_2 **27.** 3616_7 **29.** 1011_3 **31.** 121_5

33. 2403_7 **35.** 21020_6 **37.** 12233_9 **39.** 100011_2

41. 6031_7 **43.** 110_2 **45.** 31_5 **47.** 123_4

49. 103_4 R1_4 **51.** 41_5 R1_5 **53.** 45_7 R2_7 **55.** ⊖⊙$_5$

57. ⊖⊖⊙$_5$ **59.** ●●$_4$ **61.** ●●●$_4$ **63.** ●●$_4$

65. ●●●$_4$ **67.** 2302_5, 327 **69.** 13_5

71. a) 21252_8 **b)** 306 and 29 **c)** 8874 **d)** 8874 **e)** Yes

72. 5 **73.** ● = 0, ● = 1, ● = 2, ● = 3

Section 4.5, Page 201

1. Duplation and mediation, the galley method, and Napier rods

3. a) Answers will vary. **b)** 10,498

5. 713 **7.** 1458 **9.** 8260 **11.** 8649 **13.** 2250

15. 2332 **17.** 900 **19.** 204,728 **21.** 504 **23.** 406

25. 625 **27.** 60,678 **29. a)** 46 × 253 **b)** 11,638
31. a) 4 × 382 **b)** 1528 **33.** 99∩∩∩∩∩∩∩∩|||||
35. Answers will vary. **37.** 12331_5 **38. a)** 1776

Review Exercises, Page 203

1. 3103 **2.** 1211 **3.** 1311 **4.** 2114 **5.** 2314
6. 2312 **7.** *bbbbbaaaaaa* **8.** *cbbaaaaa*
9. *ccbbbbbbbbbaaa* **10.** *ddaaaaa*
11. *ddddddcccccccccbbbbba* **12.** *ddcccbaaaa*
13. 43 **14.** 27 **15.** 749 **16.** 4068 **17.** 5648
18. 6905 **19.** *hxb* **20.** *byixe* **21.** *hyfxb*
22. *czixd* **23.** *fzd* **24.** *bza* **25.** 45 **26.** 308
27. 568 **28.** 46,883 **29.** 64,481 **30.** 60,529
31. qe **32.** upb **33.** vrc **34.** BArg **35.** ODvog
36. QFvrf **37.** ʒ9999∩∩∩∩∩∩||
38. MCDLXII **39.** 一千四百六十二 **40.** $\alpha'\upsilon\xi\beta$
41. <<𐄇𐄇𐄇𐄇 <<𐄇𐄇 **42.** ⁘ **43.** 222,035 **44.** 8254
45. 685 **46.** 1991 **47.** 1277 **48.** 2690 **49.** 39
50. 5 **51.** 28 **52.** 1244 **53.** 1451 **54.** 186
55. 13033_4 **56.** 122011_3 **57.** 111001111_2 **58.** 3323_5
59. 327_{12} **60.** 717_8 **61.** 140_7 **62.** 101111_2
63. 176_{12} **64.** 1023_7 **65.** 12102_5 **66.** 12423_8
67. 3411_7 **68.** 100_2 **69.** $2E3_{12}$ **70.** 3324_5 **71.** 450_8
72. 1102_3 **73.** 212_6 **74.** 1314_5 **75.** 5656_{12}
76. 21102_3 **77.** 110111_2 **78.** 13632_8 **79.** 1011_2
80. 130_4 **81.** 23_5R1_5 **82.** 433_6 **83.** 411_6 R1_6
84. 664_8 R 2_8 **85.** 3408 **86.** 3408 **87.** 3408

Chapter Test, Page 204

1. A number is a quantity and answers the question "How many?" A numeral is a symbol used to represent a number.
2. 3646 **3.** 1275 **4.** 8090 **5.** 969 **6.** 122,142
7. 9999 **8.** 9999∩∩∩∩∩∩||| **9.** $\beta'\upsilon o$ ʒ
10. ⁘ ≡ ⁘ **11.** <<𐄇𐄇𐄇𐄇𐄇𐄇 <<<𐄇𐄇𐄇𐄇𐄇𐄇
12. MMCCCLXXVIII
13. In an additive system, the number represented by a particular set of numerals is the sum of the values of the numerals.
14. In a multiplicative system, there are numerals for each number less than the base and for powers of the base. Each numeral less than the base is multiplied by a numeral for the power of the base, and these products are added to obtain the number.
15. In a ciphered system, the number represented by a particular set of numerals is the sum of the values of the numerals. There are numerals for each number up to and including the base and multiples of the base.
16. In a place-value system, each number is multiplied by a power of the base. The position of the numeral indicates the power of the base by which it is multiplied.
17. 41 **18.** 103 **19.** 45 **20.** 305 **21.** 100100_2
22. 333_5 **23.** 1444_{12} **24.** 11365_7 **25.** 1122_5
26. 142_6 **27.** 2003_6 **28.** 220_5 **29.** 980 **30.** 8428

Chapter 5

Section 5.1, Page 216

1. Number theory is the study of numbers and their properties.
3. a) *a* divides *b* means that *b* divided by *a* has a remainder of zero.
b) *a* is divisible by *b* means that *a* divided by *b* has a remainder of zero.
5. A composite number is a natural number that is divisible by a number other than itself and 1.
7. a) The LCM of a set of natural numbers is the smallest natural number that is divisible by each number in the set.
b) Answers will vary. **c)** 80
9. Mersenne primes are prime numbers of the form $2^n - 1$, where *n* is a prime number.
11. Goldbach's conjecture states that every even number greater than or equal to 4 can be represented as the sum of two (not necessarily distinct) prime numbers.
13. The prime numbers between 1 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
15. True **17.** False; 21 is a multiple of 7.
19. False; 56 is divisible by 8. **21.** True
23. False; if a number is divisible by 3, then the sum of the digits of the number is divisible by 3.
25. True
27. 10,368 is divisible by 2, 3, 4, 6, 8, and 9.
29. 2,763,105 is divisible by 3 and 5.
31. 1,882,320 is divisible by 2, 3, 4, 5, 6, 8, and 10.
33. 60 (other answers are possible)
35. $45 = 3^2 \cdot 5$ **37.** $196 = 2^2 \cdot 7^2$

39. $303 = 3 \cdot 101$ **41.** $513 = 3^3 \cdot 19$

43. $1336 = 2^3 \cdot 167$ **45.** $2001 = 3 \cdot 23 \cdot 29$

47. a) 3 **b)** 30 **49. a)** 6 **b)** 432 **51. a)** 20 **b)** 1800

53. a) 4 **b)** 5088 **55. a)** 8 **b)** 384

57. 17, 19, and 29, 31

59. a) Yes **b)** No **c)** Yes **d)** Yes

61. $4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7,$
$12 = 5 + 7, 14 = 7 + 7, 16 = 3 + 13,$
$18 = 5 + 13, 20 = 3 + 17$

63. 70 dolls **65.** 72 cards **67.** 180 min **69.** 30 days

71. a) 4, 5, 10, 20, 25 **b)** 25, 20, 10, 5, and 4, respectively

73. A number is divisible by 15 if both 3 and 5 divide the number.

75. 5 **77.** 36 **79.** 30 **81.** No **83.** Yes

85. a) 12 **b)** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

87. For any three consecutive natural numbers, one of the numbers is divisible by 2 and another number is divisible by 3. Therefore, the product of the three numbers would be divisible by 6.

89. Yes

91. $8 = 2 + 3 + 3, 9 = 3 + 3 + 3, 10 = 2 + 3 + 5,$
$11 = 2 + 2 + 7, 12 = 2 + 5 + 5, 13 = 3 + 3 + 7,$
$14 = 2 + 5 + 7, 15 = 3 + 5 + 7, 16 = 2 + 7 + 7,$
$17 = 5 + 5 + 7, 18 = 2 + 5 + 11, 19 = 3 + 5 + 11,$
$20 = 2 + 7 + 11$

92. The answer most people select is Denmark, kangaroo, and orange.

Section 5.2, Page 225

1. Begin at zero. Represent the first addend with an arrow. Draw the arrow to the right if the addend is positive, to the left if negative. From the tip of the first arrow, represent the second addend with a second arrow. The sum of the two integers is at the tip of the second arrow.

3. To rewrite a subtraction problem as an addition problem, rewrite the minus sign as a plus sign and change the second number to its opposite.

5. The quotient of two numbers with like signs is positive. The quotient of two numbers with unlike signs is negative.

7. 3 **9.** 2 **11.** -5 **13.** 2 **15.** -21 **17.** -3

19. -10 **21.** -2 **23.** -6 **25.** -2 **27.** -20

29. 144 **31.** 96 **33.** -60 **35.** -720 **37.** 2 **39.** -1

41. -7 **43.** -15 **45.** -48

In Exercises 47–55, false answers can be modified in a variety of ways. We give one possible answer.

47. True

49. False; the difference of two negative integers may be a positive integer, a negative integer, or zero.

51. True **53.** True

55. False; the sum of a positive integer and a negative integer may be a positive integer, a negative integer, or zero.

57. 6 **59.** -17 **61.** -12 **63.** -5 **65.** -6

67. $-15, -10, -5, 0, 5, 10$

69. $-6, -5, -4, -3, -2, -1$

71. 213.8°F **73.** 210 points **75.** 1769 ft

77. a) 9 hours **b)** 2 hours **79.** -1

81. $0 + 1 - 2 + 3 + 4 - 5 + 6 - 7 - 8 + 9 = 1$

Section 5.3, Page 238

1. The set of rational numbers is the set of numbers of the form p/q, where p and q are integers and $q \neq 0$.

3. a) Divide both the numerator and the denominator by their greatest common factor.
b) $\frac{5}{9}$

5. For positive mixed numbers, multiply the denominator of the fraction by the integer preceding it. Add this product to the numerator. This sum is the numerator of the improper fraction; the denominator is the same as the denominator in the mixed number. For negative mixed numbers, temporarily ignore the negative sign, perform the conversion described above, and then add the negative sign.

7. a) The reciprocal of a number is 1 divided by the number.
b) $-\frac{1}{2}$

9. a) To add or subtract two fractions with a common denominator, perform the indicated operation on the numerators. Keep the common denominator. Reduce the new fraction to lowest terms, if possible.
b) $\frac{2}{3}$ **c)** $\frac{1}{2}$

11. Answers will vary. **13.** $\frac{2}{3}$ **15.** $\frac{2}{7}$ **17.** $\frac{21}{32}$ **19.** $\frac{7}{11}$

21. $\frac{1}{11}$ **23.** $\frac{25}{7}$ **25.** $-\frac{31}{16}$ **27.** $-\frac{79}{16}$ **29.** $\frac{17}{8}$ **31.** $\frac{15}{8}$

33. $1\frac{3}{8}$ **35.** $-12\frac{1}{6}$ **37.** $-58\frac{8}{15}$ **39.** 0.6 **41.** $0.\overline{2}$

43. 0.375 **45.** $4.\overline{3}$ **47.** $5.\overline{6}$ **49.** $\frac{25}{100} = \frac{1}{4}$ **51.** $\frac{45}{1000} = \frac{9}{200}$

53. $\frac{2}{10} = \frac{1}{5}$ **55.** $\frac{452}{1000} = \frac{113}{250}$ **57.** $\frac{1}{10{,}000}$ **59.** $\frac{2}{3}$ **61.** $\frac{2}{1}$

63. $\frac{15}{11}$ **65.** $\frac{46}{45}$ **67.** $\frac{574}{165}$ **69.** $\frac{3}{22}$ **71.** $\frac{2}{5}$ **73.** $\frac{49}{64}$ **75.** $\frac{36}{35}$

77. $\frac{5}{14}$ **79.** $\frac{13}{15}$ **81.** $\frac{21}{26}$ **83.** $\frac{23}{54}$ **85.** $\frac{17}{144}$ **87.** $-\frac{109}{600}$

89. $\frac{51}{40}$ **91.** $-\frac{1}{24}$ **93.** $\frac{19}{24}$ **95.** 1 **97.** $\frac{4}{11}$ **99.** $\frac{23}{42}$ **101.** $\frac{1}{12}$

103. $120\frac{3}{4}$ in. **105. a)** $\frac{3}{8}$ cup **b)** $\frac{3}{16}$ tsp **c)** $\frac{3}{4}$ cup

107. $\frac{59}{60}$ **109.** 27 pages **111.** $58\frac{7}{8}$ in. **113.** $12\frac{7}{16}$ in.

115. $26\frac{5}{32}$ in. **117. a)** $37\frac{5}{6}$ ft **b)** 88 ft^2 **c)** $806\frac{2}{3}$ ft^3

In Exercises 119–125, an infinite number of answers are possible. We give one answer.

119. 0.105 **121.** -2.1755 **123.** 3.1234505

125. 4.8725 **127.** $\frac{1}{2}$ **129.** $\frac{11}{200}$ **131.** $\frac{9}{40}$ **133.** $\frac{11}{200}$

135. a) $1\frac{3}{8}$ cup water (or milk) and $\frac{3}{4}$ cup oatmeal
b) $1\frac{1}{2}$ cup water (or milk) and $\frac{3}{4}$ cup oatmeal

137. a) $\frac{1}{8}$ **b)** $\frac{1}{16}$ **c)** 5 **d)** 6

Section 5.4, Page 247

1. A rational number can be written as a ratio of two integers. Real numbers that cannot be written as a ratio of two integers are irrational numbers.

3. A perfect square is any number that is the square of a natural number.

5. a) To add or subtract two or more square roots with the same radicand, add or subtract their coefficients and then multiply the sum or difference by the common radical.
b) $-\sqrt{6}$

7. a) Multiply both the numerator and denominator by a radical that will result in the radicand in the denominator becoming a perfect square.
b) $\dfrac{7\sqrt{3}}{3}$

9. Rational **11.** Rational **13.** Irrational **15.** Rational
17. Irrational **19.** 8 **21.** 10 **23.** -13 **25.** -15
27. -10 **29.** Rational number, integer, natural number
31. Rational number, integer, natural number
33. Rational number **35.** Rational number
37. Rational number **39.** $3\sqrt{2}$ **41.** $4\sqrt{3}$ **43.** $3\sqrt{7}$
45. $4\sqrt{5}$ **47.** $9\sqrt{2}$ **49.** $7\sqrt{6}$ **51.** $5\sqrt{3}$ **53.** $-13\sqrt{3}$
55. $4\sqrt{3}$ **57.** $23\sqrt{2}$ **59.** 4 **61.** $2\sqrt{15}$ **63.** $10\sqrt{2}$
65. $\sqrt{2}$ **67.** 3 **69.** $\dfrac{\sqrt{2}}{2}$ **71.** $\dfrac{\sqrt{21}}{7}$ **73.** $\dfrac{2\sqrt{15}}{3}$
75. $\dfrac{3\sqrt{2}}{2}$ **77.** $\dfrac{\sqrt{15}}{3}$

79. $\sqrt{7}$ is between 2 and 3 since 7 is between 4 and 9. $\sqrt{7}$ is between 2.5 and 3 since 7 is closer to 9 than to 4. $\sqrt{7} \approx 2.65$.

81. $\sqrt{107}$ is between 10 and 11 since 107 is between 100 and 121. $\sqrt{107}$ is between 10 and 10.5 since 107 is closer to 100 than to 121. $\sqrt{107} \approx 10.34$.

83. $\sqrt{170}$ is between 13 and 14 since 170 is between 169 and 196. $\sqrt{170}$ is between 13 and 13.5 since 170 is closer to 169 than to 196. $\sqrt{170} \approx 13.04$.

In Exercises 85–89, false answers can be modified in a variety of ways. We give one possible answer.

85. False. $\sqrt{p}$ is an irrational number for any prime number p.
87. True
89. False. The product of a rational number and an irrational number may be a rational number or an irrational number.
91. $\pi + (-\pi) = 0$ **93.** $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$
95. $\sqrt{5} \neq 2.236$ since $\sqrt{5}$ is irrational and 2.236 is rational.
97. No. π is irrational; therefore, it cannot equal $\frac{22}{7}$ or 3.14, both of which are rational.
99. $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$, $\sqrt{36} = 2 \cdot 3$, $6 = 6$
101. a) 10 mph **b)** 20 mph **c)** 40 mph **d)** 80 mph
103. a) If the result on the calculator is a terminating or repeating decimal number, then the number is rational; if the result is not a terminating or repeating decimal number, then the number is irrational.
b) Rational. $\sqrt{0.04} = 0.2$, which is a rational number.
c) Irrational. $\sqrt{0.7} = 0.8366600265\ldots$. Since the decimal number is not a terminating or a repeating decimal number, this is an irrational number.
105. a) $(44 \div \sqrt{4}) \div \sqrt{4} = 11$ **b)** $(44 \div 4) + \sqrt{4} = 13$
c) $4 + 4 + 4 + \sqrt{4} = 14$
d) $\sqrt{4}(4 + 4) + \sqrt{4} = 18$
Other answers are possible.

Section 5.5, Page 254

1. The real numbers are the union of the rational numbers and the irrational numbers.

3. If whenever the operation is performed on two elements of a set the result is also an element of the set, then the set is closed under that operation.

5. $a + b = b + a$, the order in which two numbers are added is immaterial. One example is $4 + 5 = 5 + 4$.

7. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$, when multiplying three numbers, you may place parentheses around any two adjacent numbers. One example is $(1 \times 2) \times 3 = 1 \times (2 \times 3)$.

9. Yes **11.** No **13.** Yes **15.** No **17.** Yes **19.** Yes
21. No **23.** No **25.** Yes **27.** No
29. Commutative property of addition. The only difference between the expressions on both sides of the equal sign is the order of $(3 + 4)$ and x being added.
31. $(-3)(-4) = (-4)(-3) = 12$
33. No. $3 \div 4 \neq 4 \div 3$.
35. $[(-2)(-3)](-4) = (-2)[(-3)(-4)] = -24$
37. No. $(16 \div 8) \div 2 \neq 16 \div (8 \div 2)$.
39. No. $(81 \div 9) \div 3 \neq 81 \div (9 \div 3)$.
41. Commutative property of addition
43. Associative property of multiplication
45. Associative property of addition
47. Commutative property of multiplication
49. Distributive property
51. Commutative property of addition
53. Distributive property
55. Commutative property of addition
57. $2c + 14$ **59.** $\frac{2}{3}x - 4$ **61.** $3x + 4$ **63.** $2x - 1$

65. $15 - 3\sqrt{5}$ **67.** $2 + \sqrt{6}$

69. **a)** Distributive property
b) Associative property of addition

71. **a)** Distributive property
b) Associative property of addition
c) Commutative property of addition
d) Associative property of addition

73. **a)** Distributive property
b) Commutative property of addition
c) Associative property of addition
d) Commutative property of addition

75. Yes **77.** No **79.** Yes **81.** Yes **83.** Yes **85.** Yes

87. Yes **89.** Answers will vary.

91. No. $0 \div a = 0$ (when $a \neq 0$), but $a \div 0$ is undefined.

92. **a)** No **b)** No **c)** Answers will vary.

Section 5.6, Page 264

1. The 2 is the base and the 3 is the exponent or power.

3. **a)** To multiply two exponential expressions with the same base, add the exponents and use this sum as the exponent on the common base.
b) $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 128$

5. **a)** Any nonzero expression raised to the power of 0 equals 1.
b) $7^0 = 1$

7. **a)** Any base with an exponent raised to another exponent is equal to the base raised to the product of the exponents.
b) $(3^2)^4 = 3^{2 \cdot 4} = 3^8 = 6561$

9. **a)** -1^{500} means $-(1)^{500}$ or $-1 \cdot 1^{500}$. Since 1 raised to any power equals 1, $-1^{500} = -1 \cdot 1^{500} = -1 \cdot 1 = -1$.
b) $(-1)^{500}$ means (-1) multiplied by itself 500 times. Since 500 is even, $(-1)^{500} = 1$.
c) -1^{501} means $-(1)^{501}$ or $-1 \cdot 1^{501}$. Since 1 raised to any power equals 1, $-1^{501} = -1 \cdot 1^{501} = -1 \cdot 1 = -1$.
d) $(-1)^{501}$ means (-1) multiplied by itself 501 times. Since 501 is odd, $(-1)^{501} = -1$.

11. **a)** If the exponent is positive, move the decimal point in the number to the right the same number of places as the exponent, adding zeros where necessary. If the exponent is negative, move the decimal point in the number to the left the same number of places as the exponent, adding zeros where necessary.
b) 0.000576

13. 25 **15.** 16 **17.** -9 **19.** $\frac{4}{9}$ **21.** 25 **23.** 72 **25.** 25

27. $\frac{1}{49}$ **29.** 1 **31.** 81 **33.** $\frac{1}{9}$ **35.** 4096 **37.** 121

39. 16 **41.** -16 **43.** $\frac{1}{64}$ **45.** 2.31×10^5

47. 1.5×10^1 **49.** 5.6×10^{-1} **51.** 1.9×10^4

53. 1.86×10^{-4} **55.** 4.23×10^{-6} **57.** 7.11×10^2

59. 1.53×10^{-1} **61.** 2300 **63.** 0.003901

65. 0.0000862 **67.** 0.312 **69.** 9,000,000 **71.** 231

73. 35,000 **75.** 10,000 **77.** 800,000 **79.** 0.0153

81. 320 **83.** 0.0021 **85.** 20 **87.** 6.0×10^{11}

89. 4.5×10^{-7} **91.** 2.0×10^3 **93.** 2.0×10^{-7}

95. 3.0×10^8

97. 8.3×10^{-4}; 3.2×10^{-1}; 4.6; 5.8×10^5

99. 8.3×10^{-5}; 0.00079; 4.1×10^3; 40,000

101. **a)** $\approx$\$35,590.18 **b)** $\$3.559018 \times 10^4$

103. **a)** $\approx$210,109,000,000,000,000,000 sec
b) $\approx 2.1 \times 10^{20}$ sec (about 6.7 trillion years!)

105. **a)** 18,000 hours **b)** 1.8×10^4

107. **a)** 20,000,000,000,000,000 drops **b)** 2×10^{16}

109. **a)** 18,000 times **b)** 1.8×10^4

111. **a)** $\approx$ \$17,093.02 per person **b)** \$5989.17

113. **a)** \$720,000,000 **b)** \$300,000,000 **c)** \$120,000,000
d) \$60,000,000

115. 1000 **117.** 333,333 times

119. 230,000 sec (about 2.66 days)

121. **a)** About 5.87×10^{12} (5.87 trillion) mi
b) About 500 sec or 8 min 20 sec

Section 5.7, Page 273

1. A sequence is a list of numbers that are related to each other by a given rule. One example is 1, 3, 5, 7, 9, ...

3. **a)** An arithmetic sequence is one in which each term differs from the preceding term by a constant amount. One example is 4, 7, 10, 13, 16, ...
b) A geometric sequence is one in which the ratio of any two successive terms is a constant amount. One example is 3, 6, 12, 24, ...

5. **a)** a_n is the nth term or the general term.
b) a_1 is the first term.
c) d is the common difference.
d) s_n is the sum of the first n terms.

7. 3, 5, 7, 9, 11 **9.** $-5, -2, 1, 4, 7$ **11.** 5, 3, 1, $-1, -3$

13. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$ **15.** 17 **17.** 13 **19.** $-\frac{91}{5}$ **21.** 9

23. $a_n = n$ **25.** $a_n = 2n$ **27.** $a_n = \frac{1}{3}n - 2$

29. $a_n = \frac{3}{2}n - \frac{9}{2}$ **31.** $s_{50} = 1275$ **33.** $s_{50} = 2500$

35. $s_8 = -52$ **37.** $s_8 = 60$ **39.** 3, 6, 12, 24, 48

41. 2, -4, 8, -16, 32 **43.** -3, 3, -3, 3, -3

45. -16, 8, -4, 2, -1 **47.** 3072 **49.** $\frac{3}{4}$ **51.** 8

53. $a_{10} = -39{,}366$

55. $a_n = 2^{n-1}$ **57.** $a_n = 3 \cdot (-1)^{n-1}$

59. $a_n = \frac{1}{4} \cdot (2)^{n-1}$ **61.** $a_n = 9 \cdot \left(\frac{1}{3}\right)^{n-1}$ **63.** 45

65. 27,305 **67.** $-620{,}011$ **69.** $s_{15} = -10{,}923$

71. 5050 **73.** 10,000 **75. a)** \$28,600 **b)** \$195,200
77. 12 in. **79.** 496 pinecones **81.** 52.4288 g
83. \$45,218 **85.** \$486,000 **87.** 161.4375 **89.** 267
91. 191.3568 ft

Section 5.8, Page 280

1. The first and second terms are one. Each term thereafter is the sum of the previous two terms.

3. a) The golden number is $\frac{\sqrt{5}+1}{2}$.

b) When a line segment AB is divided at a point C, such that the ratio of the whole, AB, to the larger part, AC, is equal to the ratio of the larger part, AC, to the smaller part, CB, then each of the two ratios AB/AC and AC/CB is known as the golden ratio.

c) The proportion made by using the two golden ratios, $AB/AC = AC/CB$, is known as the golden proportion.

d) A golden rectangle is one where the ratio of the length to the width is equal to the golden number.

5. Answers will vary. **7. a)** 1.618 **b)** 0.618 **c)** 1

9. $\frac{1}{1} = 1, \frac{2}{1} = 2, \frac{3}{2} = 1.5, \frac{5}{3} \approx 1.667, \frac{8}{5} = 1.6, \frac{13}{8} = 1.625, \frac{21}{13} \approx 1.615, \frac{34}{21} \approx 1.619, \frac{55}{34} \approx 1.6176, \frac{89}{55} \approx 1.6182$. The consecutive ratios alternate, increasing and decreasing about the golden ratio.

11. Each number in the Fibonacci sequence is either a prime number or is relatively prime with the number preceding or succeeding it in the sequence. Therefore, the GCF of any two consecutive Fibonacci numbers is 1.

13. Answers will vary. **15.** Answers will vary.
17. Answers will vary. **19.** Answers will vary.
21. Answers will vary. **23.** Yes; 29, 47 **25.** No
27. Yes; 105, 170 **29.** Yes; $-1, -1$
31. Answers will vary. **33.** Answers will vary.

35. a) 1, 3, 4, 7, 11, 18, 29, 47
b) $8 + 21 = 29, 13 + 34 = 47$
c) It is the Fibonacci sequence.

37. Answers will vary. **39.** Answers will vary.

Review Exercises, Page 283

1. 2, 3, 4, 6, 9 **2.** 2, 3, 4, 6, 9 **3.** $2^2 \cdot 3^2 \cdot 7$
4. $5 \cdot 7 \cdot 11$ **5.** $2^3 \cdot 3 \cdot 5 \cdot 7$ **6.** $2 \cdot 3^2 \cdot 7^2$ **7.** $2^2 \cdot 3 \cdot 11^2$
8. 15; 60 **9.** 9; 756 **10.** 5; 2250 **11.** 40; 6720
12. 4; 480 **13.** 36; 432 **14.** 45 days **15.** 3 **16.** -3
17. -4 **18.** -6 **19.** -9 **20.** 3 **21.** 0 **22.** 4 **23.** 33
24. -36 **25.** -56 **26.** 5 **27.** -2 **28.** 6 **29.** 6
30. 3 **31.** 0.3 **32.** 0.6 **33.** 0.375 **34.** 3.25
35. $0.\overline{428571}$ **36.** $0.58\overline{3}$ **37.** 0.375 **38.** 0.875
39. $0.\overline{714285}$ **40.** $\frac{9}{40}$ **41.** $\frac{9}{2}$ **42.** $\frac{2}{3}$ **43.** $\frac{235}{99}$ **44.** $\frac{83}{1000}$
45. $\frac{21}{5000}$ **46.** $\frac{211}{90}$ **47.** $\frac{19}{7}$ **48.** $\frac{25}{6}$ **49.** $-\frac{13}{4}$ **50.** $-\frac{283}{8}$
51. $2\frac{1}{5}$ **52.** $1\frac{4}{5}$ **53.** $-1\frac{5}{7}$ **54.** $-27\frac{1}{5}$ **55.** $\frac{13}{10}$ **56.** $\frac{1}{8}$
57. $\frac{17}{12}$ **58.** $\frac{3}{4}$ **59.** $\frac{35}{54}$ **60.** $\frac{53}{28}$ **61.** $\frac{1}{6}$ **62.** $\frac{13}{40}$ **63.** $\frac{8}{15}$
64. $2\frac{7}{32}$ tsp **65.** $5\sqrt{2}$ **66.** $10\sqrt{2}$ **67.** $8\sqrt{5}$ **68.** $-3\sqrt{3}$
69. $8\sqrt{2}$ **70.** $-20\sqrt{3}$ **71.** $8\sqrt{3}$ **72.** $3\sqrt{2}$ **73.** $4\sqrt{3}$
74. 3 **75.** $2\sqrt{7}$ **76.** $\frac{4\sqrt{3}}{3}$ **77.** $\frac{\sqrt{15}}{5}$ **78.** $6 + 3\sqrt{7}$
79. $4\sqrt{3} + 3\sqrt{2}$ **80.** $3\sqrt{2} + 3\sqrt{5}$
81. Commutative property of addition
82. Commutative property of multiplication
83. Associative property of addition
84. Distributive property
85. Commutative property of addition
86. Commutative property of addition
87. Associative property of multiplication
88. Commutative property of multiplication
89. Distributive property
90. Commutative property of multiplication
91. Yes **92.** No **93.** No **94.** Yes **95.** No **96.** No
97. 9 **98.** $\frac{1}{9}$ **99.** 81 **100.** 125 **101.** 1 **102.** $\frac{1}{64}$
103. 64 **104.** 81 **105.** 2.3×10^8 **106.** 1.58×10^{-5}
107. 2.75×10^{-3} **108.** 4.95×10^6 **109.** 43,000,000
110. 0.000139 **111.** 0.000175 **112.** 100,000
113. 1.4×10^{-1} **114.** 1.0×10^5 **115.** 2.1×10^1
116. 3.0×10^0 **117.** 8,000,000,000 **118.** 0.7
119. 3200 **120.** 5 **121.** ≈388 times **122.** ≈ \$5555.56
123. Arithmetic; 14, 17 **124.** Geometric; 8, 16
125. Arithmetic; $-15, -18$ **126.** Geometric; $\frac{1}{32}, \frac{1}{64}$
127. Arithmetic; 16, 19 **128.** Geometric; $-2, 2$ **129.** 15
130. -34 **131.** 25 **132.** 48 **133.** $\frac{1}{4}$ **134.** -48
135. 1365 **136.** -25 **137.** 632 **138.** 57.5 **139.** 200
140. 80 **141.** 33 **142.** -21
143. Arithmetic; $a_n = -3n + 10$
144. Arithmetic; $a_n = 3n$ **145.** Arithmetic; $a_n = -\frac{3}{2}n + \frac{11}{2}$
146. Geometric; $a_n = 3(2)^{n-1}$
147. Geometric; $a_n = 2(-1)^{n-1}$
148. Geometric; $a_n = 5\left(\frac{1}{3}\right)^{n-1}$ **149.** Yes; 13, 21
150. Yes; 17, 28 **151.** No **152.** No

Chapter Test, Page 285

1. 2, 3, 5, 6, 9, 10 **2.** $2^3 \cdot 3 \cdot 5 \cdot 7$ **3.** -7 **4.** -20
5. -175 **6.** $\frac{37}{8}$ **7.** $19\frac{5}{9}$ **8.** 0.625 **9.** $\frac{129}{20}$ **10.** $\frac{121}{240}$

11. $\frac{13}{24}$ **12.** $9\sqrt{3}$ **13.** $\frac{\sqrt{14}}{7}$

14. Yes; the product of any two integers is an integer.

15. Associative property of addition

16. Distributive property **17.** 64 **18.** 1024 **19.** $\frac{1}{81}$

20. 8.0×10^{11} **21.** $a_n = -4n + 2$ **22.** -187 **23.** 243

24. 1023 **25.** $a_n = 3(2)^{n-1}$

26. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Chapter 6

Section 6.1, Page 291

1. Letters of the alphabet used to represent numbers are called variables.

3. The solution to an equation is the number or numbers that replace the variable to make the equation a true statement.

5. a) The 4 is the base and the 5 is the exponent.
b) Answers will vary.

7. 12 **9.** 49 **11.** -9 **13.** 686 **15.** -3 **17.** 18

19. -27 **21.** $-\frac{10}{9}$ **23.** 7 **25.** 9 **27.** 0 **29.** No

31. No **33.** No **35.** Yes **37.** Yes **39.** \$12.25

41. \$426.25 **43.** 16,000,000 sec **45.** 1.71 in.

47. The two expressions are not equal.

Section 6.2, Page 302

1. The parts that are added or subtracted in an algebraic expression are called terms. In $3x - 2y$, the $3x$ and $-2y$ are terms.

3. The numerical part of a term is called its numerical coefficient. For the term $3x$, 3 is the numerical coefficient.

5. To simplify an expression means to combine like terms by using the commutative, associative, and distributive properties.

7. If $a = b$, then $a - c = b - c$ for all real numbers a, b, and c. If $2x + 3 = 5$, then $2x + 3 - 3 = 5 - 3$.

9. If $a = b$, then $a/c = b/c$ for all real numbers a, b, and c, where $c \neq 0$. If $4x = 8$, then $\frac{4x}{4} = \frac{8}{4}$.

11. A ratio is a quotient of two quantities. An example is $\frac{7}{9}$.

13. Yes. They have the same variable and the same exponent on the variable.

15. $11x$ **17.** $2x + 12$ **19.** $3x + 11y$ **21.** $-8x + 2$

23. $-5x + 3$ **25.** $13.3x - 8.3$ **27.** $-\frac{2}{15}x - 4$

29. $13x - 7y + 3$ **31.** $8s - 17$ **33.** $1.5x - 4.2$

35. $\frac{5}{12}x + \frac{3}{7}$ **37.** $4.52x - 13.5$ **39.** 5 **41.** 1 **43.** $\frac{24}{7}$

45. $\frac{2}{3}$ **47.** 3 **49.** 3 **51.** 17 **53.** -8

55. No solution **57.** All real numbers **59.** $\frac{4}{3}$ **61.** -3

63. 4 **65.** \$72.37 **67.** 59 times

69. 20,746,600 households

71. a) 1.6 kph **b)** 56.25 mph **73.** 0.3 cc

75. a) Answers will vary. **b)** -1

77. a) An equation that has no solution.
b) You will obtain a false statement.

79. a) 2 : 5 **b)** $m : m + n$

Section 6.3, Page 311

1. A formula is an equation that typically has a real-life application.

3. Subscripts are numbers (or letters) placed below and to the right of variables. They are used to help clarify a formula.

5. An exponential equation is of the form $y = a^x$, $a > 0$, $a \neq 1$.

7. 20 **9.** 56 **11.** 25 **13.** 62.8 **15.** 37.1 **17.** 2

19. 3000 **21.** 8 **23.** 25 **25.** 6 **27.** 200 **29.** 7.2

31. 14 **33.** 3240 **35.** 0.5 **37.** 14

39. $y = \frac{10x - 13}{4}$ or $y = \frac{5}{2}x - \frac{13}{4}$

41. $y = \frac{-4x + 14}{7}$ or $y = -\frac{4}{7}x + 2$

43. $y = \frac{2x + 6}{3}$ or $y = \frac{2}{3}x + 2$

45. $y = \frac{2x - z + 15}{3}$ or $y = \frac{2}{3}x - \frac{1}{3}z + 5$

47. $y = \frac{9x + 4z - 7}{8}$ or $y = \frac{9}{8}x + \frac{1}{2}z - \frac{7}{8}$

49. $R = \frac{E}{I}$ **51.** $a = p - b - c$ **53.** $B = \frac{3V}{h}$

55. $r = \frac{C}{2\pi}$ **57.** $b = y - mx$ **59.** $w = \frac{P - 2l}{2}$

61. $c = 3A - a - b$ **63.** $T = \frac{PV}{K}$

65. $C = \frac{5}{9}(F - 32)$ **67.** $s = \frac{S - \pi r^2}{\pi r}$

69. a) \$12 **b)** \$612 **71.** ≈ 18.4 in.^{3}

73. 486,000 bacteria **75.** \$300,976,658,300,000

77. ≈ 1051.47 in.^{3}

Section 6.4, Page 317

1. A mathematical expression is a collection of variables, numbers, parentheses, and operation symbols. An equation is two algebraic expressions joined by an equal sign.

3. $4 + 3x$ **5.** $6r + 5$ **7.** $15 - 2r$ **9.** $2m + 9$

11. $\frac{18 - s}{4}$ **13.** $(5y - 6) + 3$ **15.** $x - 6 = 5$; 11

17. $x - 4 = 20$; 24 **19.** $12 + 5x = 47$; 7

21. $8x + 16 = 88$; 9 **23.** $x + 11 = 3x + 1$; 5

25. $x + 10 = 2(x + 3)$; 4

27. $x + 3x = 600$; 150 tickets to nonstudents, 450 tickets to students

29. $x + 0.116x = 34.20$; ≈ 30.65 million taxpayers

31. $x - 0.10x = 15.72$; \$17.47

33. $x + 3x = 12$; Samantha: 3, Josie: 9

35. $2000 + 50x = 13{,}350$; \$227

37. a) $x + x + 3x = 45{,}000$; 9000 ft^2, 9000 ft^2, 27,000 ft^2 **b)** Yes

39. $x + 3x + 3 = 55$; United States: 13, Italy: 42

41. $3w + 2(2w) = 140$; width: 20 ft, length: 40 ft

43. $70x = 760$; ≈ 11 months

45. $\frac{r}{2} + 0.07r = 257$; \$450.88

47. Deduct \$720 from Mr. McAdams's income and \$2920 from Mrs. McAdams's income.

49.
$$x + (x + 1) + (x + 2) = 3(x + 2) - 3$$
$$3x + 3 = 3x + 6 - 3$$
$$3x + 3 = 3x + 3$$

51. $-40°$

Section 6.5, Page 326

1. Inverse variation: As one variable increases, the other decreases and vice versa.

3. Joint variation: One quantity varies directly as the product of two or more other quantities.

5. Direct **7.** Inverse **9.** Direct **11.** Inverse

13. Inverse **15.** Inverse **17.** Direct **19.** Direct

21. Answers will vary. **23. a)** $y = kx$ **b)** 15

25. a) $m = \frac{k}{n^2}$ **b)** 0.25 **27. a)** $R = \frac{k}{W}$ **b)** 0.05

29. a) $F = kDE$ **b)** 210 **31. a)** $t = \frac{kd^2}{f}$ **b)** 200

33. a) $Z = kWY$ **b)** 100 **35. a)** $H = kL$ **b)** 3

37. a) $A = kB^2$ **b)** 720 **39. a)** $F = \frac{kq_1q_2}{d^2}$ **b)** 672

41. a) $R = kL$ **b)** 0.32 ohm **43. a)** $l = \frac{k}{d^2}$ **b)** 80 dB

45. a) $R = \frac{kA}{P}$ **b)** 4800 tapes

47. a) $s = kwd^2$ **b)** 6480 pounds per square inch

49. a) $N = \frac{kp_1p_2}{d}$ **b)** $\approx 121{,}528$ calls

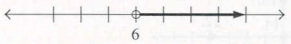
51. a) Inversely **b)** Stays 0.3 **53.** \$132.27

Section 6.6, Page 332

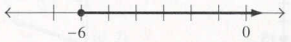
1. $a < b$ means that a is less than b, $a \le b$ means that a is less than or equal to b, $a > b$ means that a is greater than b, $a \ge b$ means that a is greater than or equal to b.

3. When both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality symbol must be reversed.

5. Yes, the inequality symbol points to the -3 in both cases.

7.


9.

11.


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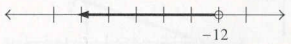

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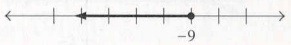

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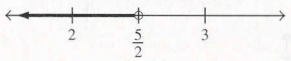

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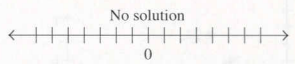

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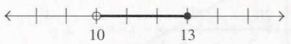

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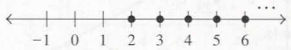

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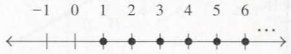

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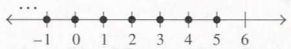

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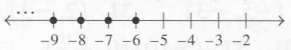

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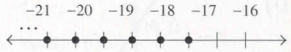

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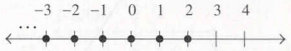

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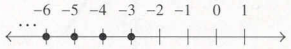


41.

43.

45. a) 2000, 2001 **b)** 1997, 1998 **c)** 1997, 1998, 1999, 2000 **d)** 1998, 1999, 2000, 2001

47. 19 videos **49.** Less than 360 mi **51.** $x \le$ \$15.57

53. $0.5 < t < 1.5$

55. $94 \le x \le 100$, assuming 100 is the highest grade possible

57. $6.875 \le x \le 11$

59. The student's answer is $x \le -12$, whereas the correct answer is $x \ge -12$. Yes, -12 is in both solution sets.

Section 6.7, Page 344

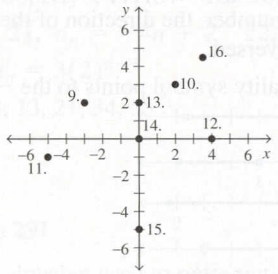
1. A graph is an illustration of all the points whose coordinates satisfy an equation.

3. To find the y-intercept, set $x = 0$ and solve the equation for y.

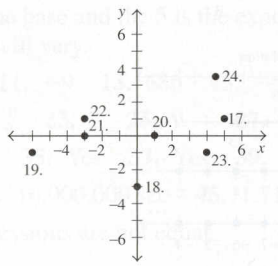
5. a) Answers will vary. **b)** $-\frac{1}{3}$

7. a) First **b)** Second

For Exercises 9–15, see the following figure.



For Exercises 17–23, see the following figure.

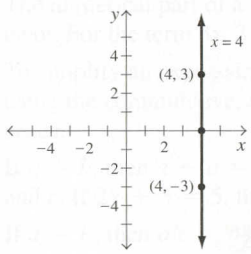


25. (0, 2) **27.** (−2, 0) **29.** (−5, −3) **31.** (2, −3)

33. (2, 2) **35.** (1, 4), (−1, 10) **37.** (5, 0), $(0, -\frac{10}{3})$

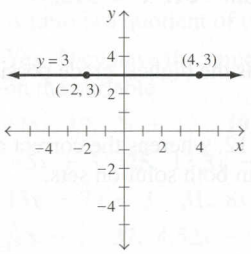
39. (−3, −2) **41.** $(0, \frac{8}{3})$, (4, 0)

43.



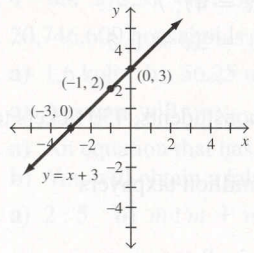
Slope: undefined

45.



Slope is 0.

47.



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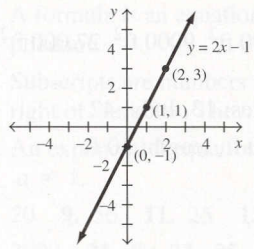

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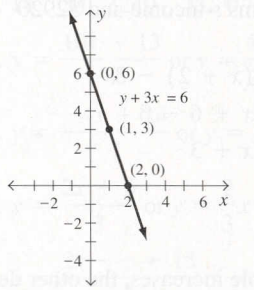

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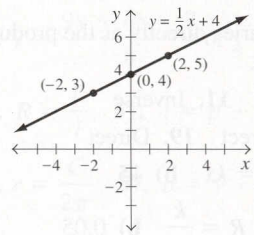

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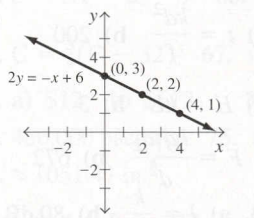

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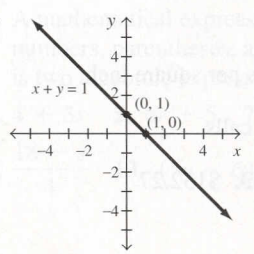

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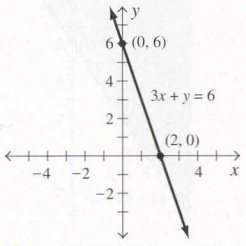

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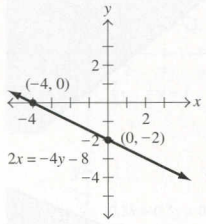

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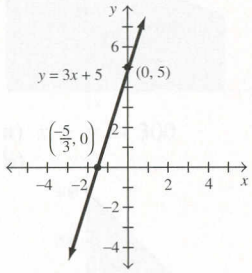

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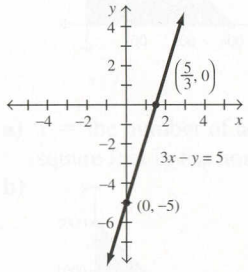
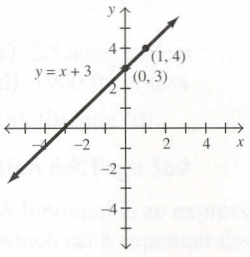


67. 2 **69.** $\frac{15}{7}$ **71.** 0 **73.** Undefined **75.** $-\frac{4}{3}$

77.



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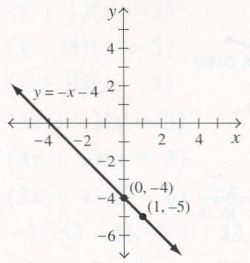

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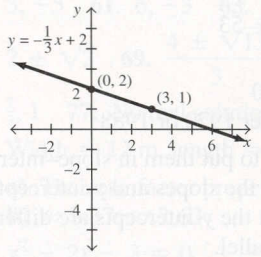

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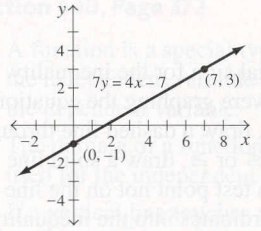

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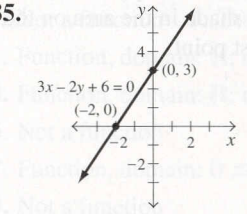
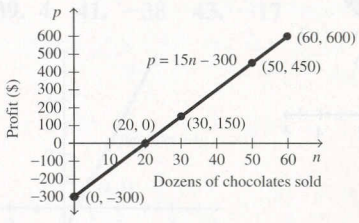


87. $y = -\frac{3}{4}x + 3$ **89.** $y = 3x + 2$

91. a) $D(-1, 2)$ **b)** $A = 10$ square units

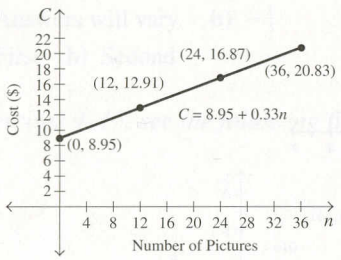
93. $(7, 2)$ or $(-1, 2)$ **95.** 3 **97.** 3

99. a)



b) $300 **c)** 20 dozens of chocolates

101. a)



b) \$15.55 **c)** 36 pictures

103. a) 10.75 **b)** $y = 10.75x + 53$
c) 85.25 **d)** ≈ 2.5 hours

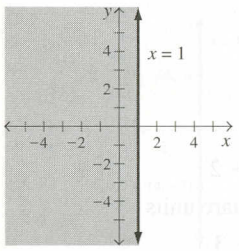
105. a) $-\frac{8}{15}$ **b)** $y = -\frac{8}{15}x + 40$
c) 32% **d)** 18.75 years after 1970 or 1988

107. a) Solve the equations for y to put them in slope–intercept form. Then compare the slopes and y-intercepts. If the slopes are equal but the y-intercepts are different, then the lines are parallel.
b) The lines are parallel.

Section 6.8, Page 350

1. (1) Mentally substitute the equal sign for the inequality sign and plot points as if you were graphing the equation. (2) If the inequality is $<$ or $>$, draw a dashed line through the points. If the inequality is $\leq$ or $\geq$, draw a solid line through the points. (3) Select a test point not on the line and substitute the x- and y-coordinates into the inequality. If the substitution results in a true statement, shade in the area on the same side of the line as the test point. If the test point results in a false statement, shade in the area on the opposite side of the line as the test point.

3.



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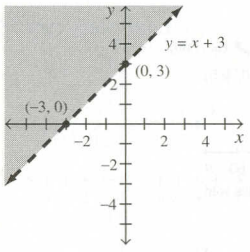

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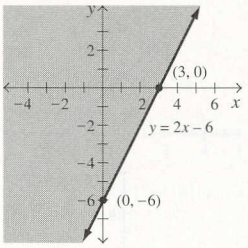

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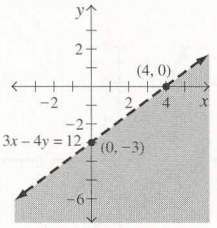

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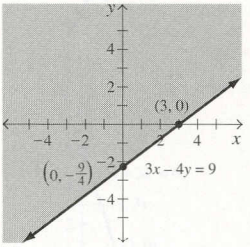

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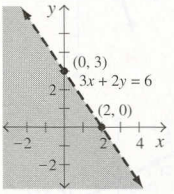

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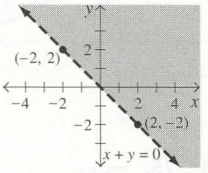

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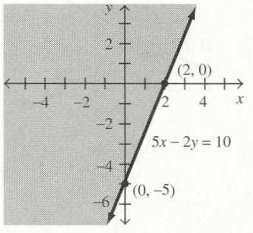

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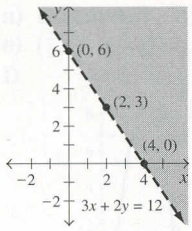

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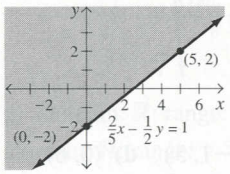

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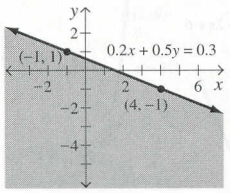
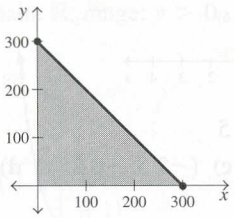


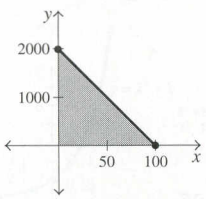
25. a) $x + y \leq 300$

b)



27. a) x = the number of acres of land, y = the number of square feet in the house

b)



c) 2.5 acres or less

d) 1900 ft^2 or less

29. (a), (b), and (d)

Section 6.9, Page 360

1. A binomial is an expression that contains two terms in which each exponent that appears on the variable is a whole number. $2x + 3, x - 7, x^2 - 9$

3. Answers will vary.

5. $ax^2 + bx + c = 0, a \neq 0$ **7.** $(x + 6)(x + 3)$

9. $(x - 3)(x + 2)$ **11.** $(x + 6)(x - 4)$

13. $(x + 1)(x - 3)$ **15.** $(x - 7)(x - 3)$

17. $(x - 5)(x + 5)$ **19.** $(x + 7)(x - 4)$

21. $(x + 9)(x - 7)$ **23.** $(2x - 5)(x + 2)$

25. $(4x + 1)(x + 3)$ **27.** $(5x + 2)(x + 2)$

29. $(4x + 3)(x + 2)$ **31.** $(4x - 3)(x - 2)$

33. $(3x + 4)(x - 6)$ **35.** $1, -2$ **37.** $-\frac{4}{3}, \frac{1}{2}$

39. $-7, -3$ **41.** $3, 1$ **43.** $5, -3$ **45.** $3, 1$ **47.** $9, -9$

49. $-9, 4$ **51.** $\frac{2}{3}, -4$ **53.** $-\frac{1}{5}, -2$ **55.** $\frac{1}{3}, 1$ **57.** $\frac{1}{4}, 2$

59. $3, -5$ **61.** $6, -3$ **63.** $9, -1$ **65.** No real solution

67. $2 \pm \sqrt{2}$ **69.** $\frac{4 \pm \sqrt{13}}{3}$ **71.** $\frac{1 \pm \sqrt{17}}{8}$ **73.** $-1, -\frac{5}{2}$

75. $\frac{7}{3}, 1$ **77.** No real solution

79. Width = 12 m, length = 22 m

81. a) The zero-factor property cannot be used.

b) $\approx 8.37, \approx 2.63$

83. $x^2 - 2x - 3 = 0$

Section 6.10, Page 372

1. A function is a special type of relation where each value of the independent variable corresponds to a unique value of the dependent variable.

3. The domain of a function is the set of values that can be used for the independent variable.

5. If a vertical line touches more than one point on the graph, then for each value of x there is not a unique value for y and the graph does not represent a function.

7. Not a function **9.** Function, domain: $\mathbb{R}$; range: $\mathbb{R}$

11. Function, domain: $\mathbb{R}$; range: $y = 2$

13. Function, domain: $\mathbb{R}$; range: $y \geq -4$

15. Not a function

17. Function, domain: $0 \leq x < 12$; range: $y = 1, 2, 3$

19. Not a function

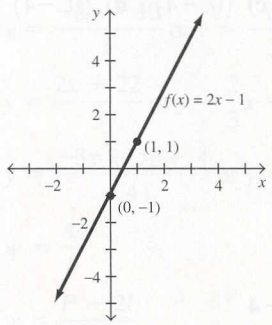
21. Function, domain: $\mathbb{R}$; range: $y > 0$

23. Not a function

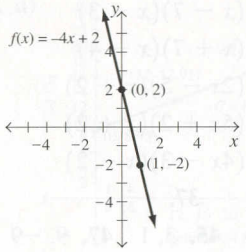
25. Yes **27.** No **29.** Yes **31.** 5 **33.** 1 **35.** -6

37. 5 **39.** 4 **41.** -38 **43.** -17

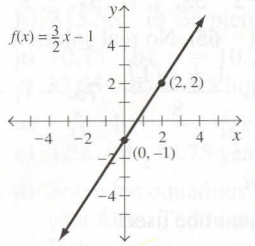
45.



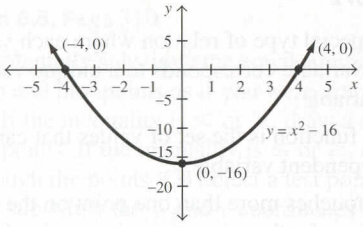
47.



49.

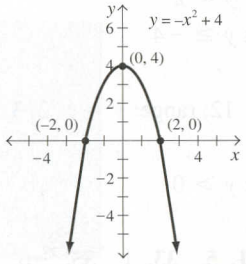


51. a) Upward **b)** $x = 0$ **c)** $(0, -16)$ **d)** $(0, -16)$
e) $(4, 0), (-4, 0)$
f)



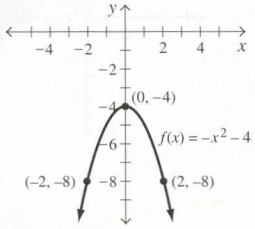
g) Domain: $\mathbb{R}$; range: $y \geq -16$

53. a) Downward **b)** $x = 0$ **c)** $(0, 4)$ **d)** $(0, 4)$
e) $(-2, 0), (2, 0)$
f)



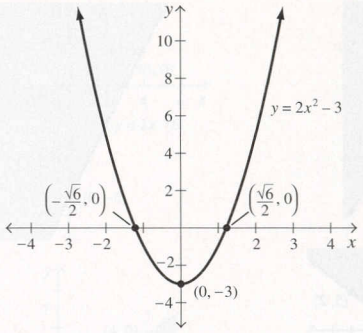
g) Domain: $\mathbb{R}$; range: $y \leq 4$

55. a) Downward **b)** $x = 0$ **c)** $(0, -4)$ **d)** $(0, -4)$
e) No x-intercepts
f)



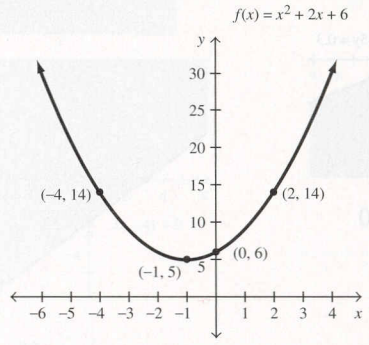
g) Domain: $\mathbb{R}$; range: $y \leq -4$

57. a) Upward **b)** $x = 0$ **c)** $(0, -3)$ **d)** $(0, -3)$
e) $(-1.22, 0), (1.22, 0)$
f)



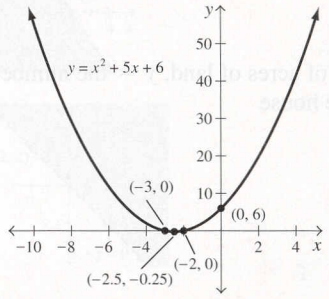
g) Domain: $\mathbb{R}$; range: $y \geq -3$

59. a) Upward **b)** $x = -1$ **c)** $(-1, 5)$ **d)** $(0, 6)$
e) No x-intercepts
f)



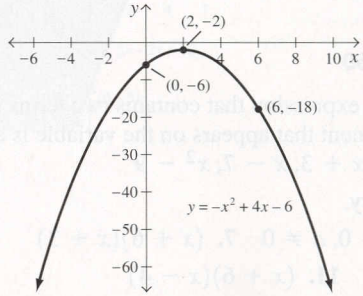
g) Domain: $\mathbb{R}$; range: $y \geq 5$

61. a) Upward **b)** $x = -\frac{5}{2}$ **c)** $(-2.5, -0.25)$ **d)** $(0, 6)$
e) $(-3, 0), (-2, 0)$
f)



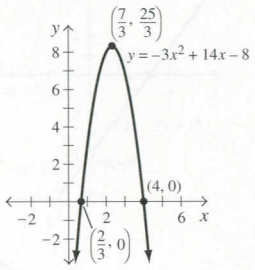
g) Domain: $\mathbb{R}$; range: $y \geq -0.25$

63. a) Downward **b)** $x = 2$ **c)** $(2, -2)$ **d)** $(0, -6)$
e) No x-intercepts
f)



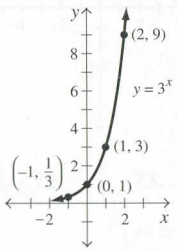
g) Domain: $\mathbb{R}$; range: $y \leq -2$

65. a) Downward **b)** $x = \frac{7}{3}$ **c)** $\left(\frac{7}{3}, \frac{25}{3}\right)$ **d)** $(0, -8)$
e) $\left(\frac{2}{3}, 0\right), (4, 0)$
f)

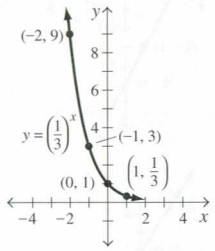


g) Domain: $\mathbb{R}$; range: $y \leq \frac{25}{3}$

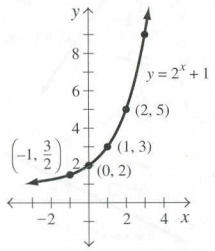
67. Domain: $\mathbb{R}$; range: $y > 0$



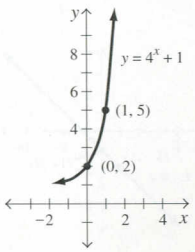
69. Domain: $\mathbb{R}$; range: $y > 0$



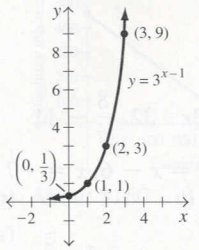
71. Domain: $\mathbb{R}$; range: $y > 1$



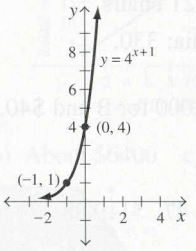
73. Domain: $\mathbb{R}$; range: $y > 1$



75. Domain: $\mathbb{R}$; range: $y > 0$



77. Domain: $\mathbb{R}$; range: $y > 0$



79. $2300

81. a) 52.23% **b)** 1997 **c)** $x \approx 4.85$; $\approx 46.67\%$

83. a) 5200 people **b)** $\approx$ 14,852 people

85. a) Yes **b)** $\approx$ 6500 scooter injuries

87. a) 23.2 cm **b)** 55.2 cm **c)** 69.5 cm

89. a) 170 beats per minute
b) $\approx$ 162 beats per minute
c) $\approx$ 145 beats per minute
d) 136 beats per minute
e) 120 years of age

Review Exercises, Page 376

1. 21 **2.** −10 **3.** 17 **4.** $\frac{1}{4}$ **5.** −65 **6.** 13
7. $4x + 1$ **8.** $13x - 8$ **9.** $7x - 3$ **10.** −10
11. 7 **12.** 11 **13.** −31 **14.** $\frac{128}{5}$ **15.** $\frac{1}{2}$ cup
16. 250 min, or 4 hr 10 min **17.** 48 **18.** ≈ 173.1
19. 101.5 **20.** 10

21. $y = \dfrac{3x - 18}{9}$ or $y = \dfrac{1}{3}x - 2$

22. $y = \dfrac{-2x + 12}{5}$ or $y = -\dfrac{2}{5}x + \dfrac{12}{5}$

23. $y = \dfrac{2x + 22}{3}$ or $y = \dfrac{2}{3}x + \dfrac{22}{3}$

24. $y = \dfrac{-3x + 5z - 4}{4}$ or $y = -\dfrac{3}{4}x + \dfrac{5}{4}z - 1$

25. $w = \dfrac{A}{l}$

26. $w = \dfrac{P - 2l}{2}$

27. $l = \dfrac{L - 2wh}{2h}$ or $l = \dfrac{L}{2h} - w$

28. $d = \dfrac{a_n - a_1}{n - 1}$

29. $8 + 2x$ **30.** $3y - 7$ **31.** $10 + 3r$ **32.** $\dfrac{8}{q} - 11$

33. $4 + 3x = 22$; $x = 6$ **34.** $3x + 8 = x - 6$; $x = -7$

35. $5(x - 4) = 45$; $x = 13$

36. $10x + 14 = 8(x + 12)$; $x = 41$

37. $x + 2x = 15{,}000$; bonds: \$5000, mutual funds: \$10,000

38. $9.50x + 15{,}000 = 95{,}000$; $\approx$ 8421 chairs

39. $x + 2x + 140 = 1130$; Philadelphia: 330, San Diego: 800

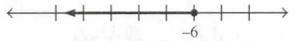
40. $x + (x + 12{,}000) = 68{,}000$; \$28,000 for B and \$40,000 for A

41. 6 **42.** 98 **43.** 20 **44.** ≈ 426.7

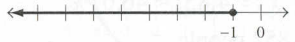
45. a) 150 lb **b)** 5 bags **46.** 4 in. **47.** \$119.88

48. 400 ft

49.



50.



51.

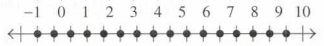
52.

53.

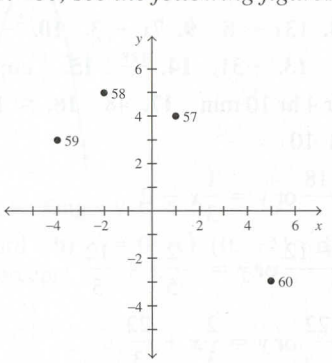
54.

55.

56.



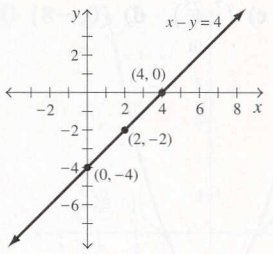
For Exercises 57–60, see the following figure.



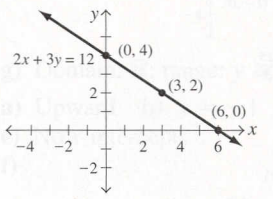
61. $D(-3, -1)$; area = 20 square units

62. $D(4, 1)$; area = 21 square units

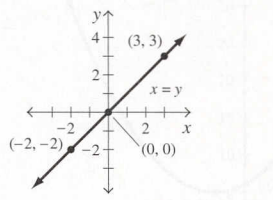
63.



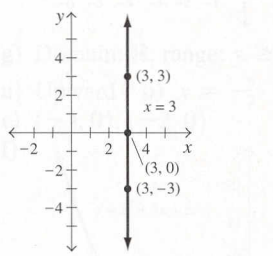
64.



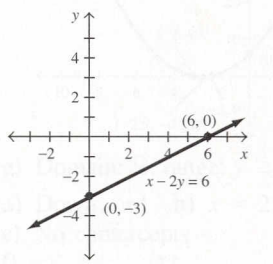
65.



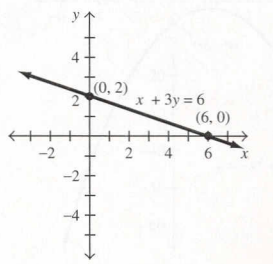
66.



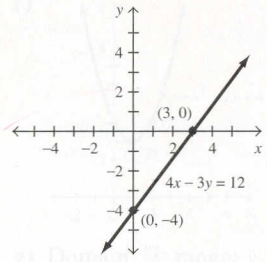
67.



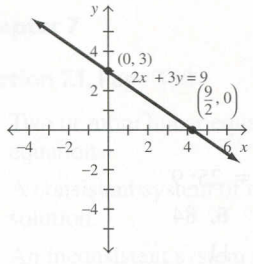
68.



69.

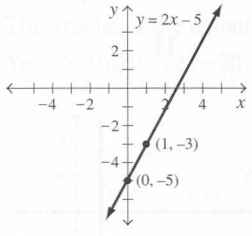


70.

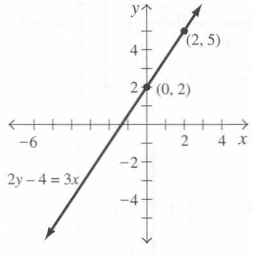


71. $\frac{2}{5}$ **72.** $-\frac{3}{2}$ **73.** $\frac{7}{3}$ **74.** Undefined

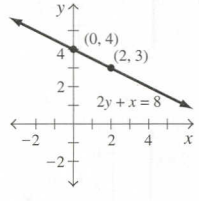
75.



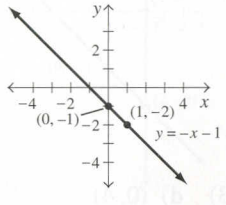
76.



77.

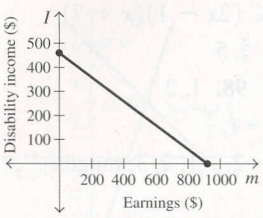


78.



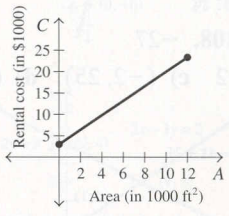
79. $y = 2x + 4$ **80.** $y = -x + 1$

81. a)



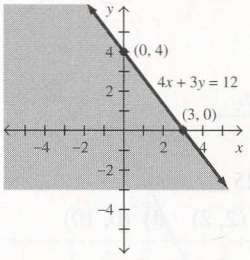
b) About \$160 **c)** About \$160

82. a)

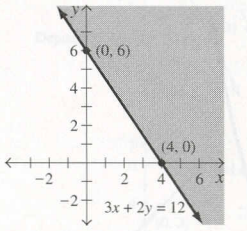


b) About \$6400 **c)** About 4120 ft^2

83.



84.



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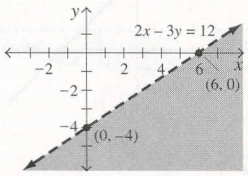
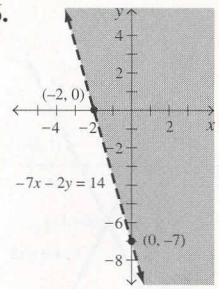


86.



87. $(x + 3)(x + 6)$ **88.** $(x + 5)(x - 4)$

89. $(x - 6)(x - 4)$ **90.** $(x - 5)(x - 4)$

91. $(3x - 1)(2x + 3)$ **92.** $(2x - 1)(x + 7)$

93. $-1, -2$ **94.** $1, 4$ **95.** $\frac{2}{3}, 5$

96. $-2, -\frac{1}{3}$ **97.** $2 \pm \sqrt{5}$ **98.** $1, 2$

99. No real solution **100.** $-1, \frac{3}{2}$

101. Function, domain: $x = -2, -1, 2, 3$; range: $y = -1, 0, 2$

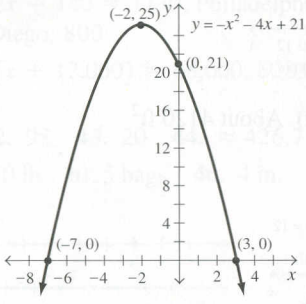
102. Not a function **103.** Not a function

104. Function, domain: $\mathbb{R}$; range: $\mathbb{R}$

105. 18 **106.** 13 **107.** 39 **108.** -27

109. **a)** Downward **b)** $x = -2$ **c)** $(-2, 25)$ **d)** $(0, 21)$ **e)** $(-7, 0), (3, 0)$

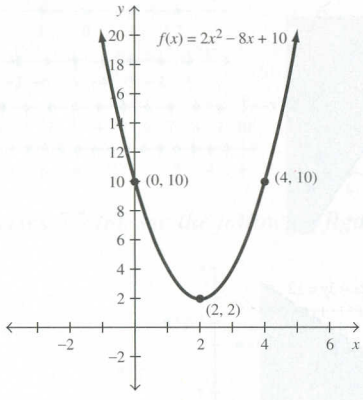
f)



g) Domain: $\mathbb{R}$; range: $y \leq 25$

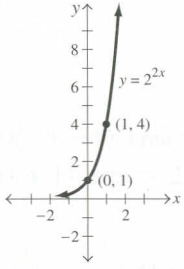
110. **a)** Upward **b)** $x = 2$ **c)** $(2, 2)$ **d)** $(0, 10)$ **e)** No x-intercepts

f)

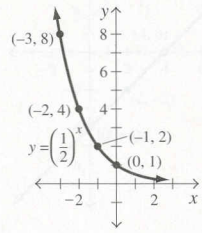


g) Domain: $\mathbb{R}$; range: $y \geq 2$

111. Domain: $\mathbb{R}$; range: $y > 0$



112. Domain: $\mathbb{R}$; range: $y > 0$



113. 22.8 mpg

114. **a)** 4208 **b)** 4250

115. $\approx 68.7\%$

Chapter Test, Page 380

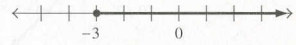
1. 3 **2.** $\frac{19}{5}$ **3.** 18 **4.** $2x + 7 = 25$; 9

5. $x + 0.07x = 26{,}750$; \$25,000 **6.** 84

7. $y = \dfrac{-3x + 11}{5}$ or $y = -\dfrac{3}{5}x + \dfrac{11}{5}$

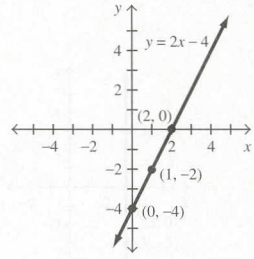
8. $3\frac{1}{3}$ **9.** 6.75 ft

10.



11. $\frac{7}{10}$

12.



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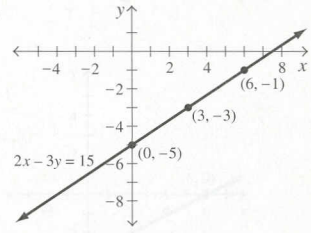

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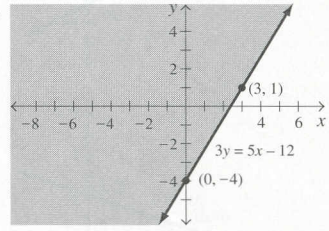
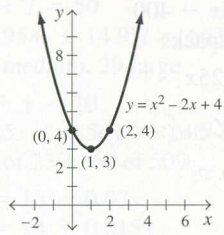


15. $7, -4$ **16.** $\frac{4}{3}, -2$

17. It is a function **18.** 11

19. **a)** Upward **b)** $x = 1$ **c)** $(1, 3)$ **d)** $(0, 4)$ **e)** No x-intercepts

f)



g) Domain: $\mathbb{R}$; range: $y \geq 3$

Chapter 7

Section 7.1, Page 388

1. Two or more linear equations form a system of linear equations.

3. A consistent system of equations is a system that has a solution.

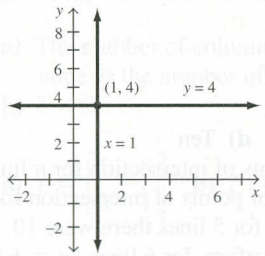
5. An inconsistent system of equations is one that has no solution.

7. The graphs of the equations will be parallel.

9. The graphs of the equations will be the same line.

11. Yes: (3, 0); no: (2, −2), (1, 2)

13.



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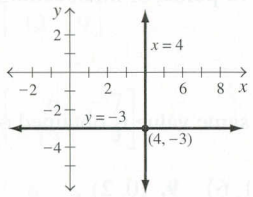

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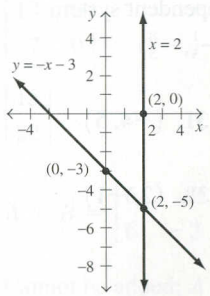

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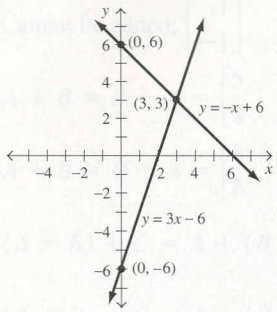

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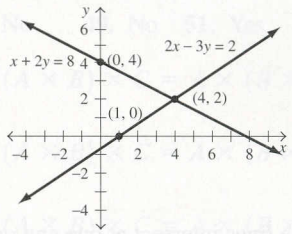

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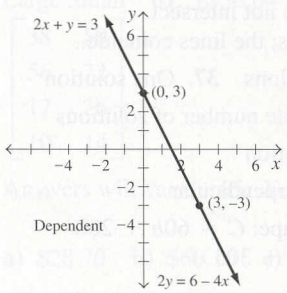

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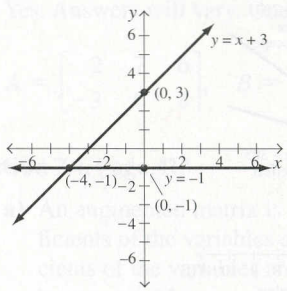

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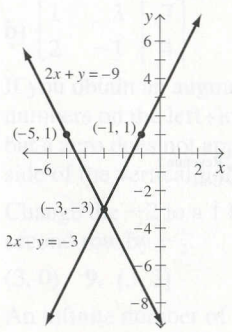

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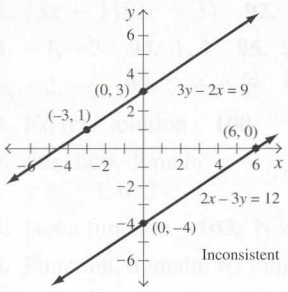

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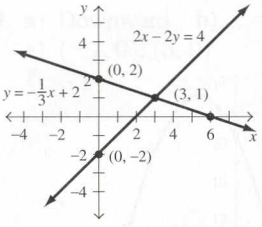


33. a) One unique solution; the lines intersect at one and only one point.
b) No solution; the lines do not intersect.
c) Infinitely many solutions; the lines coincide.

35. An infinite number of solutions **37.** One solution

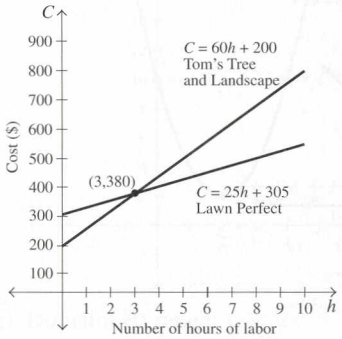
39. No solution **41.** An infinite number of solutions

43. No solution **45.** One solution

47. Not perpendicular **49.** Perpendicular

51. a) Tom's Tree and Landscape: $C = 60h + 200$
Lawn Perfect: $C = 25h + 305$

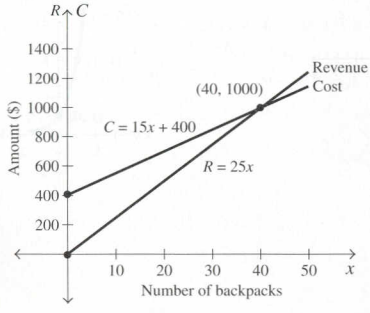
b)



c) 3 hours

53. a) $C = 15x + 400$
$R = 25x$

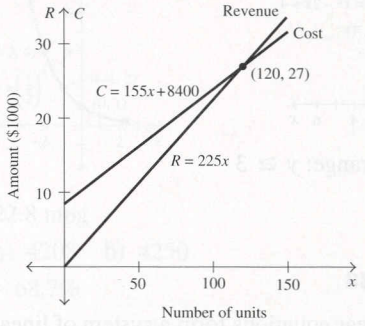
b)



c) 40 backpacks **d)** $P = 10x - 400$
e) Loss of \$100 **f)** 140 backpacks

55. a) $C = 155x + 8400$, $R = 225x$

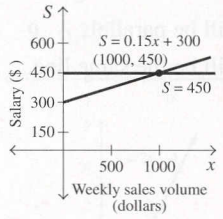
b)



c) 120 units **d)** $P = 70x - 8400$
e) Loss of \$1400 **f)** 138 units

57. a) Job 1: $s = 0.15x + 300$
Job 2: $s = 450$

b)



c) \$1000 sales volume

59. a) One **b)** Three **c)** Six **d)** Ten
e) To find the number of points of intersection for n lines, add $n - 1$ to the number of points of intersection for $n - 1$ lines. For example, for 5 lines there were 10 points of intersection. Therefore, for 6 lines ($n = 6$) there are $10 + (6 - 1) = 15$ points of intersection.

Section 7.2, Page 399

1. Answers will vary.

3. The system is dependent if the same value is obtained on both sides of the equal sign.

5. x, in the first equation **7.** $(-1, 6)$ **9.** $(0, 2)$

11. No solution; inconsistent system **13.** $(-2, \frac{8}{3})$

15. An infinite number of solutions; dependent system

17. $(-3, -6)$ **19.** $(\frac{11}{5}, -\frac{13}{5})$ **21.** $(-\frac{1}{5}, -\frac{8}{5})$

23. No solution; inconsistent system

25. $(2, 4)$ **27.** $(6, 4)$ **29.** $(-2, 0)$ **31.** $(-4, 5)$

33. $(3, 5)$ **35.** $(1, -2)$

37. No solution; inconsistent system **39.** $(2, -1)$

41. $s = 12{,}000 + 0.15p$
$s = 27{,}000 + 0.05p$
\$150,000 annual profit

43. $m + l = 50$
$10.95m + 14.95l = 663.50$
21 medium, 29 large

45. $x + y = 10$
$0.25x + 0.50y = 0.40(10)$
4ℓ of 25%, 6ℓ of 50%

47. $y = 18 + 0.02x$
$y = 24 + 0.015x$
1200 copies

49. $x + y = 20$
$3x + y = 30$
Mix 5 lb of nuts with 15 lb of pretzels.

51. $x + y = 250$
$2x + 5y = 950$
100 student tickets and 150 nonstudent tickets

53. ≈26.7 years after 1981 or in 2007

55. $(\frac{1}{2}, \frac{1}{3})$ **57.–59.** Answers will vary.

Section 7.3, page 408

1. A matrix is a rectangular array of elements.

3. A square matrix contains the same number of rows and columns.

5. 2 **7. a)** Answers will vary. **b)** $\begin{bmatrix} 4 & 9 & -7 \\ 2 & 4 & 9 \end{bmatrix}$

9. a) The number of columns of the first matrix must be the same as the number of rows of the second matrix.
b) 2×3

11. a) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b)** $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

13. $\begin{bmatrix} -4 & 2 \\ 12 & 9 \end{bmatrix}$ **15.** $\begin{bmatrix} 0 & 4 \\ 4 & 4 \\ 5 & -1 \end{bmatrix}$

17. $\begin{bmatrix} 6 & -7 \\ -12 & 4 \end{bmatrix}$ **19.** $\begin{bmatrix} 2 & 11 \\ 16 & 13 \\ -2 & 2 \end{bmatrix}$

21. $\begin{bmatrix} 6 & 4 \\ 10 & 0 \end{bmatrix}$ **23.** $\begin{bmatrix} 0 & 13 \\ 22 & 0 \end{bmatrix}$

25. $\begin{bmatrix} 13 & 0 \\ 7 & 0 \end{bmatrix}$ **27.** $\begin{bmatrix} 4 & 12 \\ 14 & 22 \end{bmatrix}$

29. $\begin{bmatrix} 15 \\ 22 \end{bmatrix}$ **31.** $\begin{bmatrix} 4 & 7 & 6 \\ -2 & 3 & 1 \\ 5 & 1 & 2 \end{bmatrix}$

33. $A + B = \begin{bmatrix} 6 & 2 & 1 \\ 6 & -2 & 4 \end{bmatrix}$; cannot be multiplied

35. Cannot be added; $A \times B = \begin{bmatrix} 26 & 38 \\ 24 & 24 \end{bmatrix}$

37. Cannot be added; $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

39. $A + B = B + A = \begin{bmatrix} 5 & 7 \\ 8 & 4 \end{bmatrix}$

41. $A + B = B + A = \begin{bmatrix} 8 & 0 \\ 6 & -8 \end{bmatrix}$

43. $(A + B) + C = A + (B + C) = \begin{bmatrix} 7 & 10 \\ 6 & 13 \end{bmatrix}$

45. $(A + B) + C = A + (B + C) = \begin{bmatrix} 5 & 5 \\ 7 & -37 \end{bmatrix}$

47. No **49.** No **51.** Yes

53. $(A \times B) \times C = A \times (B \times C) = \begin{bmatrix} 41 & 13 \\ 56 & 16 \end{bmatrix}$

55. $(A \times B) \times C = A \times (B \times C) = \begin{bmatrix} 16 & -10 \\ -24 & 2 \end{bmatrix}$

57. $(A \times B) \times C = A \times (B \times C) = \begin{bmatrix} 17 & 0 \\ -7 & 0 \end{bmatrix}$

59. Large Small **61.** [\$36.04 \$47.52]

$$\begin{bmatrix} 38 & 50 \\ 56 & 72 \\ 17 & 26 \\ 10 & 14 \end{bmatrix}$$

63. Answers will vary. **65.** Yes **67.** False

69. a) \$28.70 **b)** \$60.10 **c)** $\begin{bmatrix} 28.7 & 24.6 \\ 41.3 & 35.7 \\ 69.3 & 60.1 \end{bmatrix}$

71. Yes. Answers will vary. One example is

$$A = \begin{bmatrix} 2 & 7 & 6 \\ -3 & 0 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

Section 7.4, Page 417

1. a) An augmented matrix is a matrix formed with the coefficients of the variables and the constants. The coefficients of the variables are separated from the constants by a vertical bar.
b) $\left[\begin{array}{cc|c} 1 & 3 & 7 \\ 2 & -1 & 4 \end{array}\right]$

3. If you obtain an augmented matrix in which one row of numbers on the left side of the vertical line are all zeroes but a zero does not appear in the same row on the right side of the vertical line, the system is inconsistent.

5. Change the -2 to a 1 by multiplying the numbers in the second row by $-\frac{1}{2}$.

7. (3, 0) **9.** (3, 2)

11. An infinite number of solutions; dependent system

13. $(\frac{7}{2}, -1)$ **15.** $(\frac{1}{2}, -6)$

17. No solution; inconsistent system **19.** (3, 5)

21. 20 small flags and 35 large flags

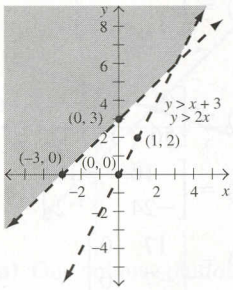
23. Truck driver: $7\frac{1}{9}$ hours; laborer: $9\frac{1}{9}$ hours

25. Nonrefillable pencils: 125; refillable pencils: 75

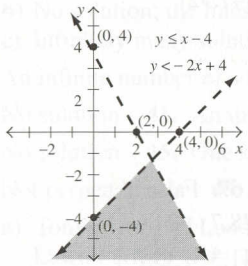
Section 7.5, Page 420

1. The solution set of a system of linear inequalities is the set of points that satisfy all inequalities in the system.

3.



5.



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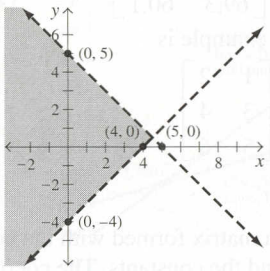

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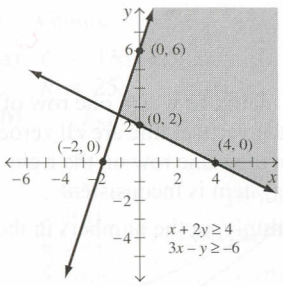

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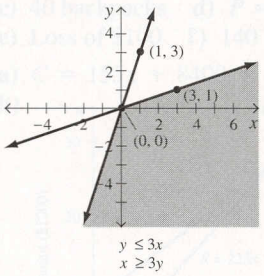

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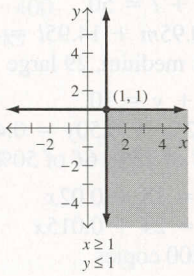

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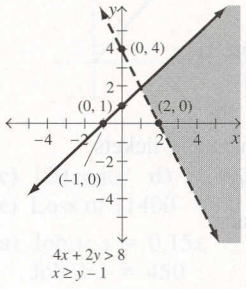

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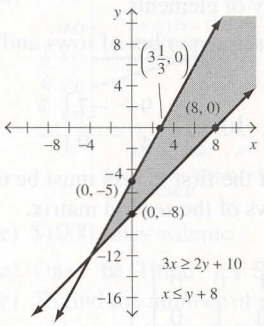
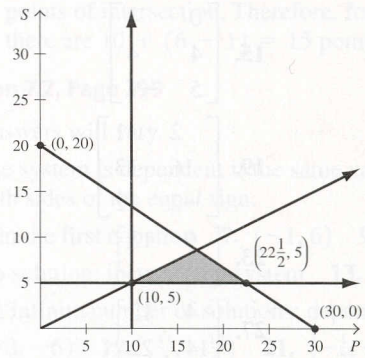


19. a) $P \geq 2S$, $P \geq 10$, $S \geq 5$, $600P + 900S \leq 18{,}000$

b)



c) One example is (15, 6); $14,400 of inventory cost

21. a) No **b)** One example: $x + y > 4$
$x + y < 1$

23. No, every line divides the plane into two half planes, only one of which can be part of the solution.

25. Answers will vary.

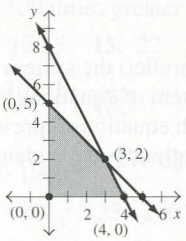
Section 7.6, Page 426

1. Constraints are restrictions that are represented as linear inequalities.

3. Vertices **5.** Answers will vary.

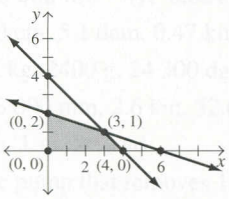
7. Maximum is 30 at (5, 0), minimum is 0 at (0, 0).

9. a)



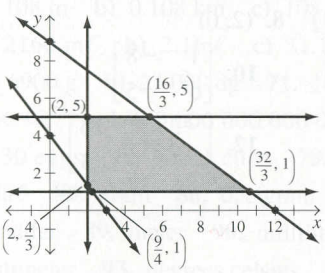
b) Maximum is 23 at (3, 2), minimum is 0 at (0, 0).

11. a)



b) Maximum is 28 at (4, 0), minimum is 0 at (0, 0).

13. a)

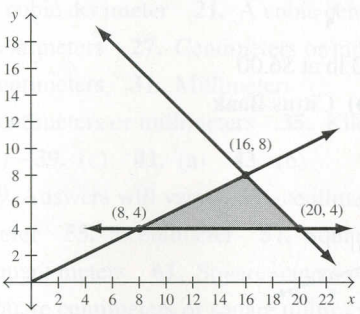


b) Maximum is ≈ 25.12 at $(\frac{32}{3}, 1)$, minimum is 6.6 at $(\frac{9}{4}, 1)$ or $(2, \frac{4}{3})$.

15. a) $x + y \le 24, x \ge 2y, y \ge 4, x \ge 0, y \ge 0$

b) $P = 0.35x + 0.50y$

c)



d) (8, 4), (16, 8), (20, 4)

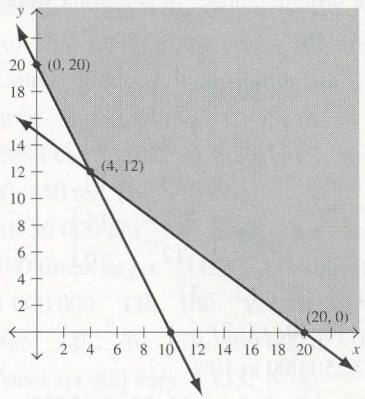
e) 16 rolls of Kodak and 8 rolls of Fuji

f) \$9.60

17. a) $3x + 4y \ge 60, 10x + 5y \ge 100, x \ge 0, y \ge 0$

b) $C = 28x + 33y$

c)



d) (0, 20), (4, 12), (20, 0)

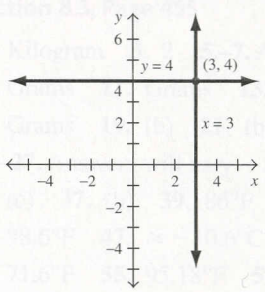
e) 4 hours for Machine I and 12 hours for Machine II

f) \$508

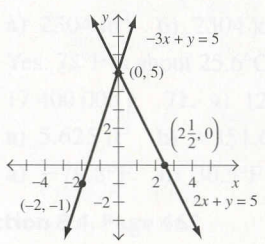
19. Three car seats and seven strollers, \$320

Review Exercises, Page 428

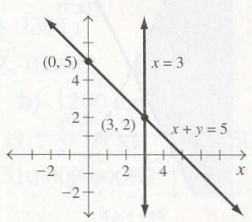
1.



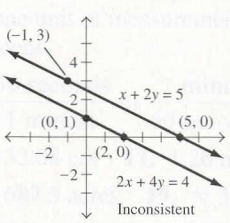
2.



3.



4.



5. An infinite number of solutions **6.** No solution

7. One solution **8.** One solution **9.** $(-9, 3)$

10. $(-1, -5)$ **11.** $(-2, -8)$

12. No solution; inconsistent **13.** $(3, 1)$ **14.** $(-7, 16)$

15. $(4, -2)$ **16.** An infinite number of solutions; dependent

17. $(30, -15)$ **18.** $(2, 0)$

19. $\begin{bmatrix} -1 & -8 \\ 8 & 7 \end{bmatrix}$ **20.** $\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$

21. $\begin{bmatrix} 2 & -6 \\ 4 & 8 \end{bmatrix}$ **22.** $\begin{bmatrix} 8 & 9 \\ -14 & -1 \end{bmatrix}$

23. $\begin{bmatrix} -20 & -14 \\ 20 & 2 \end{bmatrix}$ **24.** $\begin{bmatrix} -12 & -14 \\ 12 & -6 \end{bmatrix}$

25. $(2, 2)$ **26.** $(-2, 2)$ **27.** $(3, -3)$

28. $(1, 0)$ **29.** $(\frac{12}{11}, \frac{7}{11})$ **30.** $(1, 2)$

31. \$350,000 at 8%, \$250,000 at 10%

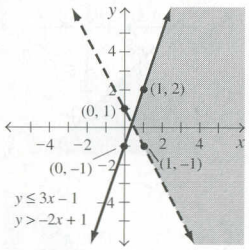
32. Mix $83\frac{1}{3}\ell$ of 80% acid solution with $16\frac{2}{3}\ell$ of 50% acid solution.

33. \$500 salary, 4% commission rate

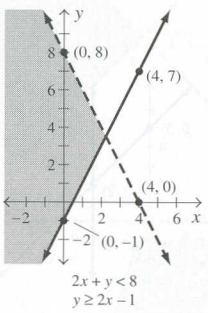
34. a) 32.5 months **b)** Model 6070B

35. a) 3 hr **b)** All-Day parking lot

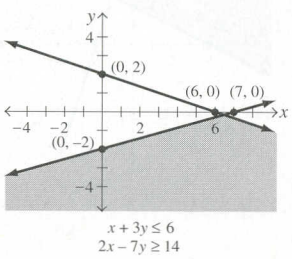
36.



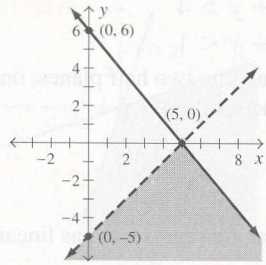
37.



38.



39.

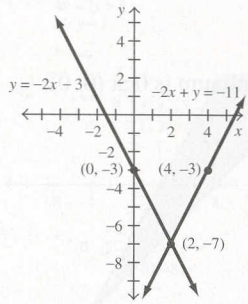


40. The maximum is 54 at (9, 0).

Chapter Test, Page 429

1. If the lines do not intersect (are parallel) the system of equations is inconsistent. The system of equations is consistent if the lines intersect. If both equations represent the same line, then the system of equations is dependent.

2.



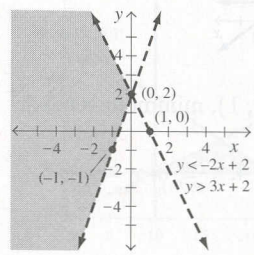
3. One solution **4.** $(2, -3)$ **5.** $(-2, -3)$

6. $(3, -1)$ **7.** $(-1, 3)$ **8.** $(2, 0)$

9. $(-2, 2)$ **10.** $\begin{bmatrix} 1 & -8 \\ 6 & 5 \end{bmatrix}$

11. $\begin{bmatrix} 7 & -12 \\ -2 & 7 \end{bmatrix}$ **12.** $\begin{bmatrix} -27 & -16 \\ 14 & 3 \end{bmatrix}$

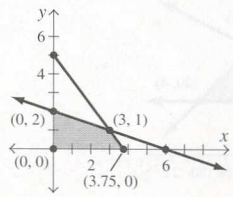
13.



14. 20 lb at \$7.50, 10 lb at \$6.00

15. a) 40 checks **b)** Citrus Bank

16. a)



b) Maximum is 18.75 at (3.75, 0), minimum is 0 at (0, 0).

Chapter 8

Section 8.1, Page 437

1. The metric system

3. It is the standard of measurement accepted worldwide. There is only one basic unit of measurement for each quantity. It is based on the number 10, which makes many calculations easier than the U.S. customary system.

5. a) Answers will vary. **b)** 0.007 146 km **c)** 30 800 dm

7. Answers will vary.

9. a) 100 times greater **b)** 100 dm **c)** 0.01 dam

11. 2 **13.** 5 **15.** 22 **17.** (d) **19.** (c) **21.** (f)

23. a) 10 **b)** $\frac{1}{100}$ **c)** $\frac{1}{1000}$ **d)** $\frac{1}{10}$ **e)** 1000 **f)** 100

25. mg; $\frac{1}{1000}$ g **27.** dg; $\frac{1}{10}$ g **29.** hg; 100 g

31. 320 000 g **33.** 200 **35.** 9.5 **37.** 0.024 26

39. 0.040 36 **41.** 13 400 **43.** 92 500 g

45. 895 000 mℓ **47.** 0.024 hg **49.** 4.0302 daℓ

51. 590 cm, 5.1 dam, 0.47 km

53. 2.2 kg, 2400 g, 24 300 dg

55. 203 000 mm, 2.6 km, 52.6 hm

57. Jim, 1 m > 1 yd

59. The pump that removes 1 daℓ per minute

61. a) 346 cm **b)** 3460 mm

63. a) 108 m **b)** 0.108 km **c)** 108 000 mm **65.** 3.2 km

67. a) 2160 mℓ **b)** 2.16 ℓ **c)** $1.13 per liter

69. a) 6900 g **b)** 23 000 dg **71.** 1000

73. $1 \times 10^{24} = 1\,000\,000\,000\,000\,000\,000\,000\,000$

75. ≈ 30 eggs **77.** ≈ 4.1 cups **79.** 5 dam **81.** 6 mg

83. 2 daℓ **85.** gram **86.** decigram **87.** liter

88. dekaliter **89.** meter **90.** milliliter **91.** kilometer

92. centimeter **93.** degrees celsius **94.** hectogram

Section 8.2, Page 446

1. Volume **3.** Area **5.** Volume **7.** Volume **9.** Area

11. Length **13.–17.** Answers will vary.

19. A cubic decimeter **21.** A cubic centimeter **23.** Area

25. Centimeters **27.** Centimeters or millimeters

29. Centimeters **31.** Millimeters

33. Centimeters or millimeters **35.** Kilometers

37. (c) **39.** (c) **41.** (a) **43.** (b)

45.–49. Answers will vary. **51.** centimeter, kilometer

53. meter **55.** a centimeter **57.** Square centimeters

59. Square meters **61.** Square meters or hectares

63. Square centimeters or square millimeters

65. Square kilometers or hectares

67. (b) **69.** (a) **71.** (c) **73.** (c)

75.–79. Answers will vary. **81.** Kiloliters

83. Milliliters **85.** Liters **87.** Cubic meters

89. Liters or milliliters **91.** (c) **93.** (c) **95.** (a)

97. (a) **99. b)** 152 561 cm^3 **101.** ≈ 0.20 m^3

103. Longer side = 4 cm, shorter side = 2.2 cm, area = 8.8 cm^2

105. 2984 cm^2 **107. a)** 5.25 km^2 **b)** 525 ha

109. a) 450 m^3 **b)** 450 kℓ

111. a) 56 000 cm^3 **b)** 56 000 mℓ **c)** 56 ℓ

113. 100 times larger **115.** 1000 times larger

117. 1 000 000 **119.** 100 **121.** 0.001 **123.** 1 000 000

125. 435 **127.** 76 **129.** 600 000 dℓ

131. Answers will vary. **133.** 6700

135. a) 4,014,489,600 sq in. **b)** Answers will vary.

137. Answers will vary.

138. a) 100 cm longer **b)** 3 times longer

139. a) Answers will vary. The average use is 5150.7 ℓ/day.

b) Answers will vary. The average use is 493.2 ℓ/day.

Section 8.3, Page 455

1. Kilogram **3.** 2 **5.–7.** Answers will vary.

9. Grams **11.** Grams **13.** Grams **15.** Kilograms

17. Grams **19.** (b) **21.** (b) **23.** (b)

25.–27. Answers will vary. **29.** (c) **31.** (b) **33.** (b)

35. (c) **37.** (b) **39.** 86°F **41.** ≈ 33.3°C **43.** ≈ 82.2°C

45. 98.6°F **47.** ≈ −10.6°C **49.** 113°F **51.** ≈ −28.9°C

53. 71.6°F **55.** 95.18°F **57.** 64.04°F–74.30°F

59. $4.34 **61.** 444 g

63. a) 2304 m^3 **b)** 2304 kℓ **c)** 2304 t

65. Yes: 78°F is about 25.6°C. **67.** 0.0042

69. 17 400 000 g **71. a)** 1200 g **b)** 1200 cm^3

73. a) 5.625 ft^3 **b)** ≈351.6 lb **c)** ≈ 42.4 gal **74.** 1500 g

75. a) −79.8°F **b)** 36.5°F **c)** 510 000 000°C

Section 8.4, Page 465

1. Dimensional analysis is a procedure used to convert from one unit of measurement to a different unit of measurement.

3. $\frac{60 \text{ seconds}}{1 \text{ minute}}$ or $\frac{1 \text{ minute}}{60 \text{ seconds}}$ **5.** $\frac{1 \text{ ft}}{30 \text{ cm}}$ **7.** $\frac{3.8\ \ell}{1 \text{ gal}}$

9. 132.08 cm **11.** 1.26 m **13.** 12 m^2 **15.** 62.4 km

17. 1687.5 acres **19.** ≈ 33.19 pints **21.** 1.52 fl oz

23. 54 kg **25.** 28 grams **27.** 0.45 kilogram

29. 2.54 centimeters, 1.6 kilometers **31.** 9 meters

33. ≈ 561.11 yd **35.** ≈ 1146.67 ft **37.** ≈ 53.13 mph
39. 43.2 m^2 **41.** ≈ 14.29 oz **43.** 240 mℓ **45.** 360 m^3
47. $0.495 per pound **49.** ≈ 9078.95 gal
51. a) ≈ 50.91 kg **b)** ≈ 113.13 lb
53. a) −8460 cm **b)** −84.6 m
55. a) 10.89 ft^2 **b)** 35.937 ft^3 **57.** 25.2 mg
59. 6840 mg, or 6.84 g **61. a)** 25 mg **b)** 900 mg
63. a) 289.2 m **b)** 76 500 t **c)** 44.8 kph
65. a) ≈ 41.1 yd **b)** 231,337.5 mi
c) 27.5 mi **d)** 2300°F
e) 209.375 mph **f)** 65,520 lb
g) 5 yd × 20 yd **h)** ≈ 45,104.21 gal/min
i) ≈ 16,733.68 gal/min **j)** 52.1 yd
k) ≈ 9.33 yd **l)** 1,406,160 lb
m) 235,871.11 lb **n)** −419.8°F
67. 7.8 lb **69.** A meter **70.** A kilogram **71.** A hectare
72. A liter **73.** A tonne **74.** A decimeter **75.** wonton
76. 1 microscope **77.** 1 kilohurtz **78.** 1 pound cake
79. 1 megaphone **80.** 2 megacycles **81.** 2 kilomockingbird
82. 1 decacards **83.** 1 decoration **84.** 1 microfiche

Review Exercises, Page 469

1. $\frac{1}{100}$ of base unit **2.** 1000 × base unit
3. $\frac{1}{1000}$ of base unit **4.** 100 × base unit
5. 10 times base unit **6.** $\frac{1}{10}$ of base unit **7.** 0.20 g
8. 320 cℓ **9.** 0.004 mm **10.** 1 kg **11.** 4620 ℓ
12. 19 260 dg **13.** 3000 mℓ, 14 630 cℓ, 2.67 kℓ
14. 0.047 km, 47 000 cm, 4700 m **15.** Centimeters
16. Grams **17.** Degrees Celsius
18. Millimeters or centimeters **19.** Square meters
20. Milliliters or cubic centimeters **21.** Millimeters
22. Kilograms or tonnes **23.** Kilometers
24. Meters or centimeters **25. a)** and **b)** Answers will vary.
26. a) and **b)** Answers will vary. **27.** (c) **28.** (b) **29.** (c)
30. (a) **31.** (a) **32.** (b) **33.** 2.5 t **34.** 6 300 000 g
35. 64.4°F **36.** 20°C **37.** ≈ −21.1°C **38.** 102.2°F
39. $l = 4$ cm, $w = 1.6$ cm, $A = 6.4$ cm^2
40. $r = 1.5$ cm, $A \approx 7.07$ cm^2
41. a) 80 m^3 **b)** 80 000 kg
42. a) 660 m^2 **b)** 0.000 66 km^2
43. a) 96 000 cm^3 **b)** 0.096 m^3
c) 96 000 mℓ **d)** 0.096 kℓ
44. 10,000 times larger **45.** ≈ 7.87 in. **46.** ≈ 233.33 lb
47. 74.7 m **48.** ≈ 111.11 yd **49.** 72 kph **50.** ≈ 42.11 qt
51. 57 ℓ **52.** ≈ 52.63 yd^3 **53.** ≈ 12.77 $in.^2$ **54.** 3.8 ℓ
55. 11.4 m^3 **56.** 99.2 km **57.** 0.9 ft **58.** 82.55 mm
59. a) 1050 kg **b)** ≈ 2333.33 lb **60.** 32.4 m^2
61. a) 190 kℓ **b)** 190 000 kg
62. a) 56 kph **b)** 56 000 meters per hour
63. a) 252 ℓ **b)** 252 kg
64. $1.58 per pound

Chapter Test, Page 471

1. 0.204 daℓ **2.** 123 000 000 mm **3.** 100 times greater
4. 2.4 km **5.** (b) **6.** (a) **7.** (c) **8.** (c) **9.** (b)
10. 10,000 times greater **11.** 1,000,000,000 times greater
12. 1148.08 cm **13.** ≈ 166.67 yd **14.** ≈ −23.33°C
15. 68°F
16. 360 cm or 365.76 cm, depending on which conversion factor you used
17. a) 3200 m^3 **b)** 3 200 000 ℓ (or 3200 kℓ)
c) 3 200 000 kg
18. $245

Chapter 9

Section 9.1, Page 482

1. a) Undefined terms, definitions, postulates (axioms), and theorems
b) First, Euclid introduced undefined terms. Second, he introduced certain definitions. Third, he stated primitive propositions called postulates about the undefined terms and definitions. Fourth, he proved, using deductive reasoning, other propositions called theorems.
3. Two lines in the same plane that do not intersect are parallel lines.
5. Two angles in the same plane are adjacent angles when they have a common vertex and a common side but no common interior points.
7. Two angles the sum of whose measure is 90° are called complementary angles.
9. An angle whose measure is greater than 90° but less than 180° is an obtuse angle.
11. An angle whose measure is 90° is a right angle.
13. Half line, $\overrightarrow{AB}$ **15.** Line segment, $\overline{AB}$ **17.** Line, $\overleftrightarrow{AB}$
19. Open line segment, $\mathring{AB}$ **21.** $\overline{BD}$ **23.** $\mathring{BD}$
25. $\{B, F\}$ **27.** $\{C\}$ **29.** $\overline{BC}$ **31.** $\overrightarrow{BC}$ **33.** $\emptyset$
35. $\overrightarrow{BC}$ **37.** $\measuredangle ABE$ **39.** $\measuredangle EBC$ **41.** $\mathring{AC}$ **43.** $\mathring{BE}$
45. Obtuse **47.** Straight **49.** Right **51.** None of these
53. 71° **55.** $57\frac{1}{4}°$ **57.** 25.3° **59.** 89° **61.** 159.5°
63. $136\frac{2}{7}°$ **65.** (d) **67.** (c) **69.** (e)

71. $m\measuredangle 1 = 47°$, $m\measuredangle 2 = 43°$

73. 134° and 46°

75. Angles 3, 4, and 7 each measure 125°; angles 1, 2, 5, and 6 each measure 55°.

77. Angles 2, 5, and 6 each measure 25°; angles 1, 3, 4, and 7 each measure 155°.

79. $m\measuredangle 1 = 70°$, $m\measuredangle 2 = 20°$

81. $m\measuredangle 1 = 33°$, $m\measuredangle 2 = 57°$

83. $m\measuredangle 1 = 115°$, $m\measuredangle 2 = 65°$

85. $m\measuredangle 1 = 29°$, $m\measuredangle 2 = 151°$

87. a) An infinite number **b)** An infinite number

89. An infinite number

For Exercises 91–97, the answers given are one of many possible answers.

91. Plane ABG and plane JCD

93. $\overleftrightarrow{BG}$ and $\overleftrightarrow{DG}$

95. Plane $AGB \cap$ plane $ABC \cap$ plane $BCD = \{B\}$

97. $\overleftrightarrow{BC} \cap$ plane $ABG = \{B\}$

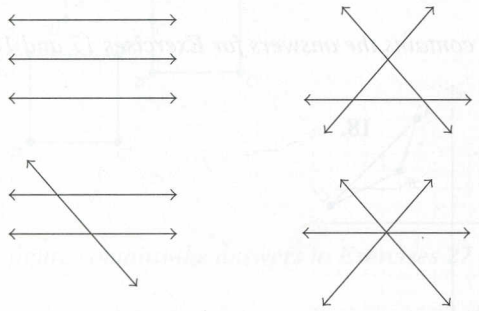
99. Always true. If any two lines are parallel to a third line, then they must be parallel to each other.

101. Sometimes true. Vertical angles are only complementary when each is equal to 45°.

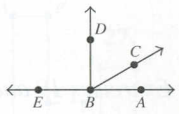
103. Sometimes true. Alternate interior angles are only complementary when each is equal to 45°.

105. No. Line m and line n may intersect.

107.



109. a)



Other answers are possible.

b) 30° **c)** 60° **d)** 90°

Section 9.2, Page 491

1. A polygon is a closed figure in a plane determined by three or more straight line segments.

3. The different types of triangles are acute, obtuse, right, isosceles, equilateral, and scalene. Descriptions will vary.

5. If the corresponding sides of two similar figures are the same length, the figures are congruent figures.

7. a) Rectangle **b)** Not regular

9. a) Hexagon **b)** Regular

11. a) Rhombus **b)** Not regular

13. a) Octagon **b)** Not regular

15. a) Scalene **b)** Right

17. a) Isosceles **b)** Obtuse

19. a) Equilateral **b)** Acute

21. a) Scalene **b)** Obtuse

23. Parallelogram **25.** Rhombus **27.** Trapezoid

29. 17° **31.** 150°

33. $m\measuredangle 1 = 50°$, $m\measuredangle 2 = 63°$, $m\measuredangle 3 = 67°$, $m\measuredangle 4 = 67°$, $m\measuredangle 5 = 50°$, $m\measuredangle 6 = 113°$, $m\measuredangle 7 = 50°$, $m\measuredangle 8 = 130°$, $m\measuredangle 9 = 67°$, $m\measuredangle 10 = 113°$, $m\measuredangle 11 = 130°$, $m\measuredangle 12 = 50°$

35. 540° **37.** 720° **39.** 3240° **41. a)** 60° **b)** 120°

43. a) 135° **b)** 45° **45. a)** 150° **b)** 30°

47. $x = 6, y = \frac{16}{5}$ **49.** $x = \frac{12}{5}, y = \frac{15}{2}$

51. $x = 1.2, y = 0.625$ **53.** 6 **55.** $\frac{20}{3}$ **57.** 14 **59.** 28

61. 28° **63.** 8 **65.** 16 **67.** 70° **69.** 55° **71.** 35°

73. 70 ft **75. a)** 246.25 mi **b)** 270.875 mi

77. $\overline{D'E'} = 4, \overline{E'F'} = 5, \overline{D'F'} = 3$

79. a) $m\measuredangle HMF = m\measuredangle TMB$, $m\measuredangle HFM = m\measuredangle TBM$, $m\measuredangle MHF = m\measuredangle MTB$

b) 44 ft

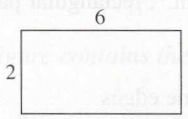
Section 9.3, Page 503

Throughout this section, we used the π key on a scientific calculator to determine answers in calculations involving π. If you use 3.14 for π, your answers may vary slightly.

1. a) Answers will vary.

b) Answers will vary.

c)



The area of this rectangle is 12 square units. The perimeter of this rectangle is 16 units.

3. a) To determine the number of square inches, multiply the number of square feet by 144.

b) To determine the number of square feet, divide the number of square inches by 144.

5. 35 in.^{2} **7.** 17.5 cm^2

9. Area = 105 ft^2; perimeter = 44 ft

11. Area = 6000 cm^2; perimeter = 654 cm

13. Area = 288 in.^{2}; perimeter = 74 in.

15. $\approx$ 153.94 in.^{2}, 43.98 in.

17. ≈ 63.62 ft^2, 28.27 ft

19. a) 9 in. **b)** 36 in. **c)** 54 in.^{2}

21. a) 26 cm **b)** 60 cm **c)** 120 cm^2

23. ≈ 21.99 cm^2 **25.** 8 in.^{2} **27.** ≈ 65.73 in.^{2}

29. ≈ 114.90 ft^2 **31.** ≈ 41.20 in.^{2} **33.** ≈ 11.89 yd^2

35. 132.3 ft^2 **37.** 234,000 cm^2 **39.** 0.1075 m^2

41. a) \$3239.50 **b)** \$4889.50 **43.** \$1700 **45.** \$2908.80

47. \$38.93 **49. a)** 177.1 m^2 **b)** 0.01771 hectare

51. $\approx$103.94 ft **53.** 40 ft

55. a) $A = s^2$ **b)** $A = 4s^2$ **c)** Four times larger

57. 24 cm^2 **59.** Answers will vary.

Section 9.4, Page 515

Throughout this section, we used the π key on a scientific calculator to determine answers in calculations involving π. If you use 3.14 for π, your answers may vary slightly.

1. Volume is a measure of the capacity of a figure.

3. A polyhedron is a closed surface formed by the union of polygonal regions. A regular polyhedron is one whose faces are all regular polygons of the same size and shape.

5. Answers will vary. **7.** 27 ft^3 **9.** 150.80 in.^{3}

11. 131.95 cm^3 **13.** 2400 in.^{3} **15.** 381.70 cm^3

17. 524.33 cm^3 **19.** 106.67 in.^{3} **21.** 59.43 m^3

23. 30.49 ft^3 **25.** 31.42 m^3 **27.** 24 ft^3 **29.** 189 ft^3

31. ≈ 5.67 yd^3 **33.** 5,900,000 cm^3 **35.** 3 m^3

37. a) 28,750 in.^{3} **b)** ≈ 16.64 ft^3 **39.** ≈ 2.50 qt

41. a) The container with the larger diameter holds more.
b) ≈ 188.50 in.^{3}

43. 82,944,000 ft^3 **45.** ≈ 283.04 in.^{3}

47. a) ≈ 323.98 in.^{3} **b)** ≈ 0.19 ft^3

49. a) Round pan base ≈ 63.62 in.^{2}; rectangular pan base $= 63$ in.^{2}
b) Round pan volume ≈ 127.24 in.^{3}; rectangular pan volume $= 126$ in.^{3}
c) Round pan

51. a) 4320 in.^{3} **b)** 2.5 ft^3 **53.** Nine edges

55. Six vertices **57.** Fourteen edges **59.** $\approx 21.46\%$

61. a)–e) Answers will vary.
f) If we double the radius of a sphere, the new volume will be eight times the original volume.

63. a) Answers will vary.
b) $V_1 = a^3$; $V_2 = a^2b$; $V_3 = a^2b$; $V_4 = ab^2$; $V_5 = a^2b$; $V_6 = ab^2$; $V_7 = b^3$
c) ab^2

64. a) 330 in.^{3} **b)** ≈ 300.84 in.^{3}

Section 9.5, Page 532

1. The act of moving a geometric figure from some starting position to some ending position without altering its shape or size is called rigid motion. The four main rigid motions studied in this section are reflections, translations, rotations, and glide reflections.

3. A reflection is a rigid motion that moves a figure to a new position that is a mirror image of the figure in the starting position.

5. A translation is a rigid motion that moves a figure by sliding it along a straight line segment in the plane.

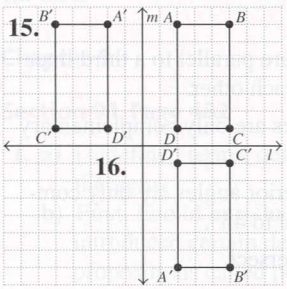
7. A rotation is a rigid motion performed by rotating a figure in the plane about a specific point.

9. A glide reflection is a rigid motion formed by performing a translation (or glide) followed by a reflection.

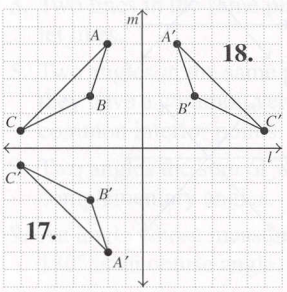
11. A geometric figure is said to have reflective symmetry if the positions of a figure before and after a reflection are identical (except for vertex labels).

13. A tessellation is a pattern consisting of the repeated use of the same geometric figures to entirely cover a plane, leaving no gaps.

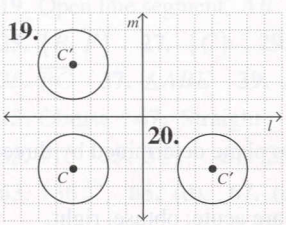
This figure contains the answers for Exercises 15 and 16.



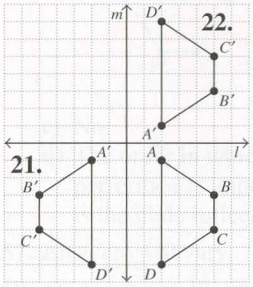
This figure contains the answers for Exercises 17 and 18.



This figure contains the answers for Exercises 19 and 20.



This figure contains the answers for Exercises 21 and 22.



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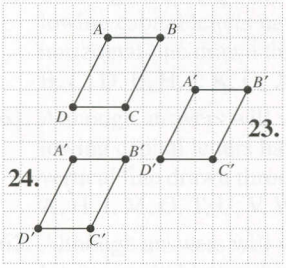
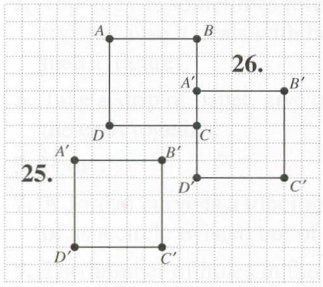


This figure contains the answers to Exercises 25 and 26.



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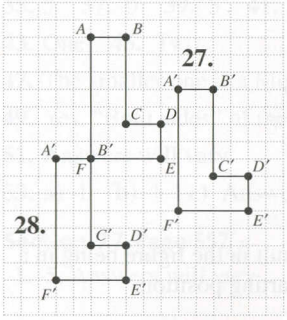
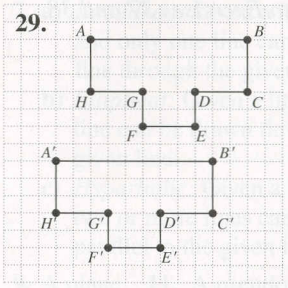
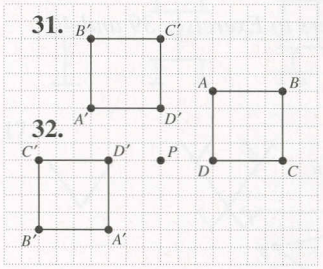
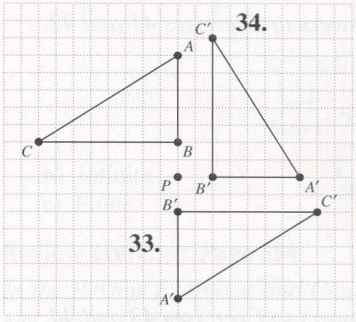




This figure contains the answers to Exercises 31 and 32.



This figure contains the answers to Exercises 33 and 34.



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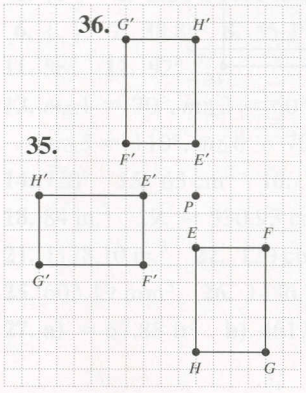

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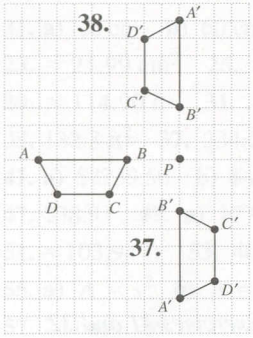
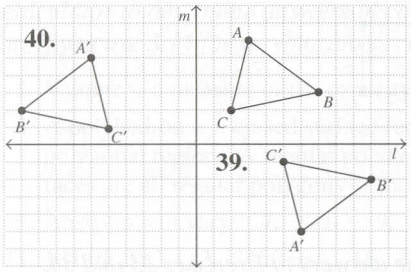


This figure contains the answers to Exercises 39 and 40.



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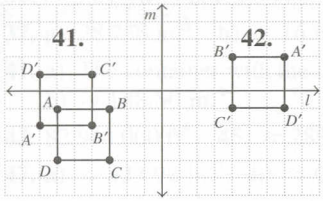

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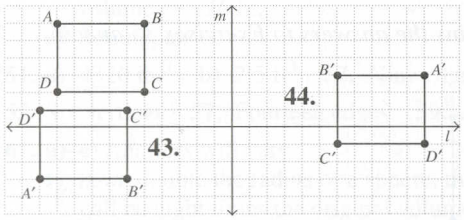

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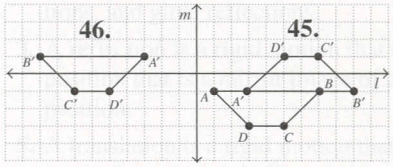
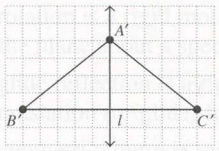
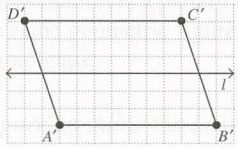


47. a)



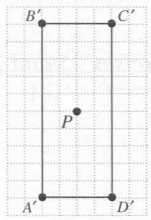
b) Yes
c) Yes

49. a)

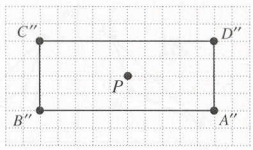


b) No
c) No

51. a)

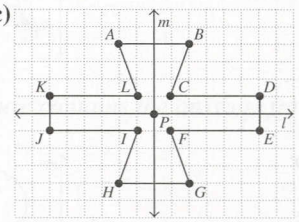


b) No
c) No
d)



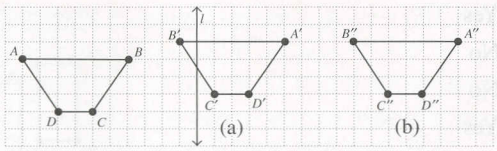
e) Yes
f) Yes

53. a)–c)



d) No. Any 90° rotation will result in the figure being in a different position than the starting position.

55. a)–b)



c) No.

d) The order in which the translation and the reflection are performed is important. The figure obtained in part (a) is the glide reflection.

57. Answers will vary.

59. a) Answers will vary.

b) A regular pentagon cannot be used as a tessellating shape.

60. Although answers will vary depending on the font, the following capital letters have reflective symmetry about a horizontal line drawn through the center of the letter: B, C, D, E, H, I, K, O, X.

61. Although answers will vary depending on the font, the following capital letters have reflective symmetry about a vertical line drawn through the center of the letter: A, H, I, M, O, T, U, V, W, X, Y.

62. Although answers will vary depending on the font, the following capital letters have 180° rotational symmetry about a point in the center of the letter: H, I, O, S, X, Z.

Section 9.6, Page 542

1. Topology is sometimes referred to as "rubber sheet geometry" because it deals with bending and stretching of geometric figures.

3. Take a strip of paper, give one end a half twist, and tape the ends together.

5. Four

7. A Jordan curve is a topological object that can be thought of as a circle twisted out of shape.

9. The number of holes in the object determines the genus of an object.

11.–19. Answers will vary. **21.** Outside **23.** Outside

25. Outside **27.** Inside **29.** 1 **31.** 1 **33.** Larger than 5

35. 5 **37.** 0 **39.** 5 **41. a)–d)** Answers will vary.

43. One **45.** Two

47. The smaller one is a Möbius strip; the larger one is not.

49. Yes. "Both sides" of the belt experience wear.

51. Answers will vary.

53. a) 1 **b)** 1 **c)** Answers will vary.

Section 9.7, Page 552

1.–5. Answers will vary.

7. a) *Euclidean:* Given a line and a point not on the line, one and only one line can be drawn parallel to the given line through the given point.

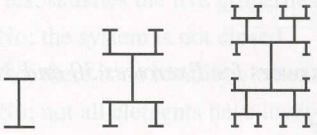
b) *Elliptical:* Given a line and a point not on the line, no line can be drawn through the given point parallel to the given line.

c) *Hyperbolic:* Given a line and a point not on the line, two or more lines can be drawn through the given point parallel to the given line.

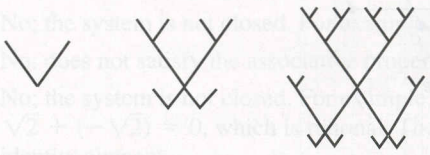
9. A plane **11.** A pseudosphere

13. Spherical: elliptical geometry; flat: Euclidean geometry; saddle-shaped: hyperbolic geometry

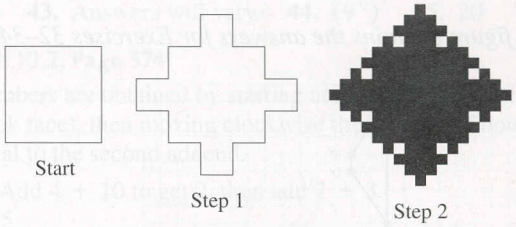
15.



17.



19. a)



b) Infinite

c) Finite

Review Exercises, Page 554

In the Review Exercises and Chapter Test questions, the π key on the calculator is used to determine answers in calculations involving π. If you use 3.14 for π, your answers may vary slightly.

1. $\{F\}$ **2.** ΔBFC **3.** $\overleftrightarrow{BC}$ **4.** $\overleftrightarrow{BH}$ **5.** $\{F\}$

6. $\{\ \}$ **7.** 38.8° **8.** 55.3° **9.** 10.2 in. **10.** 2 in.

11. 58° **12.** 92°

13. $m\measuredangle 1 = 70°$, $m\measuredangle 2 = 60°$, $m\measuredangle 3 = 120°$, $m\measuredangle 4 = 70°$, $m\measuredangle 5 = 110°$, $m\measuredangle 6 = 70°$

14. 720° **15.** 63 cm^2 **16.** 35 $in.^2$ **17.** 13 $in.^2$

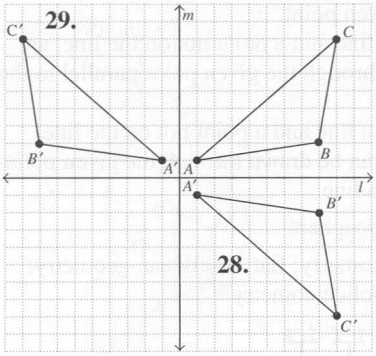
18. 84 $in.^2$ **19.** $\approx$ 530.93 cm^2 **20.** \$616

21. 1178.10 $in.^3$ **22.** 120 cm^3 **23.** 28 ft^3 **24.** 432 m^3

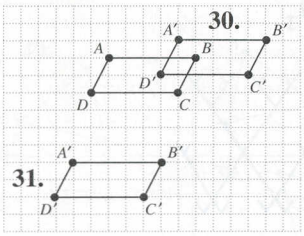
25. 603.19 mm^3 **26.** 1436.76 ft^3

27. a) $\approx$ 67.88 ft^3 **b)** 4617.5 lb; yes **c)** $\approx$ 511.14 gal

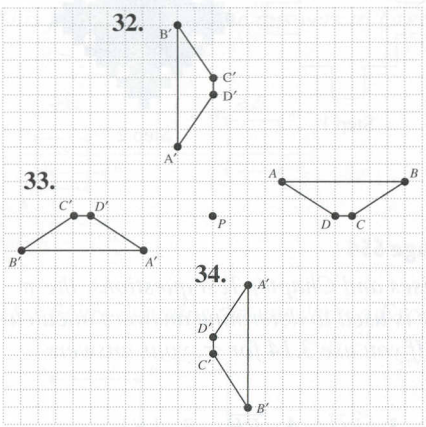
This figure contains the answers for Exercises 28 and 29.



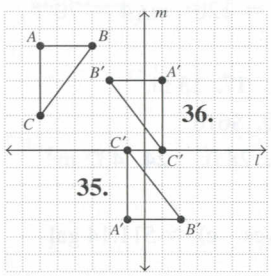
This figure contains the answers for Exercises 30 and 31.



This figure contains the answers for Exercises 32–34.



This figure contains the answers for Exercises 35 and 36.



37. Yes

38. No

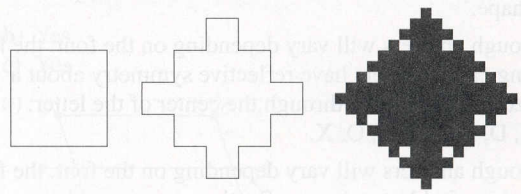
39. No

40. Yes

41. 1 **42.** Answers will vary. **43.** Outside

44. Euclidean: Given a line and a point not on the line, one and only one line can be drawn parallel to the given line through the given point. Elliptical: Given a line and a point not on the line, no line can be drawn through the given point parallel to the given line. Hyperbolic: Given a line and a point not on the line, two or more lines can be drawn through the given point parallel to the given line.

45.



Chapter Test, Page 557

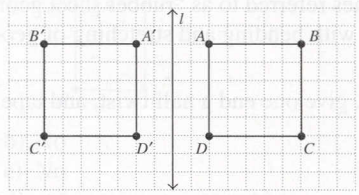
1. $\overleftrightarrow{EF}$ **2.** ΔBCD **3.** $\{D\}$ **4.** $\overleftrightarrow{AC}$ **5.** 53.1°

6. 78.5° **7.** 64° **8.** 1080° **9.** ≈ 2.69 cm

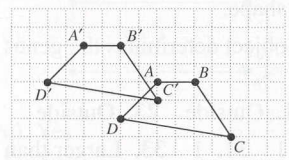
10. a) 12 in. **b)** 30 in. **c)** 30 in.^{2}

11. ≈ 2144.66 cm^3 **12.** ≈ 42.14 yd^3 **13.** 112 ft^3

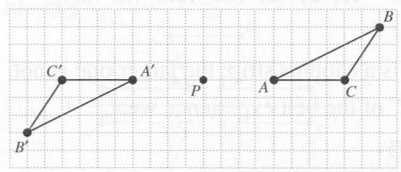
14.



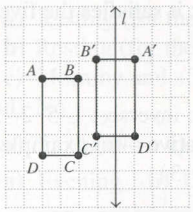
15.



16.



17.



18. a) No **b)** Yes

19. A surface with one side and one edge

20. Answers will vary.

21. Euclidean: Given a line and a point not on the line, one and only one line can be drawn parallel to the given line through the given point. Elliptical: Given a line and a point not on the line, no line can be drawn through the given point parallel to the given line. Hyperbolic: Given a line and a point not on the line, two or more lines can be drawn through the given point parallel to the given line.

Chapter 10

Section 10.1, Page 566

1. A binary operation is an operation, or rule, that can be performed on two and only two elements of a set. The result is a single element.

3. a) When we add two numbers, the sum is one number: $4 + 5 = 9$.

b) When we subtract two numbers, the difference is one number: $5 - 4 = 1$.

c) When we multiply two numbers, the product is one number: $5 \times 4 = 20$.

d) When we divide two numbers, the quotient is one number: $20 \div 5 = 4$.

5. A mathematical system is a commutative group if all five of the following conditions hold.

1. The set of elements is closed under the given operation.
2. An identity element exists for the set.
3. Every element in the set has an inverse.
4. The set of elements is associative under the given operation.
5. The set of elements is commutative under the given operation.

7. If a binary operation is performed on any two elements of a set and the result is an element of the set, then that set is *closed* under the given binary operation. For all integers a and b, $a + b$ is an integer. Therefore, the set of integers is closed under the operation of addition.

9. When a binary operation is performed on two elements in a set and the result is the identity element for the binary operation, then each element is said to be the *inverse* of the other. For the set of rational numbers, the additive inverse of 2 is -2 since $2 + (-2) = 0$ and the multiplicative inverse of 2 is $\frac{1}{2}$ since $2 \times \frac{1}{2} = 1$.

11. No; every commutative group is also a group.

13. d); The commutative property need not apply.

15. $(a + b) + c = a + (b + c)$, for any elements a, b, and c; $(3 + 4) + 5 = 3 + (4 + 5)$

17. $a \cdot b = b \cdot a$ for any elements a and b; $2 \cdot 3 = 3 \cdot 2$

19. $4 \div 2 \neq 2 \div 4$

21.
$$(6 - 4) - 1 \neq 6 - (4 - 1)$$
$$2 - 1 \neq 6 - 3$$
$$1 \neq 3$$

23. No; there is no identity element.

25. Yes; satisfies the five properties needed.

27. No; the system is not closed.

29. No; there is no identity element.

31. No; not all elements have inverses.

33. Yes; satisfies the four properties needed.

35. No; the system is not closed. For example, $\frac{1}{0}$ is undefined.

37. No; does not satisfy the associative property.

39. No; the system is not closed. For example, $\sqrt{2} + (-\sqrt{2}) = 0$, which is rational. There is also no identity element.

41. Yes **43.** Answers will vary. **44.** $(9^9)^9$ **45.** 20

Section 10.2, Page 574

1. Numbers are obtained by starting at the first addend (on a clock face), then moving clockwise the number of hours equal to the second addend.

3. a) Add $4 + 10$ to get 2, then add $2 + 3$.

b) 5

5. a) Add 12 to 5 to get $17 - 9$.

b) 8

c) Since 12 is the identity element, you can add 12 to any number without changing the answer.

7. Yes, the sum of any two numbers in clock 12 arithmetic is a number in clock 12 arithmetic.

9. Yes; 1:11, 2:10, 3:9, 4:8, 5:7, 6:6, 7:5, 8:4, 9:3, 10:2, 11:1, and 12:12

11. Yes, $6 + 9 = 9 + 6$ since both equal 3

13. a) 5 **b)** 3. If you add 5 to any number, then you get the number you started with. Thus, in clock 5 arithmetic, 5 is the identity element. Three is the additive inverse of 2 since $2 + 3 = 5$.

15. Yes; the elements are symmetric about the main diagonal.

17. Yes; C is the identity element since the row next to C is identical to the top row and the column under C is identical to the left-hand column.

19. The inverse of A is B since $A \circledcirc B = C$ and $B \circledcirc A = C$.

21. 11 **23.** 5 **25.** 4 **27.** 8 **29.** 6 **31.** 6 **33.** 3 **35.** 4 **37.** 7 **39.** 1 **41.** 12 **43.** 12

45.

+	1	2	3	4	5	6
1	2	3	4	5	6	1
2	3	4	5	6	1	2
3	4	5	6	1	2	3
4	5	6	1	2	3	4
5	6	1	2	3	4	5
6	1	2	3	4	5	6

47. 1 **49.** 3 **51.** 2 **53.** 4

55.

+	1	2	3	4	5	6	7
1	2	3	4	5	6	7	1
2	3	4	5	6	7	1	2
3	4	5	6	7	1	2	3
4	5	6	7	1	2	3	4
5	6	7	1	2	3	4	5
6	7	1	2	3	4	5	6
7	1	2	3	4	5	6	7

57. 4 **59.** 6 **61.** 4 **63.** 7

65. Yes; it satisfies the five required properties.

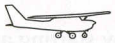
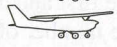
67. a) $\{0, 1, 2, 3\}$

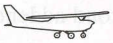
b)

c) Yes

d) Yes; 0

e) Yes; 0–0, 1–3, 2–2, 3–1

f) $(2$ $3)$  $1 = 2$
$(3$  $1)$

g) Yes; 2 $3 = 3$  2

h) Yes

69. a) $\{r, s, t, u\}$

b)

c) Yes

d) Yes; t

e) Yes; r–r, s–u, t–t, u–s

f) $(r$ $s)$ $u = r$ $(s$ $u)$

g) Yes; s $r = r$ s

h) Yes

71. a) $\{f, r, o, m\}$ **b)** **c)** Closed
d) m **e)** f **f)** f **g)** m **h)** r

73. Not associative:
$\left(M \otimes \right.$ $\left.\right) \otimes M \neq M \otimes \left(\right.$ $\otimes M\left.\right)$;
not commutative: $\otimes M \neq M \otimes$

75. No inverse for $\odot$ or for $*$, not associative

77. No identity element, no inverses, not associative, not commutative

79. a)

+	E	O
E	E	O
O	O	E

b) Yes, it is a commutative group; it satisfies the five properties.

81. Answers will vary.

83. a) Is closed; identity element is 6; inverses: 1–5, 2–2, 3–3, 4–4, 5–1, 6–6; is associative—for example,
$(2 \infty 5) \infty 3 = 2 \infty (5 \infty 3)$
$3 \infty 3 = 2 \infty 2$
$6 = 6$

b) $3 \infty 1 \neq 1 \infty 3$
$2 \neq 4$

85. a)

*	R	S	T	U	V	I
R	V	T	U	S	I	R
S	U	I	V	R	T	S
T	S	R	I	V	U	T
U	T	V	R	I	S	U
V	I	U	S	T	R	V
I	R	S	T	U	V	I

b) Yes, the associative property will hold.

c) No, it is not commutative. For example, $R * S \neq S * R$

87.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

89. Add the number in the top row and the number in the left-hand column and divide the sum by 4. The remainder is placed in the table.

Section 10.3, Page 585

1. A modulo m system consists of m elements, 0 through $m - 1$, and a binary operation.

3. 5;

0	1	2	3	4
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
.	.	.	.	.
.	.	.	.	.

5. 12 classes **7.** (b), (c) or (d) **9.** Saturday **11.** Friday
13. Saturday **15.** Thursday **17.** July **19.** March
21. September
23. July **25.** 4 **27.** 2 **29.** 3 **31.** 2 **33.** 1 **35.** 2
37. 0 **39.** 0 **41.** 6 **43.** 2 **45.** 2 **47.** 9 **49.** 5
51. 1 **53.** 2 **55.** 5 **57.** 5 **59.** $\{\ \}$
61. 1 and 6 **63.** 4 **65.** 0

67. a) 2016, 2020, 2024, 2028, 2032 **b)** 3004
c) 2552, 2556, 2560, 2564, 2568, 2572

69. **a)** Resting (for the second of two days)
b) Resting (for the second of two days)
c) Morning and afternoon practice
d) No

71. **a)** 5 **b)** No **c)** 54 weeks from this week

73. **a)** Evening **b)** Day **c)** Day

75. **a)**

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

b) Yes **c)** Yes, 0 **d)** Yes; 0–0, 1–3, 2–2, 3–1
e) $(1 + 2) + 3 = 1 + (2 + 3)$
f) Yes; $2 + 3 = 3 + 2$ **g)** Yes **h)** Yes

77. **a)**

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

b) Yes **c)** Yes, 1
d) No; no inverse for 0 or for 2, inverse of 1 is 1, inverse of 3 is 3
e) $(1 \times 2) \times 3 = 1 \times (2 \times 3)$
f) Yes, $2 \times 3 = 3 \times 2$ **g)** No

79. 2 **81.** 1, 2, 3 **83.** 0 **85.** 2 **87.** 0

88. Halfway up the mountain **89.** Math is fun.

Review Exercises, Page 589

1. A mathematical system consists of a set of elements and at least one binary operation.

2. A binary operation is an operation that can be performed on two and only two elements of a set. The result is a single element.

3. Yes; the sum of any two integers is an integer.

4. No; for example $2 - 3 = -1$ and -1 is not a natural number.

5. 7 **6.** 5 **7.** 10 **8.** 8 **9.** 9 **10.** 11

11. Closure, identity element, inverses, and associative property

12. A commutative group **13.** No; no identity element

14. No; no inverse for any integer except 1 and -1 **15.** Yes

16. No; no inverse for 0 **17.** No identity element

18. Not associative. For example, $(! \square p) \square ? \neq ! \square (p \square ?)$

19. Not every element has an inverse; not associative. For example, $(P ? P) ? 4 \neq P ? (P ? 4)$.

20. **a)** $\{ \vdash, \odot, ?, \triangle \}$ **b)** $\curlywedge$ **c)** Yes **d)** Yes; $\vdash$
e) Yes; $\vdash - \vdash, \odot - \triangle, ? - ?, \triangle - \odot$
f) $(\odot \curlywedge ?) \curlywedge \triangle = \odot \curlywedge (? \curlywedge \triangle)$
g) Yes; $\odot \curlywedge ? = ? \curlywedge \odot$ **h)** Yes

21. 0 **22.** 7 **23.** 1 **24.** 3 **25.** 4 **26.** 2 **27.** 4

28. 12 **29.** 9 **30.** 9 **31.** 4 **32.** 3 **33.** $\{ \}$ **34.** 1

35. 0, 2, 4, 6 **36.** 10 **37.** 5 **38.** 9 **39.** 7 **40.** 8

41.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Yes, it is a commutative group.

42.

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

No; no inverse for 0 or 2

43. **a)** No, she will be off.
b) Yes, she will have the evening off.

Chapter Test, Page 590

1. A set of elements and a binary operation

2. Closure, identity element, inverses, associative property, commutative property

3. No, not all elements have inverses.

4.

+	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

5. Yes, it is a commutative group. **6.** 4 **7.** 2

8. **a)** $\square$ **b)** Yes **c)** Yes, T **d)** S **e)** S

9. No, not closed. **10.** Yes, it is a commutative group.

11. Yes, it is a commutative group. **12.** 1 **13.** 3

14. 6 **15.** 2 **16.** 5 **17.** 2 **18.** $\{ \}$ **19.** 5

20. **a)**

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

b) No; no inverse for 0

Chapter 11

Section 11.1, Page 599

1. A percent is a ratio of some number to 100.
3. Divide the numerator by the denominator, multiply the quotient by 100, and add a percent sign.
5. $\text{Percent change} = \dfrac{\left(\begin{array}{c}\text{amount in}\\ \text{latest period}\end{array}\right) - \left(\begin{array}{c}\text{amount in}\\ \text{previous period}\end{array}\right)}{\text{amount in the previous period}} \times 100$

7. 50.0% **9.** 40.0% **11.** 0.8% **13.** 378% **15.** 0.04
17. 0.0134 **19.** 0.0025 **21.** 0.002 **23.** 0.01 **25.** $\approx$ 2.7%
27. 4.7 grams **29.** \$47.28 million **31.** \$159.57 million
33. \$5.6848 billion **35.** \$9.8838 billion **37.** 19.2%
39. 19.6% **41.** $\approx$ 5.8%
43. a) 17.3% **b)** 43.6% **c)** 54.3% **d)** 53.9%
45. a) 4.9% **b)** 27.0% **c)** 23.5% **d)** 10.6%
47. \$6.75 **49.** 25% **51.** 300
53. a) \$2.61 **b)** \$46.11 **c)** \$6.92 **d)** \$53.03
55. 12 students **57.** \$39,055 **59.** $\approx$ 5.3% decrease
61. $\approx$ 20.9% **63.** $\approx$ 18.6% decrease **65.** \$3750
67. He will have a loss of \$10. **69.** \$21.95

Section 11.2, Page 610

1. Interest is the money the borrower pays for the use of the lender's money.
3. Security or collateral is anything of value pledged by the borrower that the lender may sell or keep if the borrower does not repay the loan.
5. i is the *interest*, p is the *principal*, r is the interest *rate* expressed as a percent, and t is the *time*.
7. The United States rule states that if a partial payment is made on a loan, interest is computed on the principal from the first day of the loan until the date of the partial payment.

9. \$60.00 **11.** \$2.81 **13.** \$15.85 **15.** \$80.06
17. \$113.20 **19.** 10% **21.** \$600 **23.** 2 years **25.** \$1015
27. a) \$131.25 **b)** \$3631.25
29. a) \$182.50 **b)** \$3467.50 **c)** $\approx$ 7.9%
31. \$23,793.75 **33.** 168 days **35.** 266 days **37.** 264 days
39. June 14 **41.** March 24 **43.** \$1615.31 **45.** \$3635.85
47. \$5278.99 **49.** \$850.64 **51.** \$6086.82 **53.** \$2646.24
55. a) November 3, 2004 **b)** \$978.06 **c)** \$21.94 **d)** $\approx$ 4.44%
57. a) $\approx$ 409.0% **b)** $\approx$ 204.5% **c)** $\approx$ 102.3%
59. a) 6.663% **b)** $\approx$ 7.139% **c)** \$6663 **d)** \$6996.15
60. a) \$6.42 **b)** \$14.20 **c)** \$22.47 **d)** Answers will vary.

Section 11.3, page 618

1. An investment is the use of money or capital for income or profit.
3. A variable investment is one in which neither the principal nor the interest is guaranteed.
5. **a)** The effective annual yield is the simple interest rate that gives the same amount of interest as a compound rate over the same period of time.
   b) Another name for effective annual yield is annual percentage yield.

7. a) \$2122.42 **b)** \$122.42
9. a) \$3942.72 **b)** \$442.72
11. a) \$1728.28 **b)** \$228.28
13. a) \$2831.95 **b)** \$331.95
15. a) \$4806.08 **b)** \$806.08
17. \$8336.15 **19.** \$1653.36 **21.** \$2341.82
23. a) \$4195.14 **b)** \$4214.36
25. \$3106.62 **27.** \$7609.45
29. a) \$1040.60, \$40.60 **b)** \$1082.43, \$82.43
c) \$1169.86, \$169.86 **d)** No
31. a) \$1125.51, \$125.51 **b)** \$1266.77, \$266.77
c) \$1604.71, \$604.71
d) Yes; new amount $= \dfrac{(\text{old amount})^2}{1000}$

33. $\approx$ 3.53% **35.** Yes, the APY should be 2.43%.
37. He will earn more interest in the account that pays the 5% simple interest
39. a) \$129,210.47 **b)** \$134.88
41. \$23,202.23 **43.** \$12,015.94 **45.** \$1.53
47. a) 24 years **b)** 12 years **c)** 9 years **d)** 6 years **e)** 3.27%
49. \$27,550.11 **51. a)** \$55,726.01 **b)** \$55,821.15

Section 11.4, page 631

1. An open-end installment loan is one with which you can make different payments each month. A fixed installment loan is one in which you pay a fixed amount each month for a set number of months.
3. The APR is the true rate of interest charged on a loan.
5. The total installment price is the sum of all the monthly payments and the down payment, if any.
7. The unpaid balance method and the average daily balance method

9. a) \$5339.96 **b)** \$698.17
11. a) \$809.20 **b)** \$80.15
13. a) \$628.40 **b)** 9.0%

15. **a)** \$1752 **b)** 9%

17. **a)** 6% **b)** \$726.00 **c)** \$7858.00

19. **a)** \$2818.20 **b)** \$689.39 **c)** \$347.90 **d)** \$8614.17

21. **a)** \$1344.87 **b)** \$181.04 **c)** \$761.64 **d)** \$5936.84

23. **a)** \$27.63 **b)** \$1032.10

25. **a)** \$18 **b)** \$630.86

27. **a)** \$31 **b)** \$553.02

29. **a)** \$19.76 **b)** \$743.41

31. **a)** \$1.56 **b)** \$133.11

33. **a)** \$512.00 **b)** \$6.66 **c)** \$638.43

35. **a)** \$121.78 **b)** \$1.52 **c)** \$133.07
d) The interest charged using the average daily balance method is \$0.04 less than the interest charged using the unpaid balance method.

37. **a)** \$8.87 **b)** \$608.87

39. **a)** \$25 **b)** \$35.60 **c)** 8.5% **d)** 6.5%

41. **a)** 6 months **b)** \$83.95
c) The installment loan saves them \$49.45.

43. **a)** \$6872.25 **b)** \$610.37 **c)** \$2637.42 **d)** \$2501.05

45. Since Martina's billing date is June 25th, she can buy the camera from June 26th through June 29th and the purchase will appear on her July 25th bill. Since she has a 20-day grace period, she can pay for the camera on August 5th without paying interest.

Section 11.5, Page 644

1. A mortgage is a long-term loan in which the property is pledged as security for payment of the difference between the down payment and the sale price.

3. The major difference is that the interest rate for a conventional loan is fixed for the duration of the loan, whereas the interest rate for a variable-rate loan may change every period, as specified in the loan agreement.

5. A buyer's adjusted monthly income is found by subtracting any fixed monthly payments with more than 10 months remaining from the gross monthly income.

7. An amortization schedule lists payment dates and payment numbers. For each payment it lists the amount that goes to pay the interest and the principal. It also gives the balance remaining on the loan after each payment.

9. Equity is the difference between the appraised value of your home and the loan balance.

11. **a)** \$37,500 **b)** \$1625.63

13. **a)** \$21,000 **b)** \$1247.40

15. **a)** \$39,000 **b)** \$156,000 **c)** \$3120

17. **a)** \$2865 **b)** \$802.20 **c)** \$1411.50 **d)** No

19. **a)** \$187,736.40 **b)** \$112,736.40 **c)** \$38.68

21. **a)** \$31,780 **b)** \$2451.60 **c)** \$4330 **d)** \$1212.40
e) \$789.42 **f)** \$916.09 **g)** Yes. **h)** \$108.42

23. Bank B

25. **a)** 805
b)

Payment Number	Interest	Principal	Balance of Loan
1	\$750.00	\$55.00	\$99,945.00
2	\$749.59	\$55.41	\$99,889.59
3	\$749.17	\$55.83	\$99,833.76

c) 9.38%
d)

Payment Number	Interest	Principal	Balance of Loan
4	\$780.37	\$24.63	\$99,809.13
5	\$780.17	\$24.83	\$99,784.30
6	\$779.98	\$25.02	\$99,759.28

e) 9.46%

27. **a)** \$113,095.24 **b)** \$150,793.65

Review Exercises, page 647

1. 60.0% **2.** 66.7% **3.** 62.5% **4.** 4.1%

5. 0.98% ≈ 1.0% **6.** 314.1% **7.** 0.03 **8.** 0.121

9. 1.23 **10.** 0.0025 **11.** $0.008\overline{3}$ **12.** 0.0000045

13. ≈17.6% **14.** ≈11.0% **15.** 31.25% **16.** 275

17. 91.8 **18.** \$6.42 **19.** 40 people **20.** 26.7%

21. \$16.67 **22.** 9.5% **23.** \$450 **24.** 0.5 year

25. \$6214.25 **26.** **a)** \$162 **b)** \$3162

27. **a)** \$1380 **b)** \$4620 **c)** ≈14.9%

28. **a)** $7\frac{1}{2}\%$ **b)** \$830 **c)** \$941.18

29. **a)** \$1610.51, \$610.51 **b)** \$1628.89, \$628.89
c) \$1638.62, \$638.62 **d)** \$1645.31, \$645.31
e) \$1648.61, \$648.61

30. \$5076.35 **31.** 5.76% **32.** \$13,415.00

33. **a)** 6.0% **b)** 253.16 **c)** \$4150.34

34. **a)** \$109.18 **b)** \$2014.11

35. **a)** 4.5% **b)** \$32.06 **c)** \$1420.43

36. **a)** \$6.31 **b)** \$847.61 **c)** \$508.99 **d)** \$6.62
e) \$847.92

37. **a)** \$2.60 **b)** \$546.92 **c)** \$382.68 **d)** \$5.36
e) \$549.68

38. **a)** \$10,400 **b)** \$41,600 **c)** \$3040.96 **d)** 3.5%

39. **a)** \$10.44 **b)** 8.5%

40. **a)** \$33,925 **b)** \$4805.33 **c)** \$1345.49
d) \$855.93 **e)** \$1172.60 **f)** Yes

41. **a)** \$13,485 **b)** \$756.51 **c)** \$24.20 **d)** \$285,828.60
e) \$195,928.60

42. **a)** \$550.46 **b)** 8% **c)** 7.75%

Chapter Test, page 650

1. \$40 **2.** 3 years **3.** \$637.50 **4.** \$5637.50
5. \$2523.20 **6.** \$123.20 **7.** \$7961.99, \$461.99
8. \$3036.68, \$536.68 **9.** \$1997.50 **10.** \$181.46
11. 8.5% **12. a)** \$105.05 **b)** \$3155.90
13. a) 4.5% **b)** \$64.02 **c)** \$2836.28
14. a) \$12.30 **b)** \$1146.57 **c)** \$765.67 **d)** \$10.72 **e)** \$1144.99
15. \$21,675 **16.** \$6603.33 **17.** \$1848.93 **18.** \$1123.85
19. \$1428.02 **20.** Yes **21. a)** \$426,261 **b)** \$281,761

Chapter 12

Section 12.1, Page 659

1. An experiment is a controlled operation that yields a set of results.

3. Empirical probability is the relative frequency of occurrence of an event. It is determined by actual observation of an experiment.

$$P(E) = \frac{\text{number of times event has occurred}}{\text{number of times experiment was performed}}$$

5. Answers will vary.

7. No, it means that if a coin was flipped many times, about $\frac{1}{2}$ of the tosses would land heads up.

9. No, it means that the average person with traits similar to Mr. Duncan's will live another 43.21 years.

11.–13. Answers will vary.

15. a) $\frac{7}{15}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{5}$ **17. a)** $\frac{8}{19}$ **b)** $\frac{7}{19}$ **c)** $\frac{1}{19}$

19. a) The percents are relative frequencies of the events occurring.
b) 0.32 **c)** 0.22 **d)** 0.19

21. a) 1 **b)** Yes

23. a) $\frac{11}{40}$ **b)** $\frac{9}{40}$ **c)** $\frac{1}{4}$ **d)** $\frac{7}{40}$ **e)** $\frac{3}{40}$

25. a) $\frac{6}{20} = \frac{3}{10}$ **b)** $\frac{14}{20} = \frac{7}{10}$ **c)** $\frac{14}{20} = \frac{7}{10}$ **d)** $\frac{2}{20} = \frac{1}{10}$

27. a) 0 **b)** $\frac{50}{250} = 0.2$ **c)** 1

29. a) $\frac{224}{929} \approx 0.24$ **b)** $\frac{705}{929} \approx 0.76$

31. Answers will vary.

Section 12.2, Page 667

1. If each outcome of an experiment has the same chance of occurring as any other outcome, they are said to be equally likely outcomes.

3. $P(A) + P(\text{not } A) = 1$ **5.** 0.7 **7.** $\frac{7}{12}$

9. Answers will vary. **11.** 0 and 1 **13. a)** $\frac{1}{5}$ **b)** $\frac{1}{4}$

15. $\frac{1}{50}$ **17.** $\frac{1}{13}$ **19.** $\frac{12}{13}$ **21.** $\frac{1}{2}$ **23.** 1 **25.** $\frac{4}{13}$

27. a) $\frac{1}{2}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{4}$ **d)** 0

29. a) $\frac{1}{2}$ **b)** 0 **c)** $\frac{1}{3}$ **d)** $\frac{1}{6}$

31. $\frac{2}{5}$ **33.** $\frac{9}{10}$ **35.** $\frac{1}{12}$ **37.** $\frac{1}{6}$ **39.** $\frac{23}{50}$ **41.** $\frac{33}{50}$

43. $\frac{11}{17}$ **45.** $\frac{12}{17}$ **47.** $\frac{4}{11}$ **49.** $\frac{4}{11}$ **51.** 1 **53.** $\frac{1}{11}$

55. $\frac{4}{11}$ **57.** $\frac{1}{26}$ **59.** $\frac{5}{26}$ **61.** $\frac{345}{715} = \frac{69}{143}$ **63.** $\frac{533}{715} = \frac{41}{55}$

65. $\frac{97}{715}$ **67.** $\frac{50}{159}$ **69.** $\frac{66}{159} = \frac{22}{53}$ **71.** $\frac{23}{159}$ **73.** $\frac{13}{36}$

75. $\frac{1}{3}$ **77.** $\frac{23}{36}$ **79. a)** 0 **b)** 1 **81. a)** $\frac{1}{4}$ **b)** $\frac{1}{4}$ **c)** $\frac{1}{4}$

83. 29 dots

Section 12.3, Page 674

1. Answers will vary. **3.** Odds against

5. 9 to 5 **7. a)** $\frac{1}{2}$ **b)** $\frac{1}{2}$

9. a) $\frac{8}{27}$ **b)** $\frac{19}{27}$ **c)** 19 : 8 **d)** 8 : 19

11. 5 : 1 **13.** 4 : 2 or 2 : 1 **15.** 12 : 1, 1 : 12

17. 10 : 3, 3 : 10 **19.** 1 : 1 **21.** 5 : 3

23. a) 8 : 7 **b)** 7 : 8 **25.** 8 : 7 **27.** 14 : 1 **29.** 8 : 7

31. a) $\frac{5}{9}$ **b)** 4 : 5 **33.** 1 : 18 **35. a)** $\frac{8}{13}$ **b)** $\frac{5}{13}$

37. $\frac{11}{15}$ **39.** $\frac{1}{5}$ **41.** 1 : 4 **43.** 74 : 1 **45.** 0.34 **47.** 33 : 17

49. 43 : 57 **51.** 1 : 9 **53.** 7 : 1 **55. a)** $\frac{20}{21}$ **b)** 20 : 1

57. Horse 1, $\frac{2}{9}$; Horse 2, $\frac{1}{3}$; Horse 3, $\frac{1}{16}$; Horse 4, $\frac{5}{12}$; Horse 5, $\frac{1}{2}$

59. $\approx 97 : 3$

Section 12.4, Page 683

1. The expected value is the expected gain or loss of an experiment over the long run.

3. The fair price is the amount that should be charged for the game to be fair and result in an expectation of 0.

5. To obtain the fair price, add the cost to the expected value.

7. \$0.50. Since you would lose \$1.00 on average for each game you played, the price of the game should be \$1.00 less than the actual cost. Then the expectation would be \$0, and the game would be fair. The results could also be obtained from the fair price formula, fair price = expectation + cost to play.

9. −\$1.20 **11.** 176 people **13.** 70 points

15. 1.44 million viewers **17.** \$3840 **19.** \$1.60 off

21. a) ≈ −\$0.67 **b)** ≈ \$0.67

23. a) Yes, because you have a positive expectation of $\frac{1}{5}$

b) Yes, because you have a positive expectation of $\frac{1}{2}$

25. a) \$−1.20 **b)** \$0.80 **27. a)** \$−2.00 **b)** \$1.00

29. \$5.50 **31.** \$−1.25 **33. a)** \$1 **b)** \$3

35. a) \$2.25 **b)** \$4.25 **37.** 0.75 base **39.** 2.9 points

41. 381.4 employees **43.** ≈ 15.65 min **45.** 3.5

47. ≈141.51 service calls

49. a) $\frac{9}{16}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ **b)** \$11.81 **c)** \$11.81

51. An amount greater than \$1200

53. −\$0.053 or −5.3¢ **55. a)** \$458.33 **b)** \$308.33

Section 12.5, Page 692

1. If a first experiment can be performed in M distinct ways and a second experiment can be performed in N distinct ways, then the two experiments in that specific order can be performed in $M \cdot N$ distinct ways.

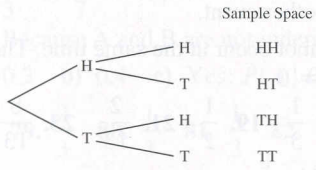
3. 14

5. The first selection is made. Then the second selection is made without the first selection being returned to the group of items being selected.

7. a) 2500 **b)** 2450 **9. a)** 216 **b)** 120

11. a) 4

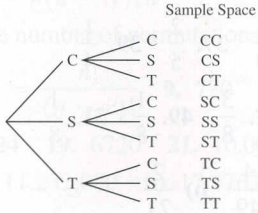
b)



c) $\frac{1}{4}$ **d)** $\frac{1}{2}$ **e)** $\frac{1}{4}$

13. a) 9

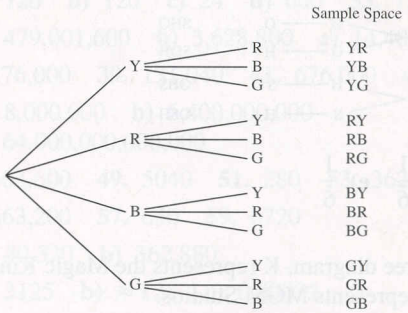
b)



c) $\frac{1}{9}$ **d)** $\frac{1}{9}$ **e)** $\frac{5}{9}$

15. a) 12

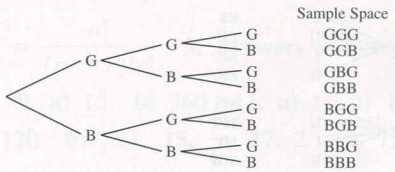
b)



c) $\frac{1}{2}$ **d)** 1 **e)** $\frac{1}{2}$

17. a) 8

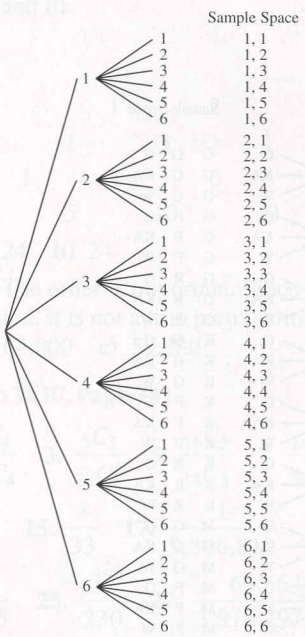
b)



c) $\frac{1}{8}$ **d)** $\frac{7}{8}$ **e)** $\frac{3}{4}$ **f)** $\frac{1}{8}$

19. a) 36

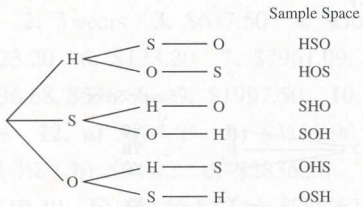
b)



c) $\frac{1}{6}$ **d)** $\frac{1}{6}$ **e)** $\frac{1}{36}$ **f)** No

21. a) 6

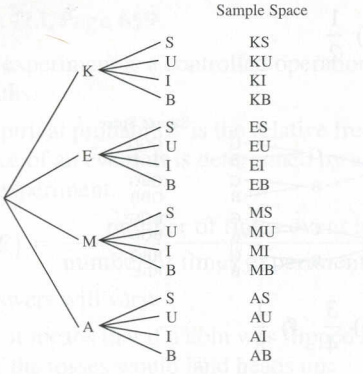
b)



c) $\frac{1}{3}$ **d)** $\frac{1}{6}$ **e)** $\frac{1}{6}$

23. a) 16

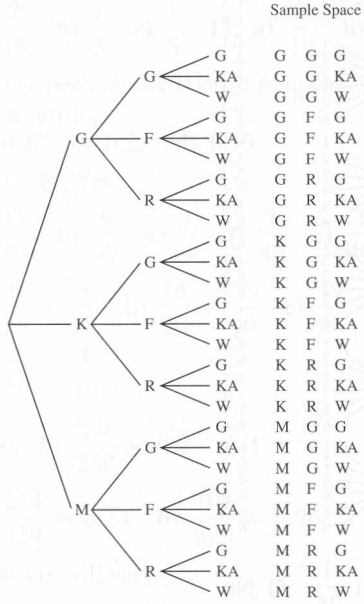
b) In the tree diagram, K represents the Magic Kingdom and M represents MGM Studios.



c) $\frac{1}{2}$ **d)** $\frac{7}{16}$ **e)** $\frac{1}{8}$

25. a) 27

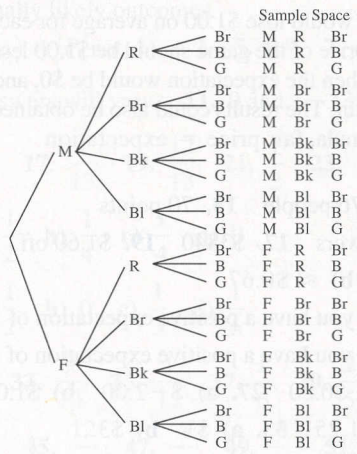
b)



c) $\frac{1}{27}$ **d)** $\frac{8}{27}$ **e)** $\frac{19}{27}$

27. a) 24

b)



c) $\frac{1}{24}$ **d)** $\frac{1}{8}$

29. a) $\frac{1}{3}$ **b)** $\frac{2}{3}$ **c)** No, the probability of selecting a red chip is not the same as the probability of selecting a white chip.

d) Answers will vary.

31. 3; 1 red, 1 blue, and 1 brown **32.** 5 faces

Section 12.6, Page 703

1. a) At least one event, A *or* B, must occur.

b) Both events, A *and* B, must occur.

3. a) Events that cannot happen simultaneously are mutually exclusive events.

b) $P(A \text{ or } B) = P(A) + P(B)$

5. We assume that event A has already occurred.

7. Two events are dependent when the probability of one item being selected has an effect on the probability of a second item being selected.

9. a) No, both events can occur at the same time.

b) Yes, the outcome of one event has no affect on the outcome of the other event.

11. Events A and B cannot occur at the same time. Therefore, $P(A \text{ and } B) = 0$.

13. 0.7 **15.** 0.5 **17.** $\frac{1}{3}$ **19.** $\frac{1}{2}$ **21.** $\frac{2}{13}$ **23.** $\frac{8}{13}$

25. $\frac{31}{52}$ **27. a)** $\frac{1}{16}$ **b)** $\frac{1}{19}$

29. a) $\frac{1}{16}$ **b)** $\frac{5}{76}$ **31. a)** $\frac{3}{80}$ **b)** $\frac{3}{76}$

33. a) $\frac{9}{25}$ **b)** $\frac{33}{95}$ **35.** $\frac{11}{20}$ **37.** $\frac{2}{5}$ **39.** $\frac{1}{4}$

41. $\frac{1}{8}$ **43.** $\frac{9}{64}$ **45.** $\frac{1}{8}$ **47.** $\frac{3}{8}$ **49.** $\frac{1}{8}$ **51.** $\frac{1}{8}$

53. a) $\frac{1}{16}$ **b)** $\frac{1}{2}$ **55. a)** $\frac{4}{49}$ **b)** $\frac{2}{21}$

57. a) $\frac{24}{49}$ b) $\frac{11}{21}$ **59.** $\frac{5}{12}$ **61.** $\frac{7}{12}$ **63.** $\frac{969}{4060}$

65. $\frac{5}{812}$ **67.** 0.7 **69.** 0.343 **71.** $\frac{1}{4}$ **73.** $\frac{27}{1024}$

75. $\frac{243}{1024}$ **77.** $\frac{3}{22}$ **79.** $\frac{1050}{1331}$ **81.** $\frac{1}{48}$ **83.** $\frac{5}{12}$

85. 0.36 **87.** 0.36

89. a) No b) 0.001 c) 0.00004 d) 0.00096
e) 0.000999 f) 0.998001

91. $\frac{32}{1000}$, or 0.032 **93.** $\frac{484}{15{,}625}$, or 0.030976 **95.** $\frac{14}{45}$

97. Favors dealer, the probability of at least one diamond is ≈ 0.44, which is less than 0.5

99. $\frac{1}{9}$ **100.** $\frac{1}{4}$ **101.** $\frac{1}{2}$ **102.** 1 **103.** Answers will vary.

Section 12.7, Page 710

1. The probability of E_2 given that E_1 has occurred

3. $\frac{1}{3}$ **5.** $\frac{1}{3}$ **7.** $\frac{2}{3}$ **9.** $\frac{2}{3}$ **11.** $\frac{3}{4}$ **13.** $\frac{2}{3}$ **15.** $\frac{2}{3}$ **17.** $\frac{1}{3}$

19. $\frac{1}{3}$ **21.** $\frac{3}{5}$ **23.** $\frac{1}{7}$ **25.** $\frac{1}{16}$ **27.** $\frac{1}{7}$ **29.** $\frac{5}{36}$

31. $\frac{1}{6}$ **33.** $\frac{2}{3}$ **35.** $\frac{107}{217}$ **37.** $\frac{25}{56}$ **39.** $\frac{6}{11}$ **41.** $\frac{4}{9}$

43. $\frac{11}{20}$ **45.** $\frac{22}{39}$ **47.** $\frac{133}{300}$ **49.** $\frac{1}{2}$ **51.** $\frac{21}{43}$

53. ≈ 0.3197 **55.** ≈ 0.0795 **57.** ≈ 0.1928 **59.** $\frac{10}{11}$

61. $\frac{3}{19}$ **63.** $\frac{44}{47}$ **65.** $\frac{11}{27}$ **67.** $\frac{10}{29}$ **69.** $\frac{11}{29}$ **71.** $\frac{93}{200}$

73. $\frac{15}{52}$ **75.** a) 140 b) 120 c) $\frac{7}{10}$ d) $\frac{3}{5}$
e) $\frac{2}{3}$ f) $\frac{4}{7}$
g) Because A and B are not independent events

77. a) 0.3 b) 0.4 c) Yes; $P(A \mid B) = P(A) \cdot P(B)$

78. $\frac{1}{3}$ **79.** $\frac{2}{3}$ **80.** $\frac{1}{3}$ **81.** $\frac{1}{3}$ **82.** 100 **83.** $\frac{1}{3}$

Section 12.8, Page 721

1. Answers will vary.

3. $n! = n(n-1)(n-2)\cdots(3)(2)(1)$

5. The number of permutations of n items taken r at a time.

7. ${}_nP_r = \frac{n!}{(n-r)!}$ **9.** 720 **11.** 30 **13.** 1 **15.** 1

17. 3024 **19.** 6720 **21.** 10,000

23. a) 11,232,000 b) 17,576,000

25. a) $5^5 = 3125$ b) $\frac{1}{3125} = 0.00032$

27. 57,106,944 **29.** 720 systems

31. a) 720 b) 120 c) 24 d) 600 **33.** 720

35. a) 479,001,600 b) 3,628,800 c) 14,400

37. 3,276,000 **39.** 131,040 **41.** 676,000 **43.** 104,000

45. a) 8,000,000 b) 6,400,000,000
c) 64,000,000,000,000

47. 3,603,600 **49.** 5040 **51.** 280 **53.** 362,880

55. 1,663,200 **57.** 630 **59.** 6720

61. a) 40,320 b) 362,880

63. a) 3125 b) ≈ 128 c) 0.00032

65. 12,600 sec, or 3.5 hr **67.** No **68.** 56 **69.** 600

Section 12.9, Page 727

1. Answers will vary.

3. ${}_nC_r = \frac{n!}{(n-r)!r!}$ **5.** Answers will vary.

7. 10 **9.** a) 15 b) 360 **11.** a) 1 b) 1

13. a) 120 b) 720 **15.** $\frac{1}{6}$ **17.** 2 **19.** 72

21. 84 **23.** 5 **25.** 210 **27.** 126 **29.** 495

31. 45 **33.** 28 **35.** 6160 **37.** 560 **39.** 294,000

41. 560 **43.** 1200 **45.** a) 45 b) 56

47. a) and b)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

49. a) 24 b) 24

51. a) The order is important. Since the numbers may be repeated, it is not a true permutation lock.
b) 64,000 c) 59,280

Section 12.10, Page 733

1. $\frac{{}_6C_4}{{}_{10}C_4}$ **3.** $\frac{{}_5C_3}{{}_{26}C_3}$ **5.** $\frac{{}_{10}C_5}{{}_{18}C_5}$ **7.** $\frac{{}_{14}C_9}{{}_{30}C_9}$ **9.** $\frac{5}{42}$ **11.** $\frac{4}{143}$

13. $\frac{1}{12}$ **15.** $\frac{4}{33}$ **17.** $\frac{1}{9{,}366{,}819}$ **19.** $\frac{3}{10}$ **21.** $\frac{7}{10}$

23. $\frac{1}{115}$ **25.** $\frac{27}{230}$ **27.** $\frac{646{,}646}{3{,}910{,}797{,}436} \approx 0.0001653$

29. $\frac{923{,}410{,}488}{3{,}910{,}797{,}436} \approx 0.236$ **31.** $\frac{20}{1001}$ **33.** $\frac{200}{1001}$

35. $\frac{1}{77}$ **37.** $\frac{5}{77}$ **39.** $\frac{5}{506} \approx 0.010$

41. a) $\frac{1}{123{,}760}$ b) $\frac{1}{30{,}940}$

43. a) $\frac{33}{54{,}145}$ b) $\frac{1}{2{,}598{,}960}$

45. a) $\frac{1}{2{,}162{,}160}$ b) $\frac{1}{6435}$

47. 1; Since there are more hairs than people, two or more people must have the same number of hairs on their head.

Section 12.11, Page 742

1. A probability distribution shows the probability associated with each specific outcome of an experiment. In a probability distribution every possible outcome must be listed and the sum of all the probabilities must be 1.

3. $P(x) = ({}_nC_x)p^x q^{n-x}$

5. 0.2646 **7.** 0.3456 **9.** 0.015625

11. a) $P(x) = ({}_nC_x)(0.14)^x(0.86)^{n-x}$

b) $P(2) = ({}_{12}C_2)(0.14)^2(0.86)^{10}$

13. 0.05954 **15.** 0.11059 **17.** 0.06877 **19.** 0.4096

21. a) 0.01024 b) 0.98976

23. a) ≈ 0.1119 b) ≈ 0.2966

25. 0; it will be midnight.

Review Exercises, Page 745

1. Answers will vary. **2.** Answers will vary.

3. $\frac{1}{5}$ **4.** Answers will vary. **5.** $\frac{2}{5}$ **6.** $\frac{1}{2}$ **7.** $\frac{7}{10}$

8. 1 **9.** $\frac{1}{5}$ **10.** $\frac{5}{24}$ **11.** $\frac{1}{6}$ **12.** $\frac{1}{3}$ **13.** $\frac{19}{24}$

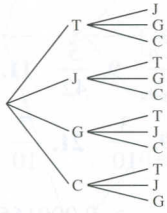
14. a) 9 : 1 b) 1 : 9 **15.** 5 : 3 **16.** $\frac{3}{85}$ **17.** 7 : 3

18. a) −$1.20 b) −$3.60 c) $0.80

19. a) −$0.23 b) $0.23 c) Lose $23.08

20. 660 people

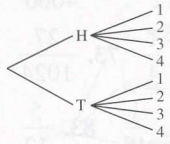
21. a)



b) Sample Space c) $\frac{1}{12}$

TJ
TG
TC
JT
JG
JC
GT
GJ
GC
CT
CJ
CG

22. a)



b) Sample Space c) $\frac{1}{4}$ d) $\frac{3}{4}$

H1
H2
H3
H4
T1
T2
T3
T4

23. $\frac{1}{4}$ **24.** $\frac{9}{64}$ **25.** $\frac{5}{16}$ **26.** $\frac{7}{8}$ **27.** 1 **28.** $\frac{3}{16}$

29. $\frac{1}{22}$ **30.** $\frac{14}{55}$ **31.** $\frac{41}{55}$ **32.** $\frac{1}{22}$ **33.** $\frac{1}{4}$

34. Against, 3 : 1; in favor, 1 : 3 **35.** $13.75

36. $\frac{1}{8}$ **37.** $\frac{5}{8}$ **38.** In favor, 3 : 5; against, 5 : 3 **39.** $3.75

40. $\frac{7}{8}$ **41.** $\frac{89}{106}$ **42.** $\frac{55}{74}$ **43.** $\frac{19}{74}$ **44.** $\frac{17}{106}$

45. $\frac{23}{40}$ **46.** $\frac{3}{17}$ **47.** $\frac{3}{4}$ **48.** $\frac{12}{17}$

49. a) 24 b) $4500 **50.** 30 **51.** 720 **52.** 504

53. 20 **54.** a) 3003 b) 3,628,800

55. a) $\frac{1}{2{,}598{,}960}$ b) $\frac{1}{135{,}145{,}920}$ **56.** 5880 **57.** 560

58. $\frac{1}{221}$ **59.** $\frac{1}{12}$ **60.** $\frac{1}{18}$ **61.** $\frac{1}{24}$ **62.** $\frac{11}{12}$ **63.** $\frac{5}{182}$

64. $\frac{45}{364}$ **65.** $\frac{2}{13}$ **66.** $\frac{11}{13}$

67. a) $P(x) = ({}_nC_x)(0.6)^x(0.4)^{n-x}$

b) $P(75) = ({}_{100}C_{75})(0.6)^{75}(0.4)^{25}$

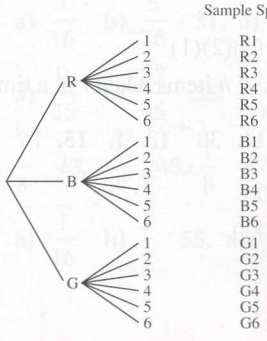
68. 0.0512 **69.** a) 0.0256 b) 0.9744

Chapter Test, Page 748

1. $\frac{11}{15}$ **2.** $\frac{2}{9}$ **3.** $\frac{5}{9}$ **4.** $\frac{7}{9}$ **5.** $\frac{1}{3}$ **6.** $\frac{1}{6}$ **7.** $\frac{1}{6}$

8. $\frac{5}{18}$ **9.** $\frac{5}{12}$ **10.** $\frac{8}{13}$ **11.** 18

12.



13. $\frac{1}{18}$ **14.** $\frac{4}{9}$ **15.** $\frac{5}{6}$ **16.** 608,400

17. a) 5 : 4 **b)** 5 : 4 **18.** $\frac{2}{7}$ **19.** \$0

20. a) $\frac{107}{228}$ **b)** $\frac{115}{228}$ **c)** $\frac{68}{115}$ **d)** $\frac{60}{107}$

21. 120 **22.** $\frac{14}{95}$ **23.** $\frac{81}{95}$ **24.** $\frac{175}{396}$ **25.** 0.0081

Chapter 13

Section 13.1, Page 756

1. Answers will vary.

3.–5. Answers will vary.

7. a) A population is all items or people of interest.
b) A sample is a subset of the population.

9. a) A random sample is a sample drawn in such a way that each item in the population has an equal chance of being selected.
b) Number each item in the population. Write each number on a piece of paper and put each numbered piece of paper in a hat. Select pieces of paper from the hat and use the numbered items selected as your sample.

11. a) A stratified sample is one that includes items from each part (or strata) of the population.
b) First identify the strata in which you are interested. Then select a random sample from each strata.

13. An unbiased sample is one that is a small replica of the entire population with regard to income, education, gender, race, religion, political affiliation, age, and so forth.

15. Stratified sample **17.** Cluster sample

19. Systematic sample **21.** Convenience sample

23. Random sample **25. a)–c)** Answers will vary.

27. President; four out of 42 U.S. presidents have been assassinated (Lincoln, Garfield, McKinley, Kennedy).

Section 13.2, Page 759

1. Answers will vary.

3. There may have been more car thefts in Baltimore, Maryland than Reno, Nevada because many more people live in Baltimore than in Reno. But, Reno may have more car thefts per capita than Baltimore.

5. Although the cookies are fat free, they still contain calories. Eating many of them may still cause you to gain weight.

7. More people drive on Saturday evening. Thus, one might expect more accidents.

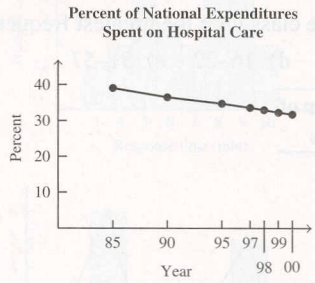
9. People with asthma may move to Arizona because of its climate. Therefore, more people with asthma may live in Arizona.

11. Although milk is less expensive at Star Food Markets than at Price Chopper Food Markets, other items may be more expensive at Star Food Markets.

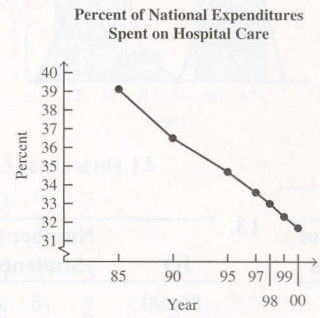
13. There may be deep sections in the pond, so it may not be safe to go wading.

15. Half the students in a population are expected to be below average.

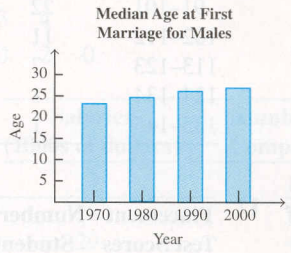
17. a)



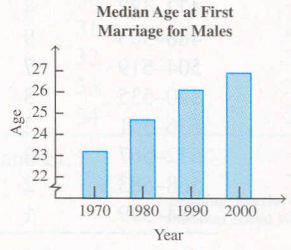
b)



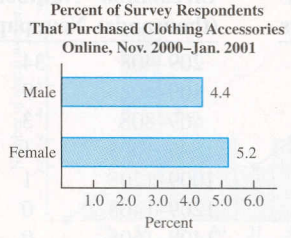
19. a)



b)



21. a)



b) Yes. Answers will vary.

23. A decimal point

Section 13.3, Page 765

1. A frequency distribution is a listing of observed values and the corresponding frequency of occurrence of each value.

3. a) 7 **b)** 16–22 **c)** 16 **d)** 22

5. The modal class is the class with the greatest frequency.

7. a) 18 **b)** 7 **c)** 19 **d)** 16–22 **e)** 51–57

9.

Number Sold	Number of Days
0	3
1	8
2	3
3	5
4	2
5	7
6	2
7	3
8	4
9	1
10	2

11.

IQ	Number of Students
78–86	2
87–95	15
96–104	18
105–113	7
114–122	6
123–131	1
132–140	1

13.

IQ	Number of Students
80–90	8
91–101	22
102–112	11
113–123	7
124–134	1
135–145	1

15.

Placement Test Scores	Number of Students
472–492	9
493–513	9
514–534	5
535–555	2
556–576	3
577–597	2

17.

Placement Test Scores	Number of Students
472–487	4
488–503	9
504–519	7
520–535	3
536–551	2
552–567	2
568–583	2
584–599	1

19.

Circulation (thousands)	Number of Newspapers
209–458	36
459–708	8
709–958	3
959–1208	1
1209–1458	0
1459–1708	0
1709–1958	1
1959–2208	1

21.

Circulation (thousands)	Number of Newspapers
209–408	34
409–608	9
609–808	3
809–1008	1
1009–1208	1
1209–1408	0
1409–1608	0
1609–1808	1
1809–2008	0
2009–2208	1

23.

Population (millions)	Number of Counties
1.4–2.1	15
2.2–2.9	6
3.0–3.7	2
3.8–4.5	0
4.6–5.3	0
5.4–6.1	1
6.2–6.9	0
7.0–7.7	0
7.8–8.5	0
8.6–9.3	0
9.4–10.1	1

25.

Population (millions)	Number of Counties
1.0–2.5	19
2.6–4.1	4
4.2–5.7	1
5.8–7.3	0
7.4–8.9	0
9.0–10.5	1

27.

Price ($)	Number of States
0.35–0.44	6
0.45–0.54	10
0.55–0.64	11
0.65–0.74	3
0.75–0.84	2
0.85–0.94	4
0.95–1.04	1
1.05–1.14	2
1.15–1.24	2
1.25–1.34	1
1.35–1.44	0
1.45–1.54	1

29.

Price ($)	Number of States
0.35–0.54	16
0.55–0.74	14
0.75–0.94	6
0.95–1.14	3
1.15–1.34	3
1.35–1.54	1

31. February, since it has the fewest numbers of days

32. a) Did You Know?, page 762: There are 6 F's.

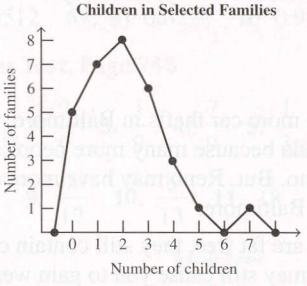
b) Answers will vary.

Section 13.4, page 774

1. Answers will vary. **3.** Answers will vary.

5. a) Answers will vary.

b)



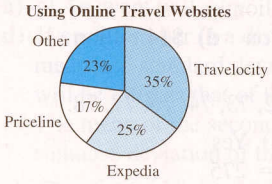
7. a) Answers will vary.

b)

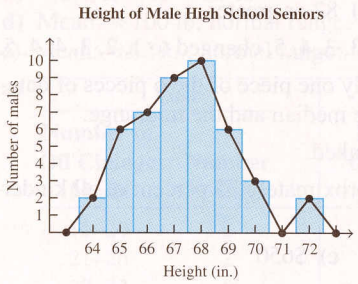
Observed values	Frequency
45	3
46	0
47	1
48	0
49	1
50	1
51	2

9. Occasionally: 295; most times: 125; every time: 35; never: 45

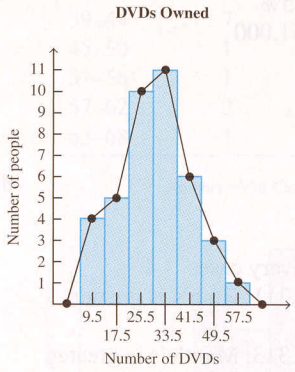
11.



13. a) and **b)**



15. a) and **b)**



17. a) 30 **b)** 4 **c)** 2 **d)** 75

e)

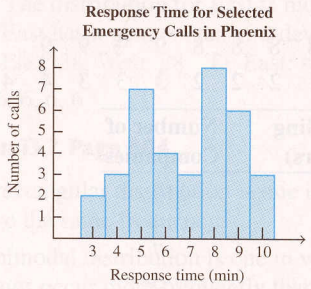
Number of Soft Drinks Purchased	Number of People
0	2
1	7
2	8
3	5
4	4
5	3
6	1

19. a) 7 **b)** 16 **c)** 36

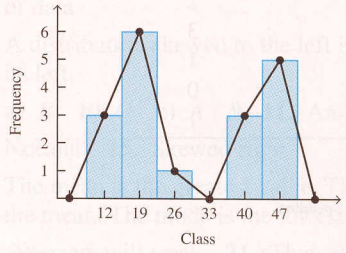
d)

Response Time (min)	Number of Calls	Response Time (min)	Number of Calls
3	2	7	3
4	3	8	8
5	7	9	6
6	4	10	3

e)



21.



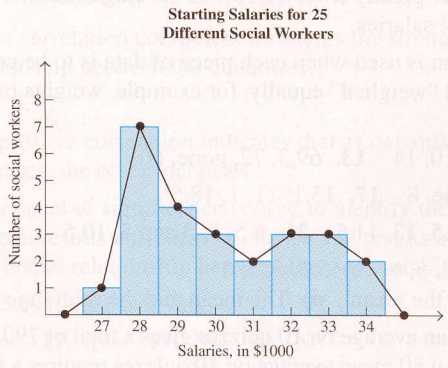
23. 1 | 5 represents 15

1	0 5 7
2	4 4
3	6 0 3
4	8 5 2 5 8
5	3 4
6	0 2 0

25. a)

Salaries (1000s of dollars)	Number of Companies
27	1
28	7
29	4
30	3
31	2
32	3
33	3
34	2

b) and **c)**



d) 2 | 3 represents 23

2	7	8	8	8	8	8	8	8	9	9	9	9	
3	0	0	0	1	1	2	2	2	3	3	3	4	4

27. a)

Advertising Spending (millions of dollars)	Number of Companies
597–905	19
906–1214	14
1215–1523	7
1524–1832	3
1833–2141	2
2142–2450	3
2451–2759	1
2760–3068	0
3069–3377	1

b) and **c)**

29. a)–e) Answers will vary.

Section 13.5, Page 784

1. Ranked data are data listed from the lowest value to the highest value or from the highest value to the lowest value.

3. The median is the value in the middle of a set of ranked data. To find the median rank the data and select the value in the middle.

5. The mode is the most common piece of data. The piece of data that occurs most frequently is the mode.

7. The median should be used when there are some values that differ greatly from the rest of the values in the set, for example, salaries.

9. The mean is used when each piece of data is to be considered and "weighed" equally, for example, weights of adult males.

11. 11, 10, 10, 14 **13.** 69.3, 72, none, 66

15. 8, 8, none, 8 **17.** 13.1, 11, 1, 18.5

19. 11.9, 12.5, 13, 11.5 **21.** 6.5, 5, 3 and 5, 10.5

23. a) 4.9, 5, 5, 6 **b)** 5.3, 5, 5, 6
c) Only the mean **d)** The mean and the midrange

25. A 79 mean average on 10 quizzes gives a total of 790 points. An 80 mean average on 10 quizzes requires a total of 800 points. Thus, Jim missed a B by 10 points, not 1 point.

27. a) 8.8 million **b)** 8.0 million **c)** None
d) 12.2 million

29. a) \$5.1 billion **b)** \$2.3 billion
c) \$2.3 billion and \$1.5 billion **d)** \$14 billion
e) Answers will vary.

31. 510 **33.** One example is 72, 73, 74, 76, 77, 78.

35. a) Yes **b)** No **c)** No **d)** Yes
e) Mean = 200; midrange = 275

37. a) 33 or greater
b) It is not possible if 100 is the maximum possible grade.
c) 22 or greater **d)** 82 or greater

39. One example: 1, 2, 3, 3, 4, 5, changed to 1, 2, 3, 4, 4, 5.

41. No, by changing only one piece of the 6 pieces of data you cannot alter both the median and the midrange.

43. The data must be ranked.

45. He is taller than approximately 35 percent of all kindergarten children.

47. a) \$430 **b)** \$350 **c)** \$650

49. Second quartile, median

51. a) \$490 **b)** \$500 **c)** 25%
d) 25% **e)** 17% **f)** \$51,000

53. a)

Ruth	Mantle
0.290	0.300
0.359	0.365
0.301	0.304
0.272	0.275
0.315	0.321

b) Mantle's is greater in every case.
c) Ruth: 0.316; Mantle: 0.311; Ruth's is greater.
d) Answers will vary.
e) Ruth: 0.307; Mantle: 0.313; Mantle's is greater.
f) Answers will vary. **g)** Answers will vary.

55. 90 **57. a)–c)** Answers will vary.

Section 13.6, Page 793

1. Range = highest value − lowest value

3. Answers will vary. **5.** Answers will vary. **7.** σ

9. Answers will vary.

11. They would be the same since the spread of data about each mean is the same.

13. a) The grades will be centered about the same number since the mean, 75.2, is the same for both classes.
b) The spread of the data about the mean is greater for the evening class since the standard deviation is greater for the evening class.

15. 11, $\sqrt{16.5} \approx 4.06$ **17.** 6, $\sqrt{4.67} \approx 2.16$

19. 11, $\sqrt{15.2} \approx 3.90$ **21.** 5, $\sqrt{3} \approx 1.73$

23. \$32, $\sqrt{137.78} \approx$ \$11.74 **25.** \$150, $\sqrt{2600} \approx$ \$50.99

27. a) \$63, $\sqrt{631.6} \approx$ \$25.13 **b)** Answers will vary.

c) Answers remain the same, range: \$63, standard deviation ≈ \$25.13.

29. a)–c) Answers will vary.

d) If each number in a distribution is multiplied by n, the mean and standard deviation of the new distribution will be n times that of the original distribution.

e) The mean of the second set is $4 \times 5 = 20$, and the standard deviation of the second set is $2 \times 5 = 10$.

31. a) The standard deviation increases. There is a greater spread from the mean as they get older.

b) ≈ 133 lb **c)** ≈ 21 lb

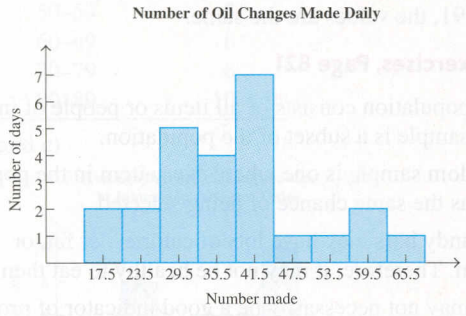
d) Mean: ≈ 100 lb; normal range: ≈ 60 to 140 lb

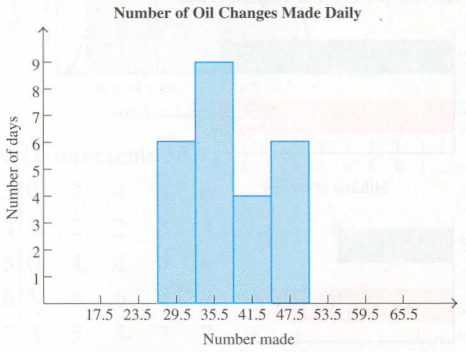
e) Mean: ≈ 62 in.; normal range: ≈ 53 to 68 in. **f)** 5%

33. a)

East		West	
Number of Oil Changes Made	Number of Days	Number of Oil Changes Made	Number of Days
15–20	2	15–20	0
21–26	2	21–26	0
27–32	5	27–32	6
33–38	4	33–38	9
39–44	7	39–44	4
45–50	1	45–50	6
51–56	1	51–56	0
57–62	2	57–62	0
63–68	1	63–68	0

b)





c) They appear to have about the same mean since they are both centered around 38.

d) The distribution for East is more spread out. Therefore, East has a greater standard deviation.

e) East: 38, West: 38 **f)** East: ≈ 12.64, West: ≈ 5.98

35. 6, 6, 6, 6, 6

Section 13.7, Page 804

1. A rectangular distribution is one in which all the values have the same frequency.

3. A bimodal distribution is one in which two nonadjacent values occur more frequently than any other values in a set of data.

5. A distribution skewed to the left is one that has "a tail" on its left.

7. a) *B* **b)** *C* **c)** *A* **9.–11.** Answers will vary.

13. Normal **15.** Skewed right

17. The mean is the greatest value. The median is lower than the mean. The mode is the lowest value.

19. Answers will vary. **21.** They all have the same value.

23. A z-score will be negative when the piece of data is less than the mean.

25. 0 **27.** 0.500 **29.** 0.818 **31.** 0.034 **33.** 0.037

35. 0.019 **37.** 0.053 **39.** 26.1% **41.** 89.8%

43. 97.1% **45.** 97.5% **47.** 2.9%

49. a) Jake, Sarah, Carol **b)** Marie, Kevin
c) Omar, Justin, Kim

51. 50% **53.** 10.6% **55.** 69.2% **57.** 24.1% **59.** 44.0%

61. 29.1% **63.** 59.9% **65.** 50.0% **67.** 10.6%

69. ≈ 21 women **71.** 86.2% **73.** 4.5% **75.** 69.2%

77. 0.6% **79.** ≈ 83 children **81.** 1.8%

83. The standard deviation is too large.

85. a) Katie: $z = 2.4$; Stella: $z = 1.7$

b) Katie. Her z-score is higher than Stella's z-score, which means her sales are further above the mean than Stella's sales.

87. Answers will vary. **89.** −1.18 **91.** 2

Section 13.8, Page 816

1. The correlation coefficient measures the strength of the relationship between the quantities.

3. 1 **5.** 0

7. A positive correlation indicates that as one quantity increases, the other increases.

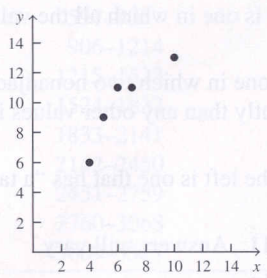
9. The level of significance is used to identify the cutoff between results attributed to chance and results attributed to an actual relationship between the two variables.

11.–13. Answers will vary.

15. Yes **17.** Yes **19.** No **21.** No

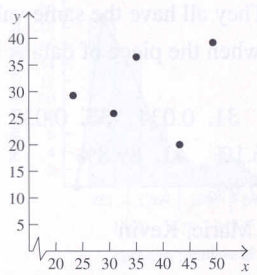
The answers in the remainder of this section may differ slightly from your answers, depending on how your answers are rounded and which calculator you used. The answers given here were obtained from a Texas Instruments TI-36x solar calculator.

23. a)



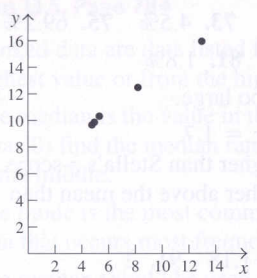
b) 0.903 **c)** Yes **d)** No

25. a)



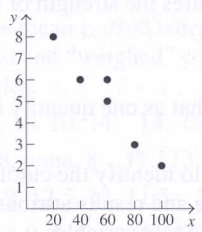
b) 0.228 **c)** No **d)** No

27. a)



b) 0.999 **c)** Yes **d)** Yes

29. a)



b) −0.968 **c)** Yes **d)** Yes

31. $y = 1.0x + 3.4$ **33.** $y = 0.2x + 23.8$

35. $y = 0.8x + 5.8$ **37.** $y = -0.1x + 9.5$

39. a) 0.960 **b)** Yes **c)** $y = 1.3x + 4.8$

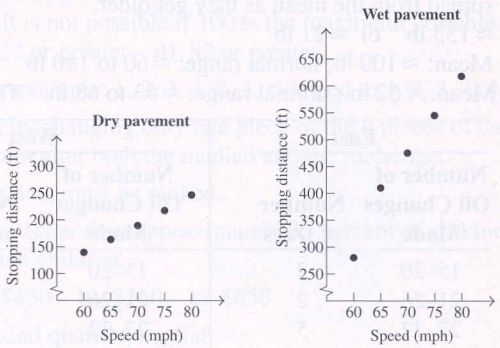
41. a) 0.950 **b)** Yes **c)** $y = 0.8x + 24.9$

43. a) −0.782 **b)** Yes **c)** $y = -0.7x + 22.3$
d) 12.5 muggings

45. a) 0.800 **b)** Yes **c)** $y = 0.2x + 2.3$ **d)** ≈ 25 units

47. a) −0.977 **b)** Yes **c)** $y = -12.9x + 99.6$
d) 41.6%

49. a) and **b)** Answers will vary.
c)



d) 0.999 **e)** 0.990 **f)** Answers will vary.
g) $y = 5.4x - 183.4$ **h)** $y = 16.2x - 669.8$
i) Dry, 232.4 ft; wet, 577.6 ft

51. Answers will vary.

53. a) 0.991 **b)** Should be the same.
c) 0.991, the values are the same.

Review Exercises, Page 821

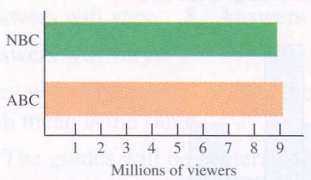
1. a) A population consists of all items or people of interest.
b) A sample is a subset of the population.

2. A random sample is one where every item in the population has the same chance of being selected.

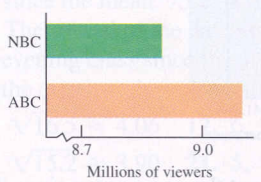
3. The candy bars may have lots of calories, or fat, or sodium. Therefore, it may not be healthy to eat them.

4. Sales may not necessarily be a good indicator of profit. Expenses must also be considered.

5. a)



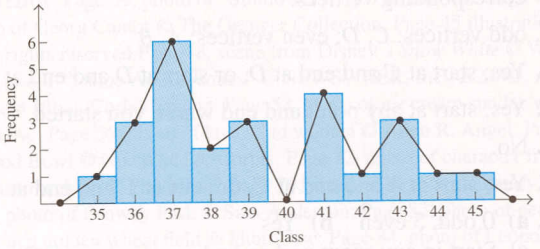
b)



6. a)

Class	Frequency
35	1
36	3
37	6
38	2
39	3
40	0
41	4
42	1
43	3
44	1
45	1

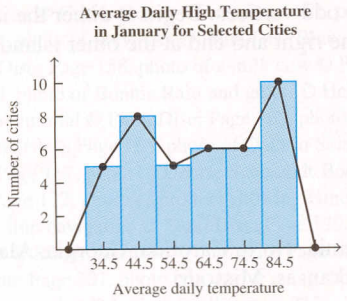
b) and c)



7. a)

High temperature	Number of Cities
30–39	5
40–49	8
50–59	5
60–69	6
70–79	6
80–89	10

b) and c)



d) 3 | 6 represents 36

3	0	3	4	5	6				
4	1	2	2	3	4	7	8	8	
5	0	4	4	5	6				
6	5	6	6	7	8	9			
7	3	5	5	7	7	9			
8	0	1	3	3	4	6	6	7	8	9

8. 80 **9.** 81 **10.** None **11.** 78 **12.** 30

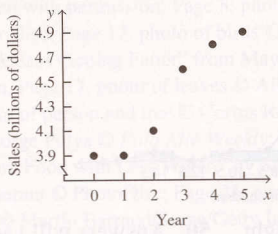
13. $\sqrt{104} \approx 10.20$ **14.** 13 **15.** 13 **16.** 7 and 12

17. 13.5 **18.** 19 **19.** $\sqrt{40} \approx 6.32$ **20.** 68.2%

21. 95.4% **22.** 94.5% **23.** 5.5% **24.** 72.6%

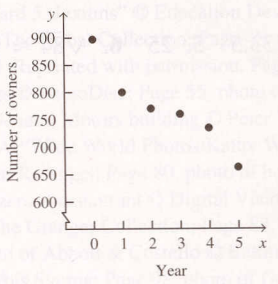
25. 34.1% **26.** 34.5% **27.** 29.0% **28.** 2.3%

29. a)



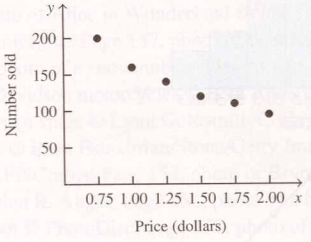
b) Yes; positive **c)** 0.964 **d)** Yes **e)** $y = 0.2x + 3.8$

30. a)



b) Yes; negative **c)** −0.952 **d)** Yes
e) $y = -39.3x + 869.0$

31. a)



b) Yes; negative **c)** −0.973 **d)** Yes
e) $y = -79.4x + 246.7$ **f)** ≈120 sold

32. 175 lb **33.** 180 lb **34.** 25% **35.** 25% **36.** 14%

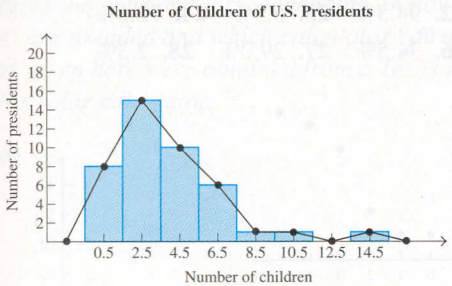
37. 18,700 lb **38.** 233 lb **39.** 145.6 lb **40.** ≈3.57

41. 2 **42.** 3 **43.** 7 **44.** 14 **45.** $\sqrt{8.105} \approx 2.85$

46.

Number of Children	Number of Presidents
0–1	8
2–3	15
4–5	10
6–7	6
8–9	1
10–11	1
12–13	0
14–15	1

47. and **48.**



49. No, it is skewed to the right. **50.** Answers will vary.

51. Answers will vary.

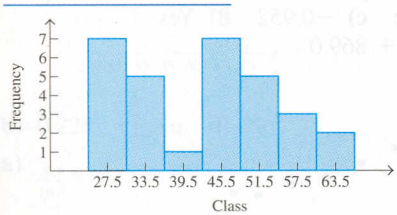
Chapter Test, Page 824

1. 36 **2.** 37 **3.** 37 **4.** 33.5 **5.** 25 **6.** $\sqrt{84} \approx 9.17$

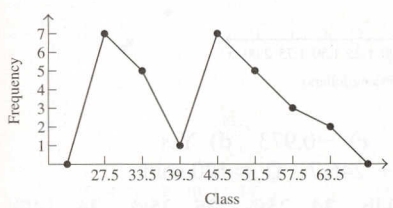
7.

Class	Frequency
25–30	7
31–36	5
37–42	1
43–48	7
49–54	5
55–60	3
61–66	2

8.



9.

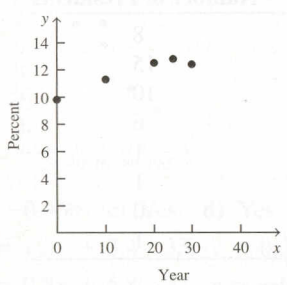


10. \$695 **11.** \$670 **12.** 75% **13.** 79%

14. \$70,000 **15.** \$740 **16.** \$640 **17.** 31.8%

18. 89.4% **19.** 10.6% **20.** ≈69 cars

21. a)



b) Yes **c)** 0.932 **d)** Yes **e)** $y = 0.1x + 10.1$ **f)** 14.1%

Appendix

Appendix, Page AA1

1. A vertex is a designated point.

3. To determine whether a vertex is odd or even, count the number of edges attached to the vertex. If the number of edges is odd, the vertex is odd. If the number of edges is even, the vertex is even.

5. 5 vertices, 7 edges **7.** 7 vertices, 11 edges

9. Each graph has the same number of edges from the corresponding vertices.

11. odd vertices: C, D; even vertices: A, B

13. Yes; start at C and end at D, or start at D and end at C.

15. Yes; start at any point and end where you started.

17. No.

19. Yes; start at A and end at C, or start at C and end at A.

21. a) 0 odd, 5 even **b)** Yes
c) Start in any room and end where you began. One path is A to D to B to C to E to A.
b) not traversable.

23. a) 2 odd, 4 even **b)** Yes
c) Start at B and end at F, or start at F and end at B. One path is B to C to F to E to D to A to B to E to F.

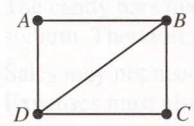
25. a) 4 odd, 1 even **b)** Not possible

27. a) 3 odd, 2 even **b)** Not possible

29. The door must be placed in room D. Room D is the only room with an odd number of doors.

31. Yes; there are two odd vertices. Begin at either the island on the left or on the right and end at the other island.

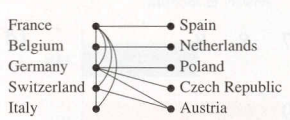
33.



35. a) Kentucky, Virginia, North Carolina, Georgia, Alabama, Mississippi, Arkansas, Missouri
b) Illinois, Arkansas, Tennessee

37. a) 4 **b)** 4 **c)** 11

39.



41. a) Yes, the graph has exactly 2 odd vertices.
b) One possiblity is C, A, B, E, F, D, G, C.

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INDEX

Note: Page numbers preceded by AA indicate material in the Appendix.

Abacus, 176
Abel, Niels, 564
Abelian groups, 561–562, 564–565
Absolute value, 812
Abstract algebra, 581
Acoustics, 791
Actuarial method, 624–626
Acute angle, 479
Acute triangle, 487
Adams, John Quincy, 460
Addition
  associative property of, 251–252, 253
  commutative property of, 251, 253
  in different base systems, 189–192
  distributive property of multiplication over, 252, 253, 293–294
  of fractions, 235–238
  of integers, 220–221
  of irrational numbers, 244–245
  of matrices, 402–403
  in modular arithmetic, 580–581
Addition formula, 696–697
Addition method, for linear equation systems, 393–398
Addition property of equality, 294
Additive identity element, 563
Additive inverse, 221–222, 563
Additive systems, 167–171
Add-on rate, 640
Adjacent angles, 479
Adjustable-rate mortgage, 635–636, 640–642
Adjusted monthly income, 636–637
Advertising, deceptive, 148, 757–759
Age, bone length and, 381
Aggregate rate cap, 641
Aleph-null, 85
Algebra, 287–302
  abstract, 581
  Boolean, 156
  checking solutions in, 288
  constants in, 288
  definition of, 288
  linear equations in, 292–317
  linear inequalities in, 329–332
  multiplication in, 288
  order of operations in, 289–291
  overview of, 288
  quadratic equations in, 350–359
  solving equations in, 288
  variables in, 288
  variation in, 320–326
  word problems in, 314–317
Algebraic equation, terms of, 292–293
Algebraic expression, 288
  evaluation of, 289
  examples of, 313–314
  factoring of, 352
  simplification of, 293
Algebraic notation, 288
Algorists, 175
Algorithm(s), 121, 351
  computer applications of, 114, 164–165, 351
  definition of, 121, 296
  Euclidian, 218
  for satisfiability problems, 121
  simplex, 421
  for weather forecasting, 365
all, as quantifier, 94–95
Allowable tolerance, 330
Alternate exterior angles, 481
Alternate interior angles, 481
American Standard Code for Informational Interchange (ASCII) code, 182
Ammann, Simon, 341
Amortization schedule, 640
Ampere, 465
Ancient Greeks, 228
and probability problems, 698–702
and statements, 61, 96–97, 101–103, 133, 160
  dominance of, 101–103
  equivalent, 133
  intersection of sets and, 61
  truth table for, 107–109
Angle(s), 478–482
  acute, 479
  adjacent, 479
  complementary, 479
  corresponding, 481, 486, 487
  definition of, 478
  degrees of, 478–479
  exterior, 481
  interior, 481
    sum of measures of, 486
  measure of, 478
  naming of, 478
  obtuse, 479
  right, 479
  of rotation, 525
  sides of, 478
  straight, 479, 486
  supplementary, 479
  trisecting of, 479
  vertex of, 478
  vertical, 481, 486
Animation, computer, 485
Annual interest rate, 604–605
Annual percentage rate, 615–617, 622, 623
Annuity, 621
Antecedent, 99, 118
Antennas, fractal, 550
Appel, Kenneth, 540–541
Approximation. *See also* Estimation
  of large number, 257
  of square root, 246–247
  symbol for, 247
Arc, in graphing, AA-2
Archimedes, 22
Architecture, golden proportion in, 277, 278
Area, 496–498
  of circle, 500–503
  metric units for, 441–443
  of parallelogram, 497
  vs. perimeter, 498
  of rectangle, 496, 498
  of square, 496, 498
  of trapezoid, 498
  of triangle, 497
  unit conversion for, 461, 462
Area sample, 754
Arguments. *See also* Logic
  invalid, 143, 148–149, 154, 160
  syllogistic, 153–157
  symbolic, 142–150
  valid, 143–147, 154–157, 160
Aristotelian logic, 93
Aristotle, 4, 92, 93, 135, 153
Arithmetic
  fundamental theory of, 211
  modular, 579–585
  universal, 294
Arithmetic mean, 753, 778
Arithmetic sequences, 268–270
Art
  Fibonacci numbers in, 279
  golden rectangle in, 278
ASCII code, 182
Assignment of truth value, 94
Associative groups, 561–562, 564, 569–570

Associative property, 251–252, 293–294
of addition, 251–252, 253
of mathematical systems, 561–562, 564, 569
of multiplication, 251–252, 253
Augmented matrix, 412–415
Average, 757
definition of, 778
mean as, 753, 778
median as, 779–780
mid-range as, 780
mode as, 780
weighted, 788
Average daily balance method, 627, 628–629
Axiom, 474
Axis
of reflection, 520
of symmetry, parabolic, 365
x-, 335, 343
y-, 335, 343
slope of line and, 335, 341–344

Babylonian numeration system, 175–178, 180
positional values in, 175–176, 181
Balloon-payment mortgage, 643
Bank discount, 605
Banker's rule, 606–608
Banks
Federal Reserve, 616
origin of, 605
Bar codes, U.S. Postal Service, 205–206
Base, 256, 289
Base systems
addition in, 189–192
base-2. *See* Binary system
base-3, 182
base-4, 182
base-5, 182, 190, 192, 193–194, 195
base-6, 182, 196
base-7, 194
base-10, 175, 182, 191–192
base-12, 182, 193
base-16, 186
base-60
Babylonian, 175, 176–178
navigational, 177
binary. *See* Binary system
computer, 182, 186
conversion between, 180, 183–186
division in, 195–196
multiplication in, 193–194
in positional-value numeration systems, 175, 181–186
subtraction in, 192–193
Bernoulli, Daniel, 654
Bernoulli, Jacob, 654
Bernoulli's theorem, 654
Biconditional statements, 100, 141, 160
definition of, 120
dominance of, 101–103
equivalent, 131
truth tables for, 120–125
Big Bang, 560, 566
Bimodal data, 780
Bimodal distribution, 796
Binary operation, 561
Binary system, 182, 186
addition in, 189–190, 189–192
for computers, 182, 186, 191
Binomial probability formula, 736–742
Binomials, multiplication of, 351
Birthdays, shared, 700
Bit, 182
Bivariate data, 809
Blackwell, David H., 799
Blood samples, 11
Blood types, 68
Bolyai, Farkos, 546
Bolyai, Janos, 546
Bonds, 618
Bone length, height and, 381
Boole, George, 93, 98, 114
Boolean algebra, 156
Boolean logic, 98
Borrowing. *See* Loan(s)
Bourbaki, Nicholas, 251
Braces, in set notation, 42
Brain, mapping of, 547
Branching, 210
Break-even analysis, 386
Break-even point, 386
Brent, Richard, 215
Buckminsterfullerene, 491
Buckyballs, 491
but statements, 96, 99
Byte, 182

Calculators
graphing, 308, 338, 381
scientific notation on, 262–264, 307, 741
square root approximation with, 246–247
Cameras, light rays and, 477
Cameron, Michael, 215
Candela, 465
Cantor, Georg, 42, 83, 84, 85, 86
Capital gains, 608
Carbon dating, formula for, 306–307
Card games, 726, 731
Cardano, Girolamo, 654
Cardinal numbers, 49
of set, 45, 85–86
Carissan, Eugene Oliver, 211
Carmody, Phil, 215
Carpet, Sierpinski, 551
Carroll, Lewis, 83, 132, 499
Cartesian coordinate system, 335–338
Cash advance, 626, 629
Catalan conjecture, 261
Cause-effect relationship, 812
Cayley, Arthur, 406
Celsius scale, 453–455, 459
Census, 751
Center of rotation, 525
centi, 433, 434
Centigrade thermometer, 453
Centimeter, 440
Cézanne, Paul, 510
Chebyshev's theorem, 807
Chinese numeration system, 170–171, 186
Chromosomes, 658
Ciphered numeration systems, 172–173
Ciphers, 582
Circle graphs, 767–768, 770
Circles, 500–503
area of, 500–503
circumference of, 500
diameter of, 500
great, 547
radius of, 500
squaring of, 479
Circuits
parallel, 164
series, 164
switching, 164
Circumference, 500
Class mark, 764
Class midpoint, 764
Class width, 762, 763
Classes, data, 762–764
Classification
taxonomic, 54
Venn diagrams in, 70
Clock arithmetic, 568–571
Closing, on house purchase, 636
Closure of sets, 250–251, 562
Cluster sample, 754
Code-breaking, 582

Coefficient
linear correlation, 808–810
numerical, 293
Pearson correlation, 812
of radical, 243
Collateral, 603
Collinear, 337
Combinations, 724–727
vs. permutations, 725–727
in solving probability problems, 730–733
Combined variation, 324–326
Comma, in statements, 97–98, 100
Common denominator, 235
Common difference, 268
Common ratio, 270
Commutative groups, 561–562, 564–565, 568–570
Commutative property, 251, 293–294
of mathematical systems, 561–562, 564–565, 568–570
Compass, 476
Complement, of set, 57, 76
Complementary angles, 479
Composite numbers, 209–210
in encryption, 214
Compound inequality, 331
Compound interest, 613–618
formula for, 614
Compound statements, 94, 95–96. *See also* Statements
truth tables for, 106–125. *See also* Truth tables
truth value of, 94, 113–115, 121–122
Computer animation, 485
Computer graphics, matrices in, 414
Computers
accuracy tests for, 245
binary system for, 182, 186, 191
e-mail and, 371
encryption for, 214
gates in, 114, 164–165
hexadecimal system for, 186
octal system for, 186
spreadsheet and, 405
in weather forecasting, 708
Conclusion, 99n
Conditional probability, 707–710
Conditional statements, 99–100, 141, 160
contrapositive of, 135–138
converse of, 135–138
conversion of to disjunction, 133–134
definition of, 118
dominance of, 101–103
inverse of, 135–138
negation of, 134
truth tables for, 118–120
variations of, 135–138, 160
Conformal mapping, 547
Congruence, 579–580
Congruent figures, 489–490
Conjecture, 3, 5, 215
Goldbach's, 215
Conjunctions, 61, 96–97, 101, 133, 160. *See also and*
dominance of, 101–103
truth table for, 107–109
Connectives, 95–103, 160
biconditional *(if and only if),* 100, 160. *See also* Biconditional statements
conditional *(if-then),* 99–100, 160. *See also* Conditional statements
conjunction *(and),* 61, 96–97, 101, 133, 160. *See also* Conjunction
disjunction *(or),* 97, 101–103, 160. *See also* Disjunction
dominance of, 101–103
logical, 93
negation *(not),* 61, 94, 96, 98, 100–103, 160. *See also* Negation
truth tables for, 106–125, 160
Consequent, 99, 118
Consistent system, 384
Constant, 288
Constant of proportionality, 320, 321–322
Constraints, 421, 423, 424
Consumer mathematics, 592–652
Contraposition, law of, 146
Contrapositive, of conditional statement, 135–138
Convenience sampling, 755
Conventional mortgage, 636–640
Converse
of conditional statement, 135–138
fallacy of, 148
Coordinate system, 335–338
Corporate bonds, 617
Correlation, 808
linear, 808–814
Correlation coefficient, 808–810
Pearson, 812
Corresponding angles, 481, 486, 487
Cosigner, 603
Countable sets, 85–86
Counterexample, 4, 562
Counting boards, 176, 412
Counting numbers, 2, 208, 219
Counting principle, 686–687, 689–697, 714–716, 727
Credit cards, 626–630
Critical thinking skills. *See also* Logic
deductive reasoning, 4–5
estimation, 8–14
inductive reasoning, 2–5
problem solving, 19–29
Critical value, 812
Cross multiplication, 300–303
Cryptology, 411
Cube
doubling of, 479
volume of, 508
Cubic units, in metric system, 443–445, 452
Cubic yards–cubic feet conversion, 514
Cubism, 510
Customary units (U.S.), 432, 440, 458
conversion of to metric units, 458–464
Cutoff value, 812
Cylinder, volume of, 508

Dantzig, George B., 421, 422
Data, 752
bimodal, 780
bivariate, 809
classes of, 762–764
grouped, 762
piece of, 761
ranked, 779–780
variability in, 789–792
Data spread, 789–792
Date of maturity
for bonds, 618
for loans, 606–610
D-Day, 442
De Morgan, Augustus, 131, 133
De Morgan's laws
in logic, 131–134, 160
in set theory, 70–71, 87, 133
Dead man's hand, 731
Debt, national, 263
deci, 434
Deciles, 788
Decimal notation, scientific notation and, 261–262
Decimal numbers, 231–234
conversion to fractions, 232–234
conversion to percent, 594
irrational numbers and, 243
nonrepeating, 242–243
nonterminating, 242
repeating, 231–234
terminating, 231–232

Decimeter, cubic, 443
Decision-making process, 26
Deductive reasoning, 4–5
Deficit, 263
Definitions
  geometric, 474
  undefined terms in, 474
Degree Celsius (^{O}C), 433
Degrees, of angles, 478–479
deka, 433, 434
De Morgan, Augustus, 131, 133, 540–541
De Morgan's laws
  in logic, 131–134, 160
  in set theory, 70–71, 87, 133
Denominator
  common, 235
  rationalization of, 246
Dependent events, 700, 707
Dependent systems, 385, 393, 396
  matrices for, 416
Dependent variables, 343, 362, 809
Descartes, René, 294, 335, 336
Descriptive statistics, 752
Detachment, law of, 144
Diameter, 500
Dice, 654, 707
Difference of two sets, 65
Digits, 175
Dimensional analysis, 458–460. *See also* Metric system
Diophantus, 294
Direct variation, 320–322
Discount note, 605
Discriminant, 361
Disjoint sets, 56
Disjunctions, 97, 160. *See also or*
  conversion of conditional statement to, 133–134
  dominance of, 101–103
  truth table for, 109–110
Disjunctive syllogism, 147
Disraeli, Benjamin, 751
Distribution(s)
  bimodal, 796
  frequency, 761–765
  Gaussian, 798
  J-shaped, 796
  normal, 798–799
  rectangular, 796
  skewed, 797
Distributive property, 252, 253, 293–294
Dividends, 608
Divine proportion, 277
Divisibility, 208, 209, 210t
Division, 208
  in base conversion, 180, 184, 185
  definition of, 224
  in different base systems, 195–196
  in Egyptian numeration system, 168
  of fractions, 235
  of integers, 224–225
  of irrational numbers, 245
  negative numbers in, 330
  in prime factorization, 210–212
  rules for, 225
  in scientific notation, 262
Division property of equality, 295–296
Divisor, 208, 228
  common, 228
    greatest, 212–213
Dodgson, Charles, 83, 132, 499
Dollars, fractions of, 233
Domain, of function, 362, 367–368
Dominant traits, 656–657
Dosage, in metric units, 445, 463, 464
Doubling the cube, 479
Down payment, 635
Drug dosages, in metric units, 445, 463, 464
Du Minuit, Peter, 614
Duodecimal system, 182
Duplation and mediation, 168, 199
Dupuit, Jules, 394

Economic models, 394
Eddington, Arthur Stanley, 560
Edge
  in graphing, AA-2
  of polyhedron, 510–511
EE key, 262–264
Effective annual yield, 615–617
Egyptian numeration system, 168
Einstein, Albert, 309, 473, 549, 566, 581
Electric current, base unit for, 465
Elements
  identity, 563
  inverse of, 563
  of set, 42, 43
Elements (Euclid), 474, 476, 545
Eléments de Mathématique (Bourbaki), 41
Elimination method, for linear equation systems, 393–398
Ellipsis, 2
  in sets, 42
Elliptical geometry, 546–547
  fifth postulate in, 547, 548
E-mail, 371
Empirical probability, 655–659
  in genetics, 656–657
Empty set, 46, 52
Encryption, 214, 582
Encyclopedia, mathematical, 251
End point, 475
Enigma, 582
Enormous theorem, 565
Equality
  addition property of, 294
  division property of, 295–296
  multiplication property of, 295
  subtraction property of, 294–295
Equality of sets, 45, 57
  verification of, 68–70
Equally likely outcomes, 663
Equations. *See also* Functions
  equivalent, 294
  exponential, 306–308
  Fermat, 310
  graphing of, 306–308, 335–344. *See also* Graphing
  inconsistent, 299
  linear, 294–302. *See also* Linear equations
  quadratic, 350–359
  solution of, 288
    checking of, 288
  solving for variable in, 309–310
Equilateral triangle, 417
Equity, 637
Equivalent equations, 294
Equivalent fractions, 236–237
Equivalent sets, 45
Equivalent statements, 128–138
  De Morgan's laws for, 131–134
  definition of, 128
  negation of conditional statement as, 134
  truth tables for, 128–131
  variations of conditional statement as, 135–138
Eratosthenes, 209
Escher, M.C., 531, 552–553
Estimation, 8–14. *See also* Approximation
  in calculations, 10
  in determining distances, 10–11
  early methods of, 12
  in medicine, 11
Euclid, 215, 474
  fifth postulate of, 545–546
Euclidian algorithm, 218
Euclidian constructions, 476
Euclidian geometry, 474, 547
  fifth postulate in, 547, 548

Euclidian parallel postulate, 545–546
Euler, Leonhard, 154, 155, 215, AA-1
Euler diagrams, 155–157
Euler's formula for networks, AA-6
Euler's polyhedron formula, 510–511
European Union, metric system and, 455
Even numbers, 2
Event, 654
Exact values, 246, 257
Exclusive *or,* 93–94
EXP key, 262–264
Expanded form, 175
Expected value (expectation), 678–683
Experiment, 654
Experimental science, logic in, 135
Exponential equations (formulas), 306–308
  decay, 306–307
  growth, 307–308
  natural, 307, 368
Exponential functions, 368–372
  decay, 369
  definition of, 369
  graphing of, 369–372
  natural, 368–369
  vertical line test for, 362–363
Exponents, 256–260, 289
  definition of, 256
  negative, 259, 260
  power rule for, 259–260
  product rule for, 257–258, 260
  quotient rule for, 258, 260
  zero, 258–259, 260
Expression. *See* Algebraic expression
Exterior angles, 481

Face, of polyhedron, 510–511
Factor(s), 209, 210t, 352
  proper, 218
Factor tree, 211
Factoring
  definition of, 352
  of quadratic equations, 355–357
  of trinomials, 351–355
Factoring machine, 211
Fahrenheit scale, 454–455, 469
Fair game, 678
Fair price, 682–683
Fallacy, 143, 148–149, 154
  of the converse, 148
  of the inverse, 148, 160
Falling objects, speed of, 4
Feasible region, 421
Fechner, Gustav, 279
Federal deficit, 263
Federal Reserve Banks, 616
Fermat, Pierre de, 215, 501, 654
Fermat equation, 310
Fermat number, 215
FHA mortgage, 642
Fibonacci, 275, 276
Fibonacci sequence, 275–279, 281
Fifth postulate, in different geometries, 547, 548
Films, target audience for, 335
Finance charges, 622
  calculation of, 624–630
Fingerprints, 3
Finite mathematical systems, 568–574
Finite sets, 44
First-degree equations. *See* Linear equations
Fixed installment loan, 622–626
Fluid
  mass of, 452, 469
  volume of, 452, 469
FOIL method, 351
Formula(s), 304–311
  and, 698–702
  addition, 696–697
  binomial probability, 736–742
  combination, 726
  compound interest, 307–308, 614
  conditional probability, 707–710
  Euler's polyhedron, 510–511
  evaluation of, 304
  exponential, 306–308
  multiplication, 699–702
  or, 695–698, 702
  permutation, 719
  permutation of duplicate objects, 721
  present value, 617
  probability, 663–664
  quadratic, 357–359
  simple interest, 304, 603
  solving for variable in, 309–310
  statistical, 305, 309–310
  subscripts in, 305
  tax free yield, 310–311
  volume, 304–305
Formula(s), Euler's network, AA-6
Four 4's, 227, 249
Four-color map problem, 540–541
Fractal antennas, 550
Fractal geometry, 549–551
Fractal tree, 550–551
Fractions, 228–231
  addition of, 235–238
  conversion of decimal numbers to, 232–234
  conversion of mixed numbers to, 229–231
  conversion to percent, 593
  division of, 235
  of dollars, 233
  in Egyptian numeration system, 168
  equivalent, 236–237
  improper, 229–231
  multiplication of, 234–235
  reduction of, 228–229
  solving equation containing, 297
  subtraction of, 235–238
  unit, 458–460
Franklin, Benjamin, 613
Free speech, advertising claims and, 148
Frequency distributions, 761–765
Frequency polygons, 762, 770–773
Friendly numbers, 213
Fuller, F. Buckminster, 491
Fullerenes, 491
Functions. *See also* Equations
  definition of, 362, 363
  domain of, 362, 367–368
  exponential, 368–372
  graphing of, 361–372, 363–364
  linear
    definition of, 363
    graphing of, 363–364
    vertical line test for, 362–363
  objective, 421–422
  quadratic
    definition of, 364
    domain of, 362, 367–368
    graphing of, 364–368
    range of, 362, 367–368
    vertical line test for, 362–363
  range of, 362, 367–368
  vertical line test for, 362–363
Fundamental law of rational numbers, 236
Fundamental theory of arithmetic, 211
Fuzzy logic, 138

Galileo, 4
Galley method, 199–200
Galois, Evariste, 564
Gambling
  odds in, 671–674
  slot machines for, 702, 714
Games
  card, 726, 731
  fair, 678
Garfield, James A., 499
Gaunt, John, 753

Gauss, Carl Friedrich, 269, 310, 546, 798
Gaussian distribution, 798
Gelosia method, 199–200
General theory of relativity, 309, 549
Genes, 657
Genetic disorders, 656, 658
  polygenetic, 706
Genetics
  empirical probability in, 656–658
  Mendelian, 656–657
Genus, of object, 542
Geodesic, 548
Geometric constructions, 475
Geometric motion. *See* Rigid motion
Geometric sequences, 270–272
Geometry, 473–559
  angles in, 478–482. *See also* Angle(s)
  area in, 496–498
  axioms in, 474
  circles in, 500–503
  classical problems in, 479
  congruent figures in, 489–490
  definition of, 474
  definitions in, 474
  direct variation in, 320
  elliptical, 546–547
  Euclidian, 474, 547
  fractal, 549–551
  history of, 474
  hyperbolic, 546, 547, 548
  joint variation in, 323–324
  Jordan curve in, 540–541
  Klein bottle in, 539–540
  lines in, 474–477
  maps in, 540–541
  Möbius strip and, 538–539
  models in, 547
  non-Euclidian, 545–551
  perimeter in, 496–498
  plane, 477
  planes in, 477–478
  points in, 474–477
  polygons in, 485–491
  postulates in, 474
  Pythagorean theorem in, 499
  Riemannian, 546–547
  rigid motion in, 519, 520–530
  rubber sheet, 538. *See also* Topology
  similar figures in, 487–489
  solid, 507–514
  spherical, 546–547
  symmetry in, 528–530
  tessellations in, 530–532
  theorems in, 474
  topologic equivalence in, 542
  transformational, 519–538
  volume in, 507–514
Germain, Sophie, 310
giga, 434
GIMPS (Great Internet Mersenne Prime Search), 215
Girard, Albert, 297
Glide, 522–524
Glide reflection, 527–528
Goldbach, Christian, 215
Goldbach's conjecture, 215
Golden number, 277
Golden proportion, 277, 278
Golden ratio, 277, 278
Golden rectangle, 278
Government revenue, 263
Graduated payment mortgage, 642–643
Gram, 451
Graph(s)
  circle, 767–768, 770
  definition of, 337
  edges in, AA-2
  frequency polygon, 762, 770–773
  histogram, 768–769
  misleading, 757–759
  statistical, 762, 767–773
  stem-and-leaf display, 772–773
  traversibility of, AA-2–3
  vertices in, AA-2
Graph theory, AA-1–6
  rule of traversibility in, AA-2–3
Graphing, 335–344
  applications of, 343–344
  Cartesian coordinates in, 335–338
  dependent/independent variables in, 343
  of exponential functions, 369–372
  of functions, 361–372
    linear, 363–364
  half-planes in, 348–349
  of inequalities, 329
    in two variables, 348–349
  of linear equation systems, 384–385
  of linear equations, 306–308, 335–344
  of linear functions, 361–372
  by plotting points, 336, 337–338
  of quadratic equations, 365–367
  by using intercepts, 338–339, 341–344
  by using slope of line and *y*-intercept, 340–344
  vertical line test in, 362–363
  x-axis in, 335, 343
  y-axis in, 335, 341–344
Graphing calculators, 308, 338, 381
Great circle, 547
Great Internet Mersenne Prime Search (GIMPS), 215
Greatest common divisor, 212–213
Greek numeration systems, 172–173
Grids, 339
Group(s), 560, 564–565. *See also* Mathematical systems
  associative, 561–562, 564, 569–570
  closure property of, 562
  commutative (abelian), 561–562, 564–565, 568–570
  criteria for, 564, 565
  definition of, 560
  identity element in, 563, 564
  inverses in, 563
  noncommutative (nonabelian), 570
Group theory, 560
  in code-breaking, 582
  Rubik's cube and, 572
  scientific applications of, 563, 566

Haken, Wolfgang, 540–541
Half line, 475
Half open line segment, 475–477
Half-plane, 348–349
Halsted, George Bruce, 546
Hamilton, William Rowan, 406
Harmonics
  Fibonacci numbers in, 279
  rational numbers in, 236
Hectare, 441
hecto, 434
Height
  bone length and, 381
  estimation of, 12
Hemophilia, 658
Hexadecimal system, 186
Hieroglyphics
  Egyptian, 168
  Mayan, 178
Hindu-Arabic numeration system, 166, 167, 168, 175, 180
  positional values in, 175, 181
Histograms, 762, 768–769
Holmes, Sherlock, 143
Home buying/ownership
  benefits of, 637
  closing in, 636
  down payment in, 635, 636
  home prices and, 779
  mortgages for, 636–643
  points in, 636
Home equity loans, 643
however statements, 96
Hung jury, 146

Huygens, Christian, 654
Hyperbolic geometry, 546, 547, 548
  fifth postulate in, 547, 548
Hypothesis, 3, 5, 99n

Identification methods, 3
Identity, 300
Identity element, 563
Identity matrix, in multiplication, 407
If and only if statements, 100, 160
  definition of, 120
  dominance of, 101–103
  equivalent, 131
  truth tables for, 120–125
If-then statements, 99–100, 160. *See also* Conditional statements
Implications, 125
Improper fractions, 229–231
Inclusive *or,* 94, 97
Income, adjusted monthly, 636–637
Inconsistent equation, 299
Inconsistent system, 385, 396
  matrix for, 415
Independent events, 700, 707
Independent variable, 343, 362, 809
Inductive reasoning, 2–5
Inequality(ies), 220
  compound, 331
  definition of, 328
  graphing of, 329, 348–349
  linear, 329–332
    systems of, 418–420
  order of, 329
  solving, 330–332
  in two variables, graphing of, 348–349
Inequality symbols, 219–220, 328
  direction of, 331, 332
Inferential statistics, 752
Infinite mathematical systems, 562
Infinite sets, 44, 83–86
  countable, 85–86
Installment loans, 621–630
  fixed, 622–626
  open-ended, 622, 626–630
Integers, 219–225
  addition of, 220–221
  associative, 561–562, 562
  commutative, 561–562
  definition of, 219
  division of, 224–225
  inverse of, 563
  in mixed numbers, 229
  multiplication of, 222–224
  negative, 219
  positive, 219
  real number line for, 219–220
  sets of, 219, 250. *See also* Sets
  subtraction of, 221–222
Intercept
  x, 338–339
  y, 338–339
Interest
  compound, 613–618
    formula for, 307–308
  credit card, 626–630
  ordinary, 603–610
  simple, 304, 603–610
    formula for, 304
  unearned, 624
  usurious, 607
Interest rate, 603
  annual, 604–605
  prime, 612
  true, 605
Interior angles, 481
  sum of measures of, 486
Internet usage, 765
Intersection of sets, 57–58, 61–62
Invalid arguments, 143, 148–149, 154, 160
Inverse, 563
  additive, 221–222
  of conditional statement, 135–138
  fallacy of, 148, 160
  multiplicative, 563
Inverse elements, 563
Inverse variation, 322–323
Ionic Greek numeration systems, 172–173
Iris-based identification, 630
Irrational numbers, 242–247
  addition of, 244–245
  approximation of, 246–247
  definition of, 242
  division of, 245
  multiplication of, 245
  in sets of real numbers, 249–250
  simplification of, 243–244
  subtraction of, 244–245
Isosceles triangle, 417

James, William, 244
Jiuzhang Suanshu, 412
Johannsen, W., 657
Joint variation, 323–324
Jordan curve, 540–541
J-shaped distribution, 796
Juries, 146

Kanada, Yasumasa, 244
Kelvin, temperatures, 465
Kepler, Johannes, 277, 278
kilo, 433, 434
Kilogram, 433, 451, 465
Kiloliter, 443
Kilometer, 440
Klein bottle, 539–540
Knot theory, 207
Koch, Helga von, 549
Koch snowflake, 549–550
Konig, Denes, AA-1
Könisberg bridge problem, AA-1–2
Kronecker, Leopold, 84
Kurowski, Scott, 215

Lagrange, Joseph, 309, 310
Laplace, Pierre-Simon de, 654
Larder, Dionysis, 394
Large numbers
  approximations of, 257
  law of, 658–659
  scientific notation for, 260–264
Lateral faces, of prism, 511
Law of contraposition, 146
Law of detachment, 144
Law of large numbers, 658–659
Law of syllogism, 147
Le Corbusier, 277
Le Grand K, 451
Least common multiple, 213–214
Least squares line, 814–816
Length
  base unit for, 465
  customary units for, 440
  metric units for, 440–441
  unit conversion for, 461–462
Leonardo of Pisa (Fibonacci), 275, 276
Les Demoiselles d'Avignon (Picasso), 510
Level of significance, 812
Liber Abacci (Fibonacci), 275, 276
Like terms, 293
Lindemann, Carl, 479
Line(s), 474–477
  of best fit, 814–816
  in different geometries, 547
  end point of, 475
  half, 475
  least squares, 814–816
  parallel, 477–478
  properties of, 475
  real number, 219–220, 328–329
  reflection, 520
  regression, 814–816
  skewed, 478

Line(s) *(continued)*
slope of
definition of, 340
determination of, 340–341
in graphing, 340–344
of symmetry, 528
transversal, 481
Line segment, 475–477
open, 475–477
Linear correlation, 808–814
Linear correlation coefficient, 808–810
Linear equations
containing decimals, 298
containing fractions, 297
definition of, 294
graphing of, 306–308, 335–344. *See also* Graphing
historical perspective on, 315
with infinite number of solutions, 299–300
with no solution, 299
in one variable
applications of, 313–317
solving, 294–302
terms in, 292–304
simultaneous, 383
slope-intercept form of, 341–344
solving, 294–300
general procedure for, 296–297
using definition of subtraction, 298–299
for variable, 309–310
systems of, 383–427
addition method for, 393–398
consistent, 384
definition of, 383
dependent, 385, 393, 396, 416
inconsistent, 385, 396, 415
matrices for, 412–417
solutions to, 383–388
substitution method for, 391–393
in two variables, 337
with variables on both sides, 297–298
for word problems, 314–317
Linear functions
definition of, 363
graphing of, 363–364
vertical line test for, 362–363
Linear inequalities, 329–332. *See also* Inequality(ies)
systems of, 382–427, 418–420
Linear programming, 421–425
fundamental principle of, 422
Linear regression, 814–816
Linnaean system, 54
Liquid
mass of, 452, 469
volume of, 452, 469
Liter, 433, 443
Lo Shu diagram, 28
Loan(s), 602–610
banker's rule for, 606–608
due date of, 606–610
discount, 605
early repayment of, 624–626, 639–640
home equity, 643
installment, 621–630
mortgage, 635–643
partial payment of, 605–610
personal, 603
principal of, 602
seasonal, 603
security (collateral) for, 603
United States rule for, 606, 608–610
Loan sharks, 607
Lobachevsky, Nikolay Ivanovich, 546
Logarithmic spiral, 276, 278
Logic, 92–160
Aristotelian, 93
Boolean, 98
connectives in, 93, 95–103, 160. *See also* Connectives
De Morgan's laws in, 131–134, 160
definition of, 92
equivalent statements in, 128–138
Euler diagrams in, 155–157
experimental applications of, 135
fuzzy, 138
history of, 93
implications in, 125
purpose of, 92
quantifiers in, 94–95, 160
self-contradictions in, 124
statements in, 94–138. *See also* Statements
syllogistic, 153–157
symbolic, 93, 133, 142–150
tautologies in, 124–125
truth tables in, 106–125
Lottery, 681, 725
Lower class limits, 763
Lowest common denominator, 235
Lucas sequence, 281
Luminosity, base unit for, 443, 465

Magic squares, 28–29
Mandelbrot, Benoit, 473
Maps, 540–541
of brain, 547
conformal, 547
Margin, 640
Mariana Trench, 222
Mars Climate Orbiter, 432
Mass
base unit for, 465
in metric system, 450–453
unit conversion for, 461, 462–463
vs. volume, 452
Mathematical encyclopedia, 251
Mathematical models, 305, 385–387
economic, 394
for weather forecasting, 365
Mathematical systems, 560
associative properties of, 561–562, 564, 569–570
binary operation in, 561
closure property of, 562
commutative properties of, 561–562, 564–565
commutative property of, 561–562, 564–565, 568–570
definition of, 561
finite, 568–574
as groups, 564–566. *See also* Group(s)
identity element in, 563
infinite, 562
inverses in, 563
modulo, 579–585
without numbers, 571–574
Mathematics
consumer, 592–652
symbolic, 294
Matrices, 402–408
addition of, 402–403
augmented, 412–415
in computer graphics, 414
definition of, 402
for dependent systems, 416
dimensions of, 402
early examples of, 412
elements of, 402
equal, 402
identity, in multiplication, 407
for inconsistent systems, 415
multiplication of, 403–408
multiplicative identity, 407, 413
row transformations for, 412–415
triangularization method for, 416–417
in spreadsheets, 405
square, 402, 407
subtraction of, 403
Mayan numeration system, 178–179
Mean, 753, 778, 781

Measurement systems
metric, 431–472. *See also* Metric system
U.S. customary, 432, 440, 458
Measures of central tendency, 778–784
mean as, 753, 778, 781
median as, 779, 781–782
midrange as, 780–781, 782
mode as, 780–781, 782
Measures of dispersion, 788–792
Measures of position, 782–784
Median, 779, 781–782
Medication dosages, metric units for, 445, 463, 464
Medicine
estimation in, 11
metric system in, 445, 463, 464
mega, 434
Members, of set, 42, 43
Mendel, Gregor, 656, 657
Mendelian genetics, 656–657
Mersenne primes, 215
Meter, 433, 440, 465
Metric system, 431–472
advantages of, 432, 439
area in, 441–443
base units in, 465
conversion to and from, 458–464
conversion within, 434–437
as decimal system, 432, 433
drug dosages in, 445, 463, 464
European Union and, 455
length in, 440–441
mass in, 450–453
in medicine, 445, 463, 464
nonadopters of, 455
prefixes in, 433–434, 435, 469
publications on, 435
temperature in, 453–455
terminology of, 431–472
volume in, 443–446, 452
Metric tonne, 451
micro, 434
Midrange, 780–781, 782
Mihailescu, Preda, 261
Military operations, linear programming in, 422
Milky Way, 566
milli, 433, 434
Milligram, 451
Millimeter, 440
Millimicron, 261
Mixed numbers, 229–231
Mixture problem, 397
Möbius strip, 538–539, 541
Klein bottle and, 539–540
Modal class, of frequency distribution, 764
Mode, 780–781, 782
Models
geometric, 547
mathematical, 305, 385–387
economic, 394
for weather forecasting, 365
Modular arithmetic, 579–585
congruence in, 579–580
Modulo classes, 580
Modulo system, 579–585
Modulor, 277
Modus ponens, 144
Modus tollens, 146
Mole, 465
Monthly Mortgage Payment Calculator, 640
Moon rocks, 364
Morain, François, 215
Mortgages, 635–643
adjustable-rate, 635–636, 640–642
amortization schedule for, 640
balloon-payment, 643
conventional, 636–640
FHA, 642
graduated payment, 642–643
monthly payment calculation for, 637–638
qualifying for, 637–638
second, 643
VA, 642
Motion
of falling object, 358
projectile, 366
rigid, 519, 520–530
Movies, target audience for, 335
Mozart, Wolfgang Amadeus, 707
Multiplication
algebraic, 288
associative property of, 251–252, 253
in base conversion, 180, 183, 185
of binomials, 351
in Chinese numeration system, 170–171
commutative property of, 251, 253
cross, 300–303
in different base systems, 193–194
in dimensional analysis, 459–460
distributive property of over addition, 252, 253, 293–294
by duplation and mediation, 168, 199
early methods of, 199–201
in Egyptian numeration system, 168
of fractions, 234–235
galley (Gelosia) method for, 199–200
of integers, 222–224
of irrational numbers, 245
of matrices, 403–408
in modular arithmetic, 581–582
Napier rods for, 200–203
negative numbers in, 329–330
in Roman numeration system, 169
rules for, 224
in scientific notation, 262
of unit fractions, 459
Multiplication formula, 699–702
Multiplication property of equality, 295
Multiplication property of zero, 222
Multiplicative identity element, 563
Multiplicative identity matrix, 413
Multiplicative inverse, 563
Multiplicative numeration systems, 170–171
Municipal bonds, 617
Music
computers in, 707
Fibonacci numbers in, 279
rational numbers in, 236
Mutual funds, 643
Mutually exclusive events, 697

Napier, John, 200
Napier rods, 200–203
National debt, 263
Natural exponential equation, 307, 368
Natural exponential formula, 307
Natural exponential functions, 368–369
Natural numbers, 2, 208, 219
closed, 250–251
factors of, 208
sets of, 250. *See also* Sets
Navigation, base 60 system in, 177
Negation, 94, 96, 98, 101
complement of sets and, 61
of conditional statement, 134
dominance of, 101–103
truth table for, 106–107, 110–111
Negative exponent, 259, 260
Negative integers, 219
Negative numbers, 297
dividing by, 330
mixed, 231
multiplying by, 329–330
as product, 224
as quotient, 225
Neilsen ratings, 752
Neither-nor statements, 99
Net amount, 680

Network, AA-1, AA-2
traversible, AA-2–3
Network theory. *See* Graph theory
Networks, Euler's formula for, AA-6
nevertheless statements, 96
Newton, Isaac, 294
Nine Chapters on the Mathematical Art, 412
Noether, Amalie Emmy, 581
Nonabelian groups, 570
Noncommutative groups, 570
none (no), as quantifier, 94–95
Non-Euclidian geometry, 545–551
Nonrepeating decimal numbers, 242–243
Nonterminating decimal numbers, 242–243
Normal curve, 796–804
finding area under, 800–804
Normal distribution, 798–799
not statements, 61, 94, 96, 98, 100–103, 160
complement of sets and, 61
of conditional statement, 134
dominance of, 101–103
truth table for, 106–107, 110–111
Notation
decimal, 261–262
scientific. *See* Scientific notation
Notes. *See* Loan(s)
Null set, 46, 52
Number(s)
cardinal, 45, 49
of set, 45, 85–86
composite, 209–210
in encryption, 214
counting, 2, 208, 219
decimal, 231–234
conversion of to fractions, 232–234
irrational numbers and, 243
nonrepeating, 242
nonterminating, 242–243
repeating, 231–234
terminating, 231–232
definition of, 167
even, 2
exact values of, 246, 257
expanded-form, 175
Fermat, 215
Fibonacci, 275–279
friendly, 213
golden, 277
irrational, 242–247
large
approximations of, 257
law of, 658–659
scientific notation for, 260–264
mixed, 229–231
natural, 2, 208, 219
closed, 250–251
factors of, 208
sets of, 250. *See also* Sets
negative, 231, 297
dividing by, 330
mixed, 231
multiplying by, 329–330
as product, 224
as quotient, 225
vs. numerals, 167
odd, 2
ordinal, 49
pentagonal, 227
perfect, 218
perfect square, 243
positive
as product, 224
as quotient, 225
power of, 256–260
prime, 208–215
rational, 228–238
real, 249–253
reciprocal, 235
small, scientific notation for, 260–264
square, 6
triangular, 6, 228
whole, 219
Number line, 219–220, 328–329
Number theory, 207–286
definition of, 208
Numerals. *See also* Numeration systems
definition of, 167
vs. numbers, 167
variations in, 179
Numeration systems, 166–203. *See also* Base systems
additive, 167–171
Babylonian, 175–178
base-10, 175, 182
binary (base 2), 182
Chinese, 170–171, 186
ciphered, 172–173
definition of, 167
Egyptian, 168
evolution of, 166, 167, 186, 195
Hindu-Arabic, 166, 167, 168, 175
Ionic Greek, 172–173
Mayan, 178–179, 182
multiplicative, 170–171
positional-value, 175–186
Roman, 168–169
tally, 167
zero in, 175
Numerator, 228
Numerical coefficient, 293

Objective function, 421–422
Obtuse angle, 479
Obtuse triangle, 417
Octal system, 186
Odd days, 210
Odd numbers, 2
Odds, 671–674
probabilities from, 674
Odds against, 671–672
Odds in favor, 672–673
On Floating Bodies (Archimedes), 22
Open line segment, 475–477
Open-ended installment loan, 622, 626–630
or
exclusive, 93–94
inclusive, 94, 97
or probability problems, 695–698, 702
or statements, 93–94, 97, 101, 160
conversion of conditional statement to, 133–134
dominance of, 101–103
equivalent, 133
truth table for, 109–110
union of sets and, 61
Ordered triple, 401
Order of operations, 289–291
Ordered arrangements. *See* Permutation
Ordered pairs, 335
in solution of linear equation system, 383–384
in solution of linear inequality system, 423
Ordinal numbers, 49
Ordinary interest, 603–606
Organon (Aristotle), 92
Origin, 335
Outcomes, 654, 655
Overlapping sets, 57, 58

Parabola, 365
Parallel circuits, 164
Parallel lines, 477–478
Parallel planes, 478
Parallelogram, 490
area of, 497
graphing of, 336
perimeter of, 498
Parentheses, in statements, 97, 98, 100, 101–102
Pascal, Blaise, 654

Pascal's triangle, 281
Pasteur, Louis, 135
Pattern analysis, 574
Pattern design, rigid motion in, 574
Paulos, John Allen, 267
Pawnbrokers, 604
Payment cap, 642
Pearson correlation coefficient, 812
PEMDAS, 290
Pentagonal numbers, 227
Pentagons, 485, 486
Percent, 593–599
  converting decimal number to, 594
  converting fraction to, 593
  rounding of, 595
  in tipping, 597
Percent change, 595–596
Percent commission, 597
Percent downpayment, 598
Percent markdown, 597–598
Percent markup on cost, 596–597
Percentile, 782
Perfect number, 218
Perfect square, 243
Perimeter, 496–498
  vs. area, 498
Periodic rate cap, 641
Permutation, 716–721
  vs. combination, 725–727
  of duplicate items, 720–721
Permutation formula, 719
Personal loans, 602–610. *See also* Loan(s)
Phi (golden number/ratio), 277, 278
Photography, 477
Physics, group theory in, 563
Pi, 500
  calculation of, 244
Picasso, Pablo, 510
Piccard, Jacques, 222
pico, 434
Pie charts, 767–768, 770
Piece of data, 761
Place-value numeration systems. *See* Positional-value numeration systems
Plane(s), 477–478, 547
  parallel, 478
  rigid motions in, 519
Plane geometry, 477
Playfair, John, 545
Playfair's postulate, 545–546
Poincaré disk, 553
Point(s), 474–477, 636
  end, 475
  noncollinear, 478
  rotation, 525
  sample, 688
  of symmetry, 529
Polya, George, 21
Polygenetic diseases, 706
Polygons, 485–491
  area of, 496–498
  congruent, 489–490
  definition of, 485
  frequency, 762, 770–773
  names of, 485
  perimeter of, 496–498
  sides of, 485
  similar, 487–489
  sum of interior angles of, 486
  vertex of, 485
Polyhedrons, 510–514
  elements of, 510–511
  regular, 511
  volume of, 511–514
Population
  mean of, 753
  sampling in, 752–755
  standard deviation of, 753
  statistical, 752
Population growth, trends in, 370
Positional-value numeration systems, 175–180
  Babylonian, 175–178, 180
  bases in, 181–186
  conversion methods for, 180, 183–186
  evolution of, 186
  Hindu-Arabic, 166, 167, 168, 175, 180
  Mayan, 178–179, 180
  vs. multiplicative systems, 171
Positive integers, 219
Positive number
  as product, 224
  as quotient, 225
Positive square root, 243
Postal bar codes, 205–206
Postnet, 205–206
Postulates, 474
Power, 256–260
  zero, 258–259
Power rule for exponents, 259–260
Prefixes, in metric system, 433–434, 435, 469
Premise, 99n
Present value, 617
Price
  fair, 682–683
  stock, 233
  total installment, 622
Prime factorization, 210–212
  by branching, 210
  by division, 210–212
  for greatest common divisor, 212–213
  for least common multiple, 213–214
Prime interest rate, 612
Prime numbers, 208–215
  in encryption, 214
  greatest common divisor, 212–213
  larger, 215
  least common multiple, 213–214
  Mersenne, 215
  product of, 210–211
  relatively prime, 213
  twin, 215
Principal, 602
Principal square root, 243
Prisms, 510
  right, 511
  volume of, 510–513
Prisoner's dilemma, 407
Probability, 653–744
  addition formula in, 696–697
  and problems in, 698–702
  binomial probability formula and, 736–742
  calculation of, 663–664
  combinations in, 724–727
  conditional, 707–710
  counting principle in, 686–687, 689–697, 727
  dependent events and, 700, 707
  empirical, 655–659
  expected value and, 678–683
  history of, 654
  important facts for, 664
  independent events and, 700, 707
  law of large numbers and, 658–659
  multiplication formula in, 699–702
  nature of, 654–656
  from odds, 674
  odds and, 671–674
  or problems in, 695–698
  permutations in, 716–721, 725–727
  statistics and, 752
  theoretical, 655, 662–667
Probability distribution, 737–738
Probability problems, combinations for, 730–733
Problem solving, 19–29
  in daily life, 26
  guidelines for, 21
Product
  factors of, 208
  negative number as, 224
  positive number as, 224
  of prime numbers, 210–211

Product rule
for exponents, 257, 260
for radicals, 243
Projectile motion, 366
Proper factors, 218
Proper subsets, 51–52
Proportion(s), 27, 300–302
golden, 277, 278
Proportionality constant, 320, 321–322
Protractor, 478
Pseudosphere, 547, 548
Purity, 593
Pyramids, 277
volume of, 513–514
Pythagoras, 242, 243, 474
Pythagorean theorem, 242, 243, 499
Pythagorean triple, 281
Pythagoreans, 243

Quadrants, 335
Quadratic equations, 350–359
graphing of, 365–367
irrational solutions to, 358–359
with no real solution, 358
solving of
by factoring, 355–357
quadratic formula for, 357–359
standard form of, 355
Quadratic formula, 357–359
Quadratic functions
definition of, 364
domain of, 362, 367–368
graphing of, 364–368
range of, 362, 367–368
vertical line test for, 362–363
Quadrilaterals, 485, 486, 490
Quality control, Six Sigma in, 800
Quantifiers, 94–95, 160
Quartile, 783–784
Queen Dido, 502
Queen Victoria, 658
Quinella bet, 728
Quotient, 225
undefined, 225
Quotient rule
for exponents, 258, 260
for radicals, 245

r (correlation coefficient), 808–810
Radical sign, 243
Radicals, 243–244
addition of, 244–245
coefficient of, 243
division of, 245
multiplication of, 243, 245
product rule for, 243
quotient rule for, 245
simplification of, 243–244
subtraction of, 244–245
Radicand, 243
Radius, 500
Rainbows, 51
Ramanujan, Srinivasa, 212
Random sample, 753–754
Range, 789
of function, 362, 367–368
Ranked data, 779–780
Rate cap, 641
Ratio
common, 270
golden, 277, 278
proportions and, 300–302
Rational numbers, 228–238
as decimal numbers, 231–234
definition of, 228
examples of, 228
fractions as, 228–231
fundamental law of, 236
improper fractions as, 229–231
mixed numbers as, 229–231
in music, 236
sets of, 250. *See also* Sets
in sets of real numbers, 249–250
Rationalization of denominator, 246
Ray, 475
Real number line, 219–220, 328–329
Real numbers, 249–253
associative property of, 251–252, 294–295
closed, 250–251
commutative property of, 251, 294–295
distributive property of, 252, 294–295
sets of. *See* Sets
Reasoning. *See also* Critical thinking skills
deductive, 4–5
inductive, 2–5
Recessive traits, 656–657
Reciprocal, 235
Recorde, Robert, 314
Rectangle, 490
area of, 496, 498
golden, 278
perimeter of, 496, 498
Rectangular coordinate system, 335–338
Rectangular distribution, 796
Rectangular solid, volume of, 507
Recursion, 549
Reducing fractions, 228–229
Reflection, 520–522
glide, 527–528
Reflective symmetry, 528–529
Regression, 808
linear, 814–816
Regression line, 814–816
Regular tessellation, 531
Relation, definition of, 361
Relative frequency, 655–659
Relatively prime numbers, 213
Relativity, theory of, 309, 549
Remainder, 208
Rent-to-own transaction, 622
Repeating decimal numbers, 231–234
conversion of to fractions, 233–234
Reproduction, Fibonacci numbers in, 279
Rhind, Henry, 168
Rhind papyrus, 168
Rhombus, 490
Riddles, 24
Riemann, G.F. Bernhard, 546
Riemannian geometry, 546–547
Right angle, 479
Right prism, 511
Right triangle, 242, 487
Rigid motion, 519, 520–530
glide reflection, 527–528
in pattern design, 574
reflection, 520–522
rotation, 524–528
symmetry and, 528–530
translation, 522–524
Rijndael Block Cipher, 214
Roman numerals, 168–169
Rosetta Stone, 168
Roster form, of sets, 42–43
Rotation, 524–528
Rotation point, 525
Rotational symmetry, 529–530
Row transformations, 412–415
triangularization method for, 416–417
Rubber sheet geometry, 538. *See also* Topology
Rubik's cube, 572
Rule of 72, 621
Rule of 78s, 624–626
Rules of Divisibility, 210
Rules of traversibility, AA-2–3
Russell, Bertrand, 290

Saccheri, Girolamo, 546
Sample
  mean of, 753, 778
  standard deviation of, 753
  unbiased, 753
Sample point, 688
Sample space, 688, 695–696
Sampling, 751–755
  cluster (area), 754
  convenience, 755
  faulty, 754
  random, 753–754
  stratified, 754–755
  systematic, 754
Satisfiability problems, 121
Savings accounts, 616–617
Scalene triangle, 487
Scatter diagram, 809–810
Schmandt-Besserat, Denise, 167
Scientific calculators
  scientific notation on, 262–264, 307, 741
  square root approximation with, 246–247
Scientific method, 3
Scientific notation, 260–264
  on calculator, 262–264
  decimal notation and, 261–262
  definition of, 260
  division in, 262
  multiplication in, 262
Scott, David, 4
Seasonal loan, 603
Second, 465
Second mortgage, 643
Security, 603
Self-contradictions, 124
Self-similarity, 549
Sequence
  arithmetic, 268–270
  definition of, 268
  Fibonacci, 275–279, 281
  geometric, 270–272
  Lucas, 281
Series circuits, 164
Set-builder (set-generation) notation, 43–44
Sets, 41–87
  applications of, 76–82
  associative property of, 561–562
  braces for, 42
  cardinal number of, 45
  closed, 250–251
  commutative property of, 561–562
  complement of, 57, 76
  countable, 85–86
  De Morgan's laws for, 70–71, 87, 133
  definition of, 42
  description of, 42
  difference of two, 65
  disjoint, 56
  elements (members) of, 42
  ellipsis in, 42
  empty (null), 46, 52
  equality of, 45, 57
    verification of, 68–70
  equivalent, 45
  finite, 44
  identity element in, 563
  infinite, 44, 83–86
  of integers, 219, 250
  intersection of, 57–59, 61–62
  in mathematical systems, 561
  of natural numbers, 250
  with one-to-one correspondence, 45
  overlapping, 57, 58
  of real numbers, 249–250
  relationships among, 250
  roster form of, 42–43
  set-builder notation for, 43–44
  statements involving, 68–70
  subsets and, 50–56. *See also* Subsets
  symbols for, 87
  union of, 59–62
  universal, 46
  Venn diagrams and, 56–82. *See also* Venn diagrams
  well-defined, 42
Sexagesimal systems
  Babylonian, 175, 176–178
  navigational, 177
Shallit, Jeffrey, 211
Shannon, Claude, 114
Shareholder, 608
Shares, 608
SI system, 432. *See also* Metric system
Sierpinski, Waclaw, 551
Sierpinski carpet, 551
Sierpinski triangle, 551
Sieve of Eratosthenes, 209
Similar figures, 487–489
Simple interest, 304, 603–606
  banker's rule for, 606–608
  United States rule for, 606, 608–610
Simple statements, 94. *See also* Statements
Simplex method, 421
Simson, Robert, 277
Simultaneous linear equations, 383
Six Sigma, 800
Skewed distribution, 797
Skewed lines, 478
Slope of line
  definition of, 340
  determination of, 340–341
  in graphing, 340–344
Slope-intercept form, of linear equation, 341–344
Slot machines, 702, 714
Small numbers, scientific notation for, 260–264
Solid geometry, 507–514
Solution, checking of, 288
some, as quantifier, 94–95
Space
  sample, 688, 695–696
  in theory of relativity, 549
Space-time, 549
Spam, 371
Speed, of falling objects, 4
Sphere, 547
  volume of, 508
Spherical geometry, 546–547
Spreadsheet, 405
Square, 490
  area of, 496, 498
  magic, 28–29
  perfect, 243
  perimeter of, 498
Square feet–square inches conversion, 502
Square matrix, 402
  multiplicative identity of, 407
Square numbers, 6
Square root
  calculator approximation of, 246–247
  principal (positive), 243
Square units, in metric system, 441–443
Squaring the circle, 479
Standard deviation, 753, 789–792
Standard (z) scores, 305, 799–804
Stanines, 788
Statements, 94–138
  and, 61, 96–97, 101–103, 133, 160
  biconditional, 100–103, 120–125, 131
  but, 96, 99
  comma in, 97–98
  compound, 94, 95–96
  conditional, 99–103, 118–120. *See also* Conditional statements
  connectives in, 95–103, 101–103, 160

Statements *(continued)*
De Morgan's laws for, 70–71, 131–134
equivalent, 128–138. *See also* Equivalent statements
if and only if, 100–103, 120, 131, 160
if-then, 99–100, 101, 160
involving sets, 68–70
negation of, 94, 96, 98, 101–103
neither-nor, 99
not, 61, 94, 96, 98, 100–103, 160
or, 61, 93–94, 97, 101–103, 109–110, 133–134, 160
parentheses in, 97, 98, 100, 101–102, 101–103
quantifiers in, 94–95
simple, 94
symbols for, 96, 101
truth tables for, 106–125. *See also* Truth tables
truth value of, 94, 113–115, 121–122
Statistical data, 752. *See also* Data
Statistical graphs, 762, 767–773
Statistical population, 752
Statistics, 751–821
average in, 757
definition of, 752
descriptive, 752
display of, 757, 762, 767–773
frequency distributions in, 761–765
graphs of, 762, 767–773
inferential, 752
linear correlation in, 808–814
linear regression and, 814–816
mean in, 753
measures of central tendency in, 778–784
measures of dispersion in, 788–792
measures of position in, 782–784
misuse of, 757–759
probability and, 752
sampling techniques and, 751–755
standard deviation in, 753
Stem-and-leaf display, 772–773
Stevenson, Branson Graves, 540
Stock exchange, 608
Stock prices, 233
Stocks, 608
mutual funds and, 643
Straight angle, 479, 486
Straightedge, 476
Stratified sample, 754–755
Subscripts, 305
Subsets, 50–56
definition of, 50
number of, 52–54
proper, 51–52
in Venn diagrams, 57
Substitution method, for linear equation systems, 391–393
Subtraction
definition of, 221, 298
in different base systems, 192–193
of fractions, 235–238
of integers, 221–222
of irrational numbers, 244–245
of matrices, 403
in modular arithmetic, 581–582
in Roman numeration system, 169
Subtraction property of equality, 294–295
Summation, symbol for, 778
Supplementary angles, 479
Switching circuits, 164
Syllogism, law of, 147
Syllogistic arguments, 153–157
disjunctive, 147
Euler diagrams and, 155–157
vs. symbolic arguments, 154, 160
validity of, 154–157, 160
Sylvester, James, 406
Symbolic arguments, 142–150
conclusion in, 143
definition of, 143
determining logical conclusion in, 150
fallacies, 143, 148
premises of, 143
vs. syllogistic arguments, 154, 160
with three premises, 149–150
truth tables for, 143–146, 149–150
validity of, 143–147, 160
Symbolic logic, 93, 133
Symbolic mathematics, 294
Symbols
evolution and standardization of, 314
inequality, 219–220, 328–332
Symmetry, 528–530
axis of, 365
line of, 528
in pattern design, 574
point of, 529
reflective, 528–529
rotational, 529–530
Système international d'unités (SI system), 432. *See also* Metric system
base units of, 465
Systems. *See* Mathematical systems
Tally sticks, 606
Tautologies, 124–125
Tax free yield formula, 310–311
Taxonomy, 54
Temperature
base unit for, 465
Celsius, 453–455, 469
Fahrenheit, 454–455, 469
tera, 434
Terminating decimal numbers, 231
conversion of to fractions, 232–233
Terms, 292–293
definition of, 268
Tessellations, 530–532
Thales, 474
Theorem(s), 474
Bernoulli's, 654
Chebyshev's, 807
definition of, 211
enormous, 565
Pythagorean, 242, 243, 499
Theoretical probability, 655, 662–667
Theory of relativity, 309, 549
Third-order systems, 412
Thoburn, Steve, 453
Tiling, 530–532
Time
base unit for, 465
interest and, 603
in theory of relativity, 549
Tipping, 597
Titius-Bode law, 270
Topologic equivalence, 542
Topology, 538, AA-1
genus in, 542
Jordan curve and, 540–541
Klein bottle and, 539–540
maps and, 540–541
Möbius strip and, 538–550, 541
Total installment price, 622
Transformational geometry, 519–538
Transformations, 519, 563
Translation, 522–524
Translation vector, 522
Transversal, 481
Trapezoid, 490
area of, 498
perimeter of, 498
Treasury bills, 612
add-on rate (margin) for, 640
Treasury bonds, 617
Tree diagrams, 687–692
Triangles, 485–486
acute, 487
area of, 497

congruent, 489–490
construction of, 475, 485
equilateral, 487
isosceles, 487
obtuse, 487
Pascal's, 281
perimeter of, 498
right, 242, 487
scalene, 487
Sierpinski, 551
similar, 487–489
sum of interior angles of, 486
Triangular numbers, 6, 228
Triangularization method, 416–417
Trinomials
definition of, 351
factoring of, 351–355
Trisecting an angle, 479
Triskaidekaphobia, 202
True interest rate, 605
Truth tables, 106–125, 160
for biconditional statements, 120–125
cases in, 108, 112–113
columns in, 108
for conditional statements, 118–120
for conjunction, 107–109
construction of, 107–109, 111
definition of, 106
for disjunction, 109–110
for equivalent statements, 128–131
for negation, 106–107, 110–111
for symbolic arguments, 143–146, 149–150
Truth value
assignment of, 94
determination of, 113–115, 121–122. *See also* Truth tables
Truthfulness, in advertising, 148
Twin prime conjecture, 215
Twin primes, 215

Undefined quotient, 225
Underbakke, David, 215
Unearned interest, 624
Union of sets, 59–62
Unit fraction, 458–460
United States customary system, 432, 440, 458
conversion to and from metric system, 458–464
United States Postal Service bar codes, 205–206
United States rule, 606, 608–610
Units, 209
metric, 431–472
U.S. customary, 432, 440, 458–464
Universal arithmetic, 294
Universal sets, 46
Universe, expansion of, 560, 565
Unlike terms, 293
Unpaid balance method, 627–628
Upper class limits, 763
Usury, 607

VA mortgage, 642
Valid arguments, 143–147, 154–157, 160
Variable, 288
dependent, 343, 362, 809
independent, 343, 362, 809
isolation of, 294
solving for, 309–310
Variable-rate mortgage, 635–636, 640
Variance, 789n, 791n
Variation, 320–326
combined, 324–326
definition of, 320
direct, 320–322
inverse, 322–323
joint, 323–324
Vector, translation, 522
Velocity, calculation of, 358
Venn diagrams, 56–82, 154
applications of, 70
complement of set in, 57
construction of, 76–80
intersection of sets in, 57–59, 61–62
labeling method for, 65
with three sets, 66–68
types of sets in, 57
union of sets in, 59–62
Vertex
of angle, 478
even, AA-2
of feasible region, 421
in graphing, AA-2
odd, AA-2
parabolic, 365
of polygon, 485
of polyhedron, 510–511
Vertical angles, 481, 486
Vertical line test, 362–363
Vibrations, 236
Viète, François, 294
Virgil, 219, 502
Volume
of cone, 508
of cube, 508
of cylinder, 508
formula for, 304–305
vs. mass, 452
metric units for, 443–446, 469
of polyhedron, 510–514, 511–514
of prism, 510–513
of pyramid, 513–514
of sphere, 508
unit conversion for, 461–462

Walsh, Don, 222
Wantzel, Pierre, 479
Water
mass of, 452, 469
volume of, 452, 469
Weather forecasting, 708
algorithm(s), 365
Weight, vs. mass, 450–451
Weighted average, 788
Weights and measures
metric, 431–472
U.S. customary, 432, 440, 458
Welch, Jack, 800
Whitehead, Alfred North, 287
Whittaker, Andrew "Jack," Jr., 725
Whole numbers, 219
Wiles, Andrew J., 501
William of Ockham, 71
William the Conqueror, 751
Woltman, George, 215
Word problems, solving, 314–317
Wundt, Wilhelm, 279

x-axis, 335, 343
x-intercept, 338–339

y-axis, 335, 343
slope of line and, 335, 341–344
Yield, tax free, 310–311
y-intercept, 338–339

Zeno's Paradox, 87
Zero
in Hindu-Arabic numeration system, 175
multiplication property of, 222
Zero exponent rule, 258–259, 260
Zero-factor property, 355–356
Z-scores, 305, 799–804

Arthritis, 677
Body Mass Index, 38, 312
Calcium Tablets, 438
Calories and Exercise, 16
Cholesterol Level, 806
Cosmetic Surgery, 127
Cough Syrup, 464
Determining Amount of Insulin, 302, 303
Determining Dosage by Weight, 464
Dolphin Drug Care, 743
Experimental Drug, 705
Flu Serum, 734
Gaining Weight, 16
Health Care Costs, 29, 333
Height and Weight Distribution, 794
Hospital Care Expenditures, 760
Hospital Expansion, 56
Infant Mortality Rate, 760
Laboratory Research, 400, 728
Medical Tests, 676
Medicine, 463, 467, 747
Nursing Question, 468
Office Visit, 743
Pediatric Dosage, 36, 467, 472
Physical Therapy, 586
Potassium, 599
Prader-Willi Syndrome, 660
Proper Dosage, 303, 445
Reduced Fat Milk, 600
Sales of Botox, 371
SARS Cases, 595
Smoking and Cancer, 5, 467, 472
Stomach Ache Remedy, 467
Target Heart Rate, 375
Vision Surgery, 743
Weight, 37, 655, 806
X-Rays, 32

Home And Garden

A Branching Plant, 286
A Dinner Party, 728
A Dinner Toast, 729
Amount of Recipe Ingredients, 25, 32, 237, 240, 241, 283, 286, 303, 377
Area of a Garden, 449, 506
Buying Carpet, 466, 470, 505
Buying Plants, 39
Chlorine in a Swimming Pool, 819
Cornflakes, 466
Cost of Flooring, 505, 554
Crop Storage, 240
Dimensions/Area of a Deck, 318
Dimensions of a Bookcase, 319
Dimensions of a Fenced Area, 316
Dimensions of a Room, 241
Drying Time, 292
Enclosing Two Pens, 318
Estimating Bushels of Grapes, 9
Estimating Number of Grubs in a Lawn, 14
Farmland Area, 449
Fertilizer Coverage, 29, 378, 501
Fish Tank Capacity, 449, 452, 457, 470, 471, 512, 517
Floor Area, 318
Floor Molding, 241
Flower Box Volume, 518
Flower Garden, 350, 360
Flower Probabilities, 701
Food Mixtures, 400, 457
Gasoline Containers, 516
Grass Growth, 292
Grass Seed Coverage, 303
Hanging a Picture, 241
Hanging Wallpaper, 346
Height of a Computer Stand, 240
Height of a Tree, 29, 489
Installing Tile, 502, 505, 554
Irrigation System, 22
Landscape Service, 385, 389, 505
Leaking Faucet, 31, 518
Making Cookies, 467
Orange Orchards, 292
Painting a House, 334
Pet Supplies, 318
Preparing Insecticide, 27
Quilt Design, 6
Rose Garden Top Soil, 517
Security Systems, 389
Selecting Cereal, 723
Silage Storage, 509
Sprinkler System, 240
Stairway Height, 240
Supporting a Jacuzzi, 559
Swimming Pool Volume, 444, 449, 470, 514, 517
Table Seating Arrangements, 729
Thistles, 240
Volume of a Bread Pan, 516
Volume of a Freezer, 516
Volume of a Water Trough, 555
Walkway Area, 449
Wasted Water, 31
Water Heaters, 445, 457, 743
Water Restrictions, 31

Labor

Airline Hiring, 686
Annual Pay Raises, 274
Categorizing Contracts, 75
Daily Tips, 784
Disability Income, 378
Distance to Work, 777
Employee Increase, 602
Employment Assignments, 731
Hours of Work, 29, 30, 805
Job Interviews, 127
Job Offers, 390
Labor Union Membership, 596
Middle Class, 18
Number of Sick Days Taken, 774
Nursing Shifts, 587
Occupations, 63
On the Job, 417
Overtime, 315, 676
Promotions, 676
Real Estate, 80
Salaries, 274, 318, 374, 428, 602, 648, 775, 777, 785, 788, 824
Salary Options, 334, 686
Sales Commission, 318
Sales Staff Turnover, 783
Salesperson Promotion, 807
Transportation to Work, 743
Where Teens Work, 774
Work Schedules, 217, 584, 587, 590
Work Shift, 735
Years Until Retirement, 793

Numbers

A Square Pattern, 6
A Triangular Pattern, 6
Aerial Tram Load, 438
Codes/IDs/Passwords, 722, 723, 749
Combinations, 725, 726, 727, 728, 731, 732
Counting Principle, 689, 714
Digits, 745
Discovering an Error, 82
License Plates, 722, 723
Magic Squares, 28, 33, 37, 39
Number Puzzle, 33
Palindromes, 33, 37
Pascal's Triangle, 729
Permutations, 716, 717, 719, 720, 723
Possible Phone Numbers, 723
Social Security Numbers, 722, 778, 819
Sum of Numbers, 37
Using the Addition Formula, 696

Probability

A Marked Deck, 736
Apples from a Bucket, 749
Bean Bag Toss, 670
Choosing One Bird from a List, 664, 670
Choosing Two Aces, 748
Coin Toss, 655, 660, 693, 746
Color Chips, 748
Dealing Cards, 734
Design an Experiment, 662
Drawing from a Hat, 711, 734
Expectation and Fair Price, 683
Expected Value, 684
Finding Probabilities, 663, 674
Green and Red Balls, 734
Marbles, 670, 676, 693
Picking Chips, 685, 686, 695, 706
Raffles, 668, 681, 685, 745
Rock Toss, 662
Rolling Dice, 660, 671, 676, 684, 686, 693, 711
Roulette, 677
Selecting Balls, 688
Selecting Cards from a Deck, 666, 668, 684, 699, 700, 743, 749
Selecting Colored Balls With Replacement, 739

Selecting Items at Random, 692
Slot Machine, 705
Spin the Spinner, 668, 675, 685, 704
Spinning a Wheel, 706, 711, 746
Using Conditional Probability Formula, 709
Wheel of Fortune, 668, 687

Sports and Recreation

A Home Run, 439
Art Show, 317
Athletes' Salaries, 795
Attending Plays, 728
Barbie and Ken Dolls, 217
Baseball, 274, 595, 685, 733, 734
Basketball, 63, 400, 684
Batting, 724, 787
Bicycles, 734, 745
Big Screen TVs, 728
Billiard Balls, 675
Bowl Games, 675
Broadway Shows, 777
Card Games, 706, 707, 731, 735, 745
Chess Tournaments, 598
Chorus and Band, 65
Climbing a Mountain Range, 588
Club Membership, 648, 722, 725, 735, 746
Daily Double/Trifecta, 721, 722
Darts, 669
Dogsled, 747
Dominos, 33
Entertainers, 728
Estimating Weights, 14
Expected Movie Attendance, 684
Fishing, 601, 748, 793
Fitness Equipment, 76
Fitness Test Scores, 805
Football Yardage, 227
Game of Pit Score, 226
Golf, 400, 704
Hiking, 818
Hitting a Bulls-Eye, 661
Horse Racing, 677, 745
Jazz Concert, 775
Jet Ski Rental, 36
Logic Game, 165
Lottery, 5, 30, 675, 677, 684, 687, 706, 729, 734, 747
Magazines, 748
Most Improved Baseball Record, 595
Movie Ratings, 819
Music, 723, 728
Olympic Medals, 72
Oscars, 36
Painting, 724, 728
Park Visits by Families, 766
Photography, 328
Plasma Television, 506
Playing Bingo, 676
Professional Teams, 81
Remote Control, 693
Rock Concert, 676
Running, 438
Sand Volleyball Court, 509
School Play Tickets, 400
Scooter Injuries, 375
Scrabble, 724
Seating Capacity at Lambeau Field, 647
Selecting Ticket Winners, 690
Soccer League Games, AA-5
Sports Puzzle, 34
Stacking Trading Cards, 217
Swimming Event, 723
Television, 72, 89, 303, 684, 745, 747, 806, 825
Television News, 745, 821
Tennis, 749
Theatre Seating, 735
Ticket Sales, 318
Toy Car Collection, 217
Track and Field, 439
TV Game Show, 734
Videotapes and DVDs, 712, 775
Wagering Strategy, 275
Walking, 438
Water Park, 786
Winning a Race, 676, 677
Workout Schedule, 586

Statistics And Demographics

Aging Population, 16
Baby Gender, 662
Change in the Data, 784, 786
Chebyshev's Theorem, 807
County Population, 766
Creating a Data Set, 785, 786
Data from a Questionnaire, 756
Decreasing and Increasing Population, 600
Diversity of Doctors, 30
Expected Growth, 375
Fastest-Growing Cities, 81
Find the Mean, Median, Midrange, Mode, 779, 780, 781
Frequency Distribution of Daily Newspaper Circulation, 763
Frequency Distribution of Family Income, 764
Heights of High School Senior Males, 775
IQ Scores, 766
Life Expectancy, 784
Married Householders With Children, 347
Measures of Central Tendency, 781
Median Age of First Marriage, 760
Men's Weight, 823
Number of Children per Family, 693, 704, 756, 769, 770, 774
Percentiles, 786
Population Growth, 308
Population of Mexico, 599
Poverty in the United States, 602
Ranking of Metropolitan Areas, 73
Survey of Visitors at the Grand Canyon, 61
The Day of Six Billion, 267
The Elderly U.S. Population, 824
U.S. Population, 333
World Population, 265

Travel

A Drive in Pennsylvania, 15
A Drive in Mississippi, 15
Airline Routes, 735
Airline Seat Availability, 726
Airport Activity, 217
Bus Stations and Loops, 724
Busiest Ports, 44
Cincinnati to Columbus, 466
Class Trip, 619
Common Borders, AA-5
Commuting Time, 817
Cost of a Tour, 292
Cost of a Vacation, 14
Crossing a Moat, 499
Crossing Time Zones, 37, 227
Depth of a Submarine, 303
Determining Distances, 10, 374, 466, 469
Disney Magic, 467
Disney Tickets, 599
Distances in Minnesota, 494
Distances in Texas, 494
Docking a Boat, 506
Drive to the Beach, 39
Flight Schedules, 586
Flying West, 37
Frequent Flier Miles, 14
Going on Vacation, 40
Labor Day Travel, 767
Map Reading, 378
National Parks, 745, 785
Online Travel Web Sites, 775
Railroad Car Estimation, 36
Reading a Map, 29
Resorts, 81
Road Signs, 439
Ski Vacation, 18
Taxi Ride, 728
Train Stops, 283
Travel Expenses, 632
Travel Preferences, 81
Vacation Days, 318
Vacationing in Florida, 694
Vertical Distance Traveled, 227
Visiting California, 64
Visits to the San Diego Zoo, 776
Walking Path, 36

Weather

Aruba, 743
Costliest Hurricanes, 785
Expected Attendance, 746
Extreme Temperatures, 226
Forecasts, 741
High Temperatures in 40 U.S. Cities, 822
Seattle Greenery, 684

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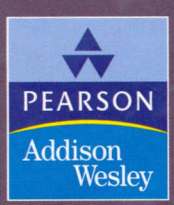

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