



San Francisco, CA

## CRITICAL THINKING SKILLS

Life constantly presents new problems. The more sophisticated our society becomes, the more complex the problems. We as individuals are constantly solving problems and do so daily. For example, when we consider ways of reducing our expenses or when planning a trip, we need to make problem-solving decisions. Businesses are constantly trying to solve problems that involve making a profit for the company and keeping customers satisfied.

The goal of this chapter is to help you master the skills of reasoning, estimating, and problem solving. These skills will aid you in solving problems in the remainder of this book as well as problems that you will encounter in everyday life.

Engineers, scientists, and others who worked in the energy industry in California had many problems to solve in the years 2000 and 2001 when the entire state felt the power shortage and experienced many blackouts. Even with Californians cutting back on power, power companies still barely met demands. Under the utilities' rotating system, often during the blackouts, when one group of customers got its electricity back, another group had its power shut off. And when federal energy regulators lifted price caps so that the companies involved could increase their access to electrical supplies, customers' electric bills rose dramatically.

Effective January 2002, California customers were free to choose their energy supplier. Two organizations, the California Power Exchange and the California Independent System Operator, now coordinate the buying, selling, and transmitting of electricity. This system will create many problem-solving opportunities for the employees of these two organizations, as well as for California residents who now can choose electricity suppliers.



## 1.1 INDUCTIVE REASONING

The goal of this chapter is to help you improve your reasoning and problem-solving skills. This section introduces inductive and deductive reasoning, which are used in problem solving. The next section introduces the concept of estimation. Estimation is a technique that can be used to determine if an answer obtained for a problem or from a calculation is “reasonable.” Section 1.3 introduces and applies problem-solving techniques.

Before looking at some examples of inductive reasoning and problem solving, let us first review a few facts about certain numbers. The **natural numbers** or **counting numbers** are the numbers 1, 2, 3, 4, 5, 6, 7, 8, . . . . The three dots, called an **ellipsis**, mean that 8 is not the last number but that the numbers continue in the same manner. A word that we sometimes use is “divisible.” If  $a \div b$  has a remainder of zero, then  $a$  is *divisible by*  $b$ . The counting numbers that are divisible by 2 are 2, 4, 6, 8, . . . . These are called the **even counting numbers**. The numbers that are not divisible by 2 are 1, 3, 5, 7, 9, . . . . These are the **odd counting numbers**. When we refer to **odd numbers** or **even numbers**, we mean odd or even counting numbers.

Recognizing patterns is sometimes helpful in solving problems, as Examples 1 and 2 illustrate.

### EXAMPLE 1 The Product of Two Odd Numbers

If two odd numbers are multiplied together, will the product always be an odd number?

**SOLUTION:** To answer this question, we will examine the products of several pairs of odd numbers to see if there is a pattern.

$1 \times 3 = 3$	$3 \times 5 = 15$	$5 \times 7 = 35$
$1 \times 5 = 5$	$3 \times 7 = 21$	$5 \times 9 = 45$
$1 \times 7 = 7$	$3 \times 9 = 27$	$5 \times 11 = 55$
$1 \times 9 = 9$	$3 \times 11 = 33$	$5 \times 13 = 65$

All the products are odd numbers. Thus, we might predict from these examples that the product of any two odd numbers is an odd number. ▲

### EXAMPLE 2 The Sum of an Odd Number and an Even Number

If an odd number and an even number are added, will the sum be an odd or an even number?

**SOLUTION:** Let's look at a few examples where one number is odd and the other number is even.

$3 + 4 = 7$	$9 + 6 = 15$	$23 + 18 = 41$
$5 + 12 = 17$	$5 + 14 = 19$	$81 + 32 = 113$

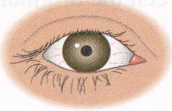
All these sums are odd numbers. Therefore, we might predict that the sum of an odd and an even number is an odd number. ▲

In Examples 1 and 2, we cannot conclude that the results are true for all counting numbers. From the patterns developed, however, we can make predictions. This type of reasoning process, arriving at a general conclusion from specific observations or examples, is called **inductive reasoning**, or **induction**.



## DID YOU KNOW

### The Eyes Tell It All



Most of us realize that thus far, no two people have been found to have the same fingerprints or DNA, so fingerprints and DNA have become forms of identification. As technology improves, so do identification techniques. Today, when you purchase a season pass at Disney World, you place your forefinger and middle finger in a device that measures and records your finger size, shape, and density. Then you are given a seasonal pass. Each time you enter Disney World, you put those two fingers in the device, which indicates whether you are allowed or not allowed entry under the seasonal pass that you show.



Automated teller machines (ATMs) are now experimenting with determining identity using a person's eye. When you open an account, a photo of your eye is taken and entered into a computer. When you use an ATM, a powerful camera automatically (without you even knowing it) checks the veins in your eye against the computer's files. Shortly, we will no longer need personal identification numbers (PINs) to withdraw money from our accounts. All these techniques are based on inductive reasoning. Based on past experiences, no two people have been found to have the same fingerprints; finger size, shape, and density; DNA; or eyes.

**Inductive reasoning** is the process of reasoning to a general conclusion through observations of specific cases.

Induction often involves observing a pattern and from that pattern predicting a conclusion. Imagine an endless row of dominoes. You knock down the first, which knocks down the second, which knocks down the third, and so on. Assuming the pattern will continue uninterrupted, you conclude that eventually all the dominoes will fall, even though you may not witness the event.

Inductive reasoning is often used by mathematicians and scientists to predict answers to complicated problems. For this reason, inductive reasoning is part of the **scientific method**. When a scientist or mathematician makes a prediction based on specific observations, it is called a **hypothesis** or **conjecture**. After looking at the products in Example 1, we might conjecture that the product of two odd numbers will be an odd number. After looking at the sums in Example 2, we might conjecture that the sum of an odd number and an even number is an odd number.

Examples 3 and 4 illustrate how we arrive at a conclusion using inductive reasoning.

#### EXAMPLE 3 Fingerprints and DNA

What reasoning process has led to the conclusion that no two people have the same fingerprints or DNA? This conclusion has resulted in fingerprints and DNA being used in courts of law as evidence to convict persons of crimes.

**SOLUTION:** In millions of tests, no two people have been found to have the same fingerprints or DNA. By induction, then, we believe that fingerprints and DNA provide a unique identification and can therefore be used in a court of law as evidence. Is it possible that, sometime in the future, two people will be found who do have exactly the same fingerprints or DNA?

#### EXAMPLE 4 Divisibility by 4

Consider the conjecture “If the last two digits of a number are divisible by 4, then the number itself is divisible by 4.” We will test several numbers to see if the conjecture appears true or false.

**SOLUTION:** Let's look at some numbers whose last two digits are divisible by 4.

Number	Are the Last Two Digits Divisible by 4?	Is the Number Divisible by 4?
344	yes, $44 \div 4 = 11$	yes, $344 \div 4 = 86$
4312	yes, $12 \div 4 = 3$	yes, $4312 \div 4 = 1078$
10,528	yes, $28 \div 4 = 7$	yes, $10,528 \div 4 = 2632$
20,104	yes, $04 \div 4 = 1$	yes, $20,104 \div 4 = 5026$

In each case, we find that if the last two digits of a number are divisible by 4, then the number itself is divisible by 4. From these examples, we might be tempted to generalize that the conjecture “If the last two digits of a number are divisible by 4, then the number itself is divisible by 4” is true.\*

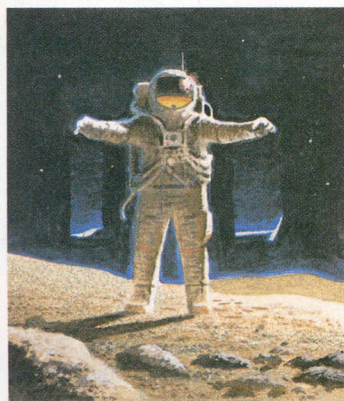
\*This statement is in fact true, as is discussed in Section 5.1.





## DID YOU KNOW

### *An Experiment Revisited*



Apollo 15 astronaut David Scott used the moon as his laboratory to show that a heavy object (a hammer) does indeed fall at the same rate as a light object (a feather). Had Galileo dropped a hammer and feather from the Tower of Pisa, the hammer would have fallen more quickly to the ground and he still would have concluded that a heavy object falls faster than a lighter one. If it is not the object's mass that is affecting the outcome, then what is it? The answer is air resistance or friction: Earth has an atmosphere that creates friction on falling objects. The moon does not have an atmosphere; therefore, no friction is created.

### EXAMPLE 5 *Pick a Number, Any Number*

Pick any number, multiply the number by 4, add 6 to the product, divide the sum by 2, and subtract 3 from the quotient. Repeat this procedure for several different numbers and then make a conjecture about the relationship between the original number and the final number.

**SOLUTION:** Let's go through this one together.

Pick a number:	say, 5
Multiply the number by 4:	$4 \times 5 = 20$
Add 6 to the product:	$20 + 6 = 26$
Divide the sum by 2:	$26 \div 2 = 13$
Subtract 3 from the quotient:	$13 - 3 = 10$

Note that we started with the number 5 and finished with the number 10. If you start with the number 2, you will end with the number 4. Starting with 3 would result in a final number of 6, 4 would result in 8, and so on. On the basis of these few examples, we may conjecture that when you follow the given procedure, the number you end with will always be twice the original number. ▲

The result reached by inductive reasoning is often correct for the specific cases studied but not correct for all cases. History has shown that not all conclusions arrived at by inductive reasoning are correct. For example, Aristotle (384–322 B.C.) reasoned inductively that heavy objects fall at a faster rate than light objects. About 2000 years later, Galileo (1564–1642) dropped two pieces of metal—one 10 times heavier than the other—from the Leaning Tower of Pisa in Italy. He found that both hit the ground at exactly the same moment, so they must have traveled at the same rate.

When forming a general conclusion using inductive reasoning, you should test it with several special cases to see whether the conclusion appears correct. If a special case is found that satisfies the conditions of the conjecture but produces a different result, such a case is called a **counterexample**. A counterexample proves that the conjecture is false because only one exception is needed to show that a conjecture is not valid. Galileo's counterexample disproved Aristotle's conjecture. If a counterexample cannot be found, the conjecture is neither proven nor disproven.

A second type of reasoning process is called **deductive reasoning**, or **deduction**. Mathematicians use deductive reasoning to *prove* conjectures true or false.

**Deductive reasoning** is the process of reasoning to a specific conclusion from a general statement.

### EXAMPLE 6 *Pick a Number, $n$*

Prove, using deductive reasoning, that the procedure in Example 5 will always result in twice the original number selected.

**SOLUTION:** To use deductive reasoning, we begin with the *general* case rather than specific examples. In Example 5, specific cases were used. Let's select the letter  $n$  to represent *any number*.



Pick any number:  $n$

Multiply the number by 4:  $4n$  ( $4n$  means 4 times  $n$ )

Add 6 to the product:  $4n + 6$

Divide the sum by 2:  $\frac{4n + 6}{2} = \frac{4n}{2} + \frac{6}{2} = 2n + 3$

Subtract 3 from the quotient:  $2n + 3 - 3 = 2n$

Note that, for any number  $n$  selected, the result is  $2n$ , or twice the original number selected. ▲

In Example 5, you may have *conjectured*, using specific examples and inductive reasoning, that the result would be twice the original number selected. In Example 6, we *proved*, using deductive reasoning, that the result will always be twice the original number selected.

## SECTION 1.1 EXERCISES

### Concept/Writing Exercises

1. a) List the natural numbers.  
b) What is another name for the natural numbers?
2. a) What does it mean to say, " $a$  is divisible by  $b$ ," where  $a$  and  $b$  represent natural numbers?  
b) List three natural numbers that are divisible by 4.  
c) List three natural numbers that are divisible by 9.

In Exercises 3–6, explain your answer in one or two sentences.

3. What is a conjecture?
4. What is inductive reasoning?
5. What is deductive reasoning?
6. What is a counterexample? Explain.
7. Which type of reasoning is generally used to arrive at a conjecture?
8. Which type of reasoning is used to prove a conjecture?
9. You have purchased one lottery ticket each week for many months and have not won more than \$5.00. You decide, based on your past experience, that you are not going to win the grand prize and so you stop playing the lottery. What type of reasoning did you use? Explain.
10. In the 1950s, doctors noticed that many of their lung cancer patients were also cigarette smokers. Doctors reasoned that cigarette smoking increased a person's chance of getting lung cancer. What type of reasoning did the doctors use? Explain.

### Practice the Skills

In Exercises 11–14, use inductive reasoning to predict the next line in the pattern.

11.

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1

```

12.

```

10 = 101
100 = 102
1000 = 103
10,000 = 104

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13.

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1 × 9 = 9
2 × 9 = 18
3 × 9 = 27
4 × 9 = 36

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14.


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11 × 11 = 121
11 × 12 = 132
11 × 13 = 143

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In Exercises 15–18, draw the next figure in the pattern (or sequence).

15.  ... 16.  ...

17.  ...

18.  ...



In Exercises 19–28, use inductive reasoning to predict the next three numbers in the pattern (or sequence).

19. 3, 6, 9, 12, ...      20. 26, 20, 14, 8, ...  
 21. 1, -1, 1, -1, 1, ...      22. 5, 3, 1, -1, -3, ...  
 23.  $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$       24. 2, -6, 18, -54, ...  
 25. 1, 4, 9, 16, 25, ...      26. 0, 1, 3, 6, 10, 15, ...  
 27. 1, 1, 2, 3, 5, 8, 13, 21, ...      28.  $5, -\frac{10}{3}, \frac{20}{9}, -\frac{40}{27}, \dots$

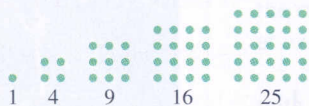
### Problem Solving

29. Find the letter that is the 118th entry in the following sequence. Explain how you determined your answer.

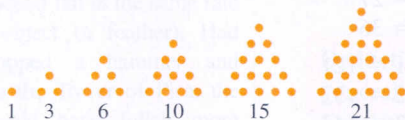
Y, R, R, Y, R, R, Y, R, R, Y, R, R, Y, R, R, ...

30. a) Select a variety of one- and two-digit numbers between 1 and 99 and multiply each by 9. Record your results.  
 b) Find the sum of the digits in each of your products in part (a). If the sum is not a one-digit number, find the sum of the digits again until you obtain a one-digit number.  
 c) Make a conjecture about the sum of the digits when a one- or two-digit number is multiplied by 9.

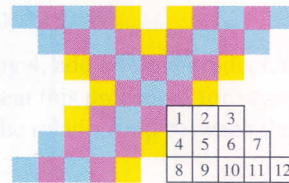
31. **A Square Pattern** The ancient Greeks labeled certain numbers as **square numbers**. The numbers 1, 4, 9, 16, 25, and so on are square numbers.



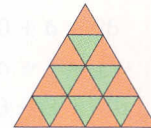
- a) Determine the next three square numbers.  
 b) Describe a procedure to determine the next five square numbers without drawing the figures.  
 c) Is 72 a square number? Explain how you determined your answer.
32. **A Triangular Pattern** The ancient Greeks labeled certain numbers as **triangular numbers**. The numbers 1, 3, 6, 10, 15, 21, and so on are triangular numbers.



- a) Can you determine the next two triangular numbers?  
 b) Describe a procedure to determine the next five triangular numbers without drawing the figures.  
 c) Is 72 a triangular number? Explain how you determined your answer.
33. **Quilt Design** The pattern shown above on the right is taken from a quilt design known as a triple Irish chain. Complete the color pattern by indicating the color assigned to each square.

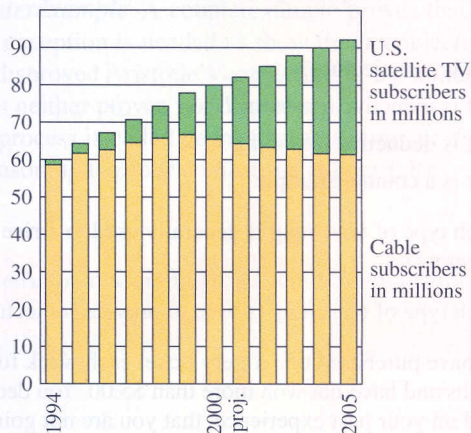


34. **Triangles in a Triangle** Four rows of a triangular figure are shown.



- a) If you added six additional rows to the bottom of this triangle, using the same pattern displayed, how many triangles would appear in the 10th row?  
 b) If the triangles in all 10 rows were added, how many triangles would appear in the entire figure?
35. **Satellite vs. Cable**
- a) The graph shows the past and expected future trend in the number of satellite and cable subscribers from 1994 to 2005. If you had to make a prediction for the number of cable subscribers in the year 2010, what would you predict?  
 b) Using the graph, predict the number of satellite subscribers in 2010.  
 c) Explain how you are using inductive reasoning in determining your answer.

Satellite TV's Big Gains

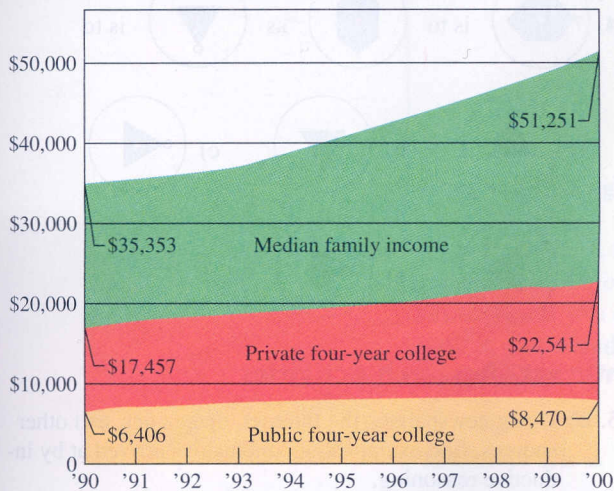


Source: *Fortune*, Feb. 5, 2001

36. **Income and Tuition** The graph on the top of the next page shows the cost of a year's tuition at the average public four-year college, average private four-year college, and the median family income from 1990 to 2000.



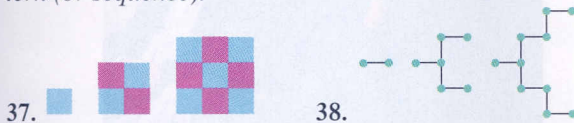
Income and Tuition



Note: College costs include tuition, room, and board  
 Source: Census Bureau, College Board  
 U.S. News and World Report, Oct. 30, 2000

- Using the graph, make a prediction of the cost of tuition at the average private four-year college in 2005.
- Using the graph, predict the median family income in 2005.
- Explain how you are using inductive reasoning in determining your answer.

In Exercises 37 and 38, draw the next diagram in the pattern (or sequence).



- Pick any number, multiply the number by 4, add 8 to the product, divide the sum by 4, and subtract 2 from the quotient. See Example 5.
  - What is the relationship between the number you started with and the final number?
  - Arbitrarily select some different numbers and repeat the process, recording the original number and the result.
  - Can you make a conjecture about the relationship between the original number and the final number?
  - Try to prove, using deductive reasoning, the conjecture you made in part (c). See Example 6.
- Pick any number and multiply the number by 10. Add 5 to the product. Divide the sum by 5 and subtract 1 from the quotient.
  - What is the relationship between the number you started with and the final answer?
  - Arbitrarily select some different numbers and repeat the process, recording the original number and the results.
  - Can you make a conjecture about the relationship between the original number and the final number?
  - Try to prove, using deductive reasoning, the conjecture you made in part (c).
- Pick any number and add 1 to it. Find the sum of the new number and the original number. Add 9 to the sum. Divide the sum by 2 and subtract the original number from the quotient.
  - What is the final number?
  - Arbitrarily select some different numbers and repeat the process. Record the results.
  - Can you make a conjecture about the final number?
  - Try to prove, using deductive reasoning, the conjecture you made in part (c).
- Pick any number and add 10 to the number. Divide the sum by 5. Multiply the quotient by 5. Subtract 10 from the product. Then subtract your original number.
  - What is the result?
  - Arbitrarily select some different numbers and repeat the process, recording the original number and the result.
  - Can you make a conjecture regarding the result when this process is followed?
  - Try to prove, using deductive reasoning, the conjecture you made in part (c).

In Exercises 43–48, find a counterexample to show that each of the statements is incorrect.

- The product of 2 three-digit numbers is a five-digit number.
- The sum of 3 two-digit numbers is a three-digit number.
- When a counting number is added to 3 and the sum is divided by 2, the quotient will be an even number.
- The product of any two counting numbers is divisible by 2.
- The difference of any two counting numbers will be a counting number.
- The sum of any two odd numbers is divisible by 4.

#### 49. Interior Angles of a Triangle

- Construct a triangle and measure the three interior angles with a protractor. What is the sum of the measures?
- Construct three other triangles, measure the angles, and record the sums. Are your answers the same?
- Make a conjecture about the sum of the measures of the three interior angles of a triangle.

#### 50. Interior Angles of a Quadrilateral

- Construct a quadrilateral (a four-sided figure) and measure the four interior angles with a protractor. What is the sum of their angle measures?
- Construct three other quadrilaterals, measure the angles, and record the sums. Are your answers the same?
- Make a conjecture about the sum of the measures of the four interior angles of a quadrilateral.



## Challenge Problems/Group Activities

51. Complete the following square of numbers. Explain how you determined your answer.

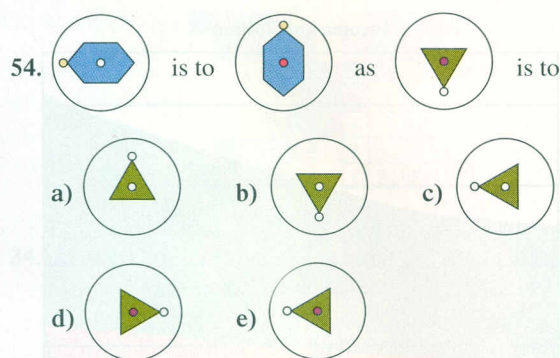
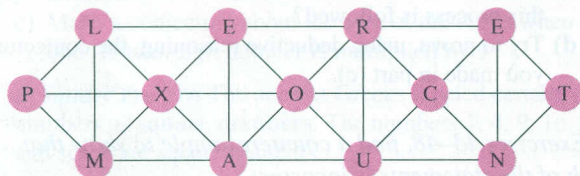
1	2	3	4
2	5	10	17
3	10	25	52
4	17	52	?

52. Find the next three numbers in the sequence.

1, 8, 11, 88, 101, 111, 181, 1001, 1111, ...

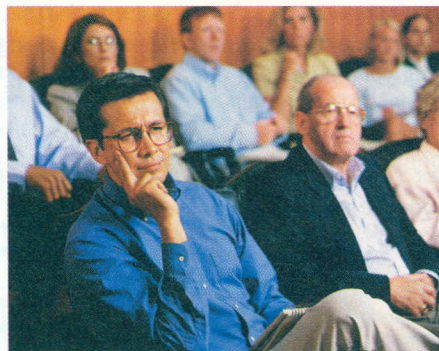
## Recreational Mathematics

53. Trace out a 13-letter word that has been highlighted in this section by moving along the lines. You will need to use all the letters, and you cannot retrace a line.



## Internet/Research Activities

55. a) Using newspapers, the Internet, magazines, and other sources, find examples of conclusions arrived at by inductive reasoning.  
b) Explain how inductive reasoning was used in arriving at the conclusion.
56. When a jury decides the guilt or innocence of a defendant, do the jurors collectively use primarily inductive reasoning, deductive reasoning, or an equal amount of each? Write a brief report supporting your answer.



## 1.2 ESTIMATION



An important step in solving mathematical problems—or, in fact, *any* problem—is to make sure that the answer you’ve arrived at makes sense. One technique for determining whether an answer is reasonable is to estimate. **Estimation** is the process of arriving at an approximate answer to a question. This section demonstrates several estimation methods.

To estimate, or approximate, an answer, we often round numbers as illustrated in the following examples. The symbol  $\approx$  means *is approximately equal to*.

### EXAMPLE 1 Estimating the Cost of Doughnuts

For a division meeting, Judy Wood decides to treat her colleagues to fresh doughnuts. Estimate her cost if she purchases 38 doughnuts at \$0.49 each.



**SOLUTION:** We may round the amounts as follows to obtain an estimate.

Number	Number rounded
38 →	40
$\times \$0.49 \rightarrow$	$\times \$0.50$
	<u>\$20.00</u>

Thus, the 38 doughnuts would cost approximately \$20.00, written  $\approx \$20$ .

In Example 1, the true cost is  $\$0.49 \times 38$ , or \$18.62. *Estimates are not meant to give exact values for answers but are a means of determining whether your answer is reasonable.* If you calculated an answer of \$18.62 and then did a quick estimate to check it, you would know that the answer is reasonable because it is close to your estimated answer.

### EXAMPLE 2 Two Ways to Estimate

At a local supermarket, Kaitlyn purchased ice cream for \$5.19, lettuce for \$1.09, bread for \$1.98, a frozen dinner for \$4.79, ground beef for \$4.26, steaks for \$15.37, and a green onion for \$0.92. The total bill was \$44.08. Use estimation to determine whether this amount is reasonable.

**SOLUTION:** The most expensive item is \$15.37, and the least expensive is \$0.92. How should we estimate? We will estimate two different ways. First, we will round the cost of each item to the nearest 10 cents. For the second method, we will round the cost of each item to the nearest dollar. Rounding to the nearest 10 cents is more accurate. To determine whether the total bill is reasonable, however, we may need to round only to the nearest dollar.

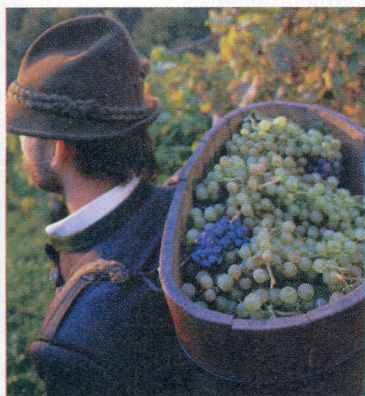
	Round to the nearest 10 cents		Round to the nearest dollar	
Ice cream	\$5.19 →	\$5.20	\$5.19 →	\$5.00
Lettuce	1.09 →	1.10	1.09 →	1.00
Bread	1.98 →	2.00	1.98 →	2.00
Frozen dinner	4.79 →	4.80	4.79 →	5.00
Ground beef	4.26 →	4.30	4.26 →	4.00
Steaks	15.37 →	15.40	15.37 →	15.00
Onion	0.92 →	<u>0.90</u>	0.92 →	<u>1.00</u>
		\$33.70		\$33.00

Using either estimate, we find that the bill of \$44.08 is quite high. Therefore, Kaitlyn should check the bill carefully before paying it. Adding the prices of all seven items gives the true cost of \$33.60.

### EXAMPLE 3 Select the Best Estimate

The number of bushels of grapes produced at a vineyard are 71,309 Cabernet Sauvignon, 123,879 French Colombard, 106,490 Chenin Blanc, 5960 Charbono, and 12,104 Chardonnay. Select the best estimates of the total number of bushels produced by the vineyard.

- a) 500,000      b) 30,000      c) 300,000      d) 5,000,000





**SOLUTION:** Following are suggested roundings. On the left, the numbers are rounded to thousands. For a less close estimate, round to ten thousands, as illustrated on the right.

Round to the nearest thousand		Round to the nearest ten thousand	
71,309 →	71,000	71,309 →	70,000
123,879 →	124,000	123,879 →	120,000
106,490 →	106,000	106,490 →	110,000
5,960 →	6,000	5,960 →	10,000
12,104 →	<u>12,000</u>	12,104 →	<u>10,000</u>
	319,000		320,000

Either rounding procedure indicates that the best estimate is (c), or 300,000. ▲

#### EXAMPLE 4 Using Estimation in Calculations

The odometer of an automobile reads 48,213.7 miles.

- If the automobile averaged 22.1 miles per gallon (mpg) for that mileage, estimate the number of gallons of gasoline used.
- If the cost of the gasoline averaged \$1.59 per gallon, estimate the total cost of the gasoline.

**SOLUTION:**

- To estimate the number of gallons, divide the mileage by the number of miles per gallon.

$$\frac{48,213.7}{22.1}$$

Round these numbers to obtain an estimate.

$$\frac{50,000}{20} = 2500$$

Therefore, the car used approximately 2500 gallons (gal) of gasoline.

- Rounding the price of the gasoline to \$1.60 per gallon gives the cost of the gasoline as  $2500 \times \$1.60$ , or \$4000. ▲

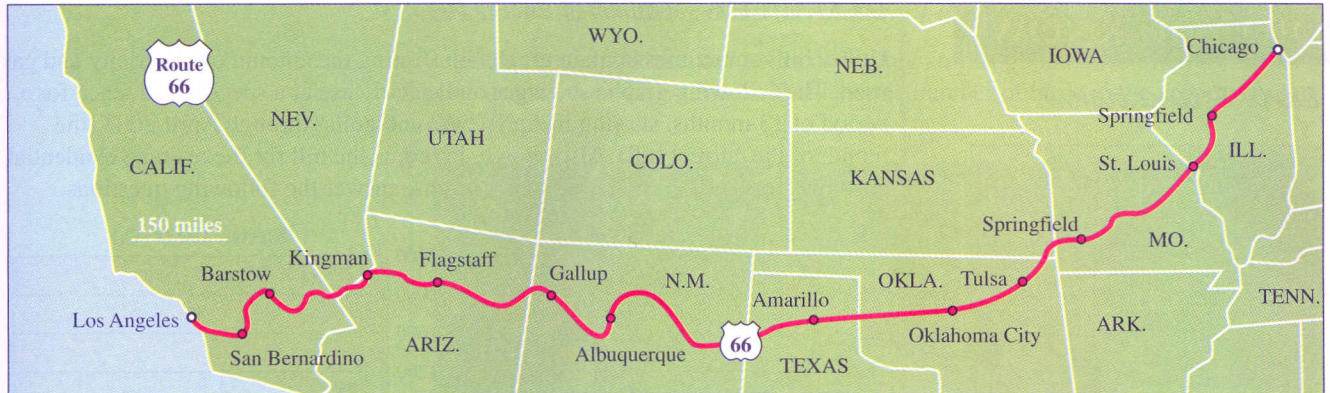
Now let's look at some different types of estimation problems.

#### EXAMPLE 5 Estimating Distances on Route 66

Route 66, also called America's Main Street, or the Mother Road, is probably the most famous road in the United States. Its 75th birthday was officially on November 11, 2001. About 85% of the road is still drivable through 8 states. A map of Route 66 in its heyday, before the opening of the Interstate bypasses, is shown on the next page.

- Using Route 66, estimate the distance from Springfield, Illinois, to Tulsa, Oklahoma.
- Using Route 66, estimate the distance from Chicago, Illinois, to Los Angeles, California.





Source: Gannett News Service

### SOLUTION:

- a) Using a ruler and the scale given on the map, we can determine that approximately  $\frac{1}{2}$  inch represents 150 miles (mi). One way to estimate the distance between Springfield, Illinois, and Tulsa, Oklahoma, is to mark off  $\frac{1}{2}$ -inch intervals along Route 66. If you do so you may obtain about three intervals. Thus, the distance is  $3 \times 150$  mi, or about 450 mi.

Sometimes on a map like this one, it may be difficult to get an accurate estimate because of the curves on the map. To get a more accurate estimate, you may want to use a piece of string. Place the beginning of the string at Springfield, Illinois, and, using tape or pins, align the string with the road. Indicate on the string where the road ends. Then remove the string and make interval markings on the string (or measure the length of string you have marked off). If, for example, your string from Springfield, Illinois, to Tulsa, Oklahoma, measures  $3\frac{1}{8}$  intervals, then the distance is about  $3.125 \times 150$  mi, or about 469 mi.

- b) Using the procedure discussed in part (a), we estimate that the distance from Chicago, Illinois, to Los Angeles, California is about 8 in., or about 16 half-inches. Thus, the distance of Route 66 is about  $16 \times 150$ , or about 2400 mi. ▲

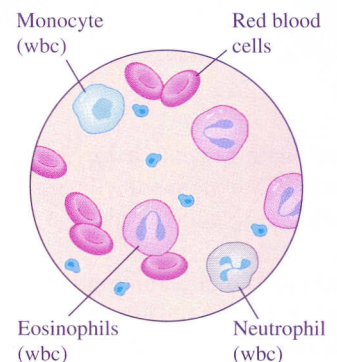
## DID YOU KNOW

### Estimating Techniques in Medicine

Estimating is one of the diagnostic tools used by the medical profession. Physicians take a small sample of blood, tissue, or body fluids to be representative of the body as a whole. Human blood contains different types of white blood cells that fight infection. When a bacterium or virus gets into the blood, the body responds by producing more of the type of white blood cell whose job it is to destroy that particular invader. Thus, an increased level of white blood cells in a sample of blood not only indicates the presence of an infection but also helps identify its type. A trained medical technician estimates the relative number of each kind of white blood cell found in a count of 100 white blood cells. An increase in any one kind indicates the type of infection present. The accuracy of this diagnostic tool is impressive when you consider that there are normally 5000 to 9000 white blood cells in a dropful of blood (1 cubic millimeter).



Most analysis of blood samples done today is performed using computers.

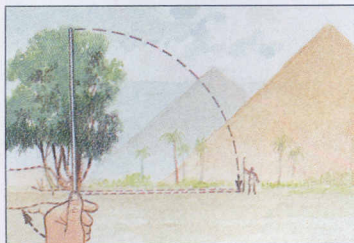


A high count of eosinophils (a particular kind of white blood cell, or wbc) can be an indicator of an allergic reaction.



## DID YOU KNOW

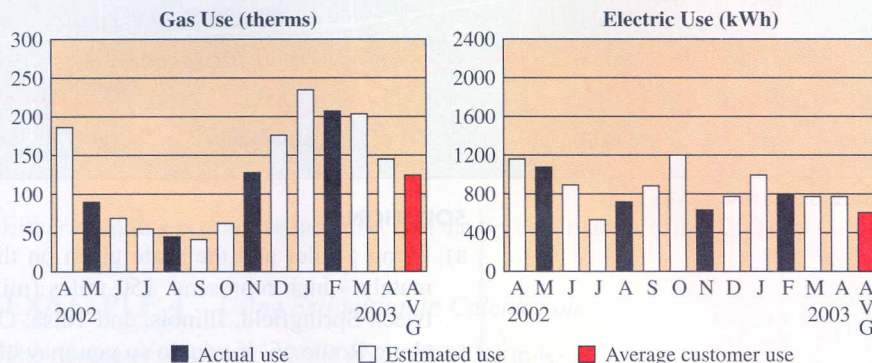
## Early Estimation



The ancient Egyptians and Greeks used sticks to estimate heights. To measure height, a person held a stick in his or her outstretched arm until it just covered the height of the object. The person then turned the stick through a right angle. Measuring the distance the stick appeared to cover on the ground provided an estimate of the object's height.

## EXAMPLE 6 Estimated Energy Use

Utility bills sometimes contain graphs illustrating the amount of electricity and gas used. The following graphs show gas and electric use at a specific residence for a period of 13 months, starting in April 2002 and going through April 2003 (the month of the current bill). Also shown, in red, is the bill for the average residential customer for April 2003. Using these graphs, answer the following questions.



- How often was an actual gas and electric reading made?
- Estimate the number of therms of gas used by the average residential customer in April 2003.
- Estimate the amount of gas used by the resident in April 2003.
- If the cost of gas is 69.3672 cents per therm, estimate the gas bill in November 2002.
- In which month was the most electricity used? How many kilowatt hours (kWh) were used in this month?
- If the cost of electricity is 9.0983 cents per kilowatt hour, estimate the cost of electricity in February 2003.

## SOLUTION:

- Actual readings were made in only four months, May, August, November, and February.
- Approximately 125 therms were used, as shown by the height of the red bar.
- Approximately 145 therms were used (slightly less than 150).
- In November, about 125 therms were used. The rate, 69.3672 cents per therm, is the same as \$0.693672 per therm. To get a rough approximation, round the rate to \$0.70 per therm.

$$0.70 \times 125 = 87.50^*$$

Thus, the cost of gas used was about \$88.

- The most electricity was used in October. Approximately 1200 kWh were used.
- In February, about 800 kWh were used. Write 9.0983 cents as \$0.090983. Rounding the rate to \$0.09 per kilowatt hour and multiplying by 800 yields an estimate of \$72.

$$0.09 \times 800 = \$72$$

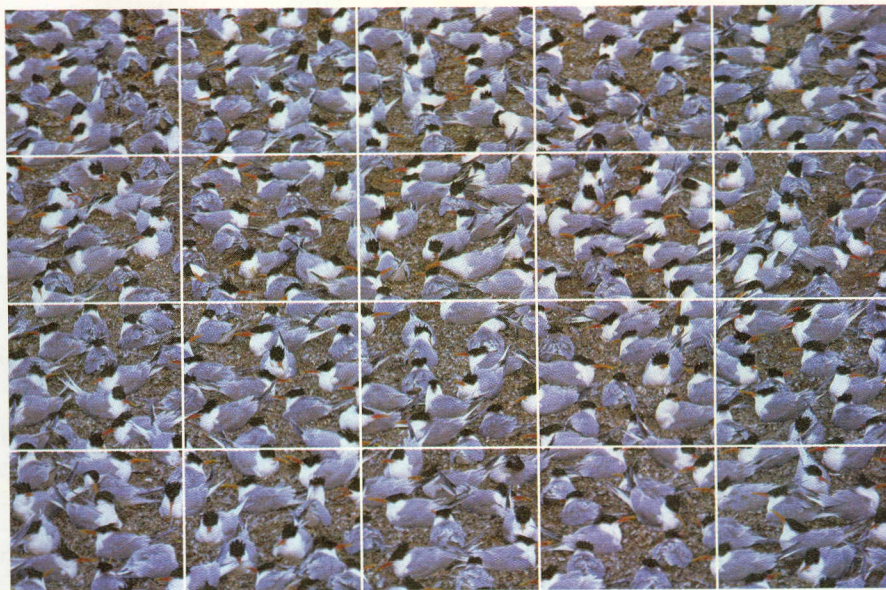
Thus, the cost of electricity in February 2003 was about \$72.

\*The amounts here and in part (f) do not include the basic monthly charge, fuel adjustment, taxes, and other extra charges often included on utility bills.



**EXAMPLE 7** *Estimating the Number of Birds in a Photo*

Scientists who are concerned about dwindling animal populations often use aerial photography to make estimates. Estimate the number of birds in the accompanying photograph.



**SOLUTION:** To estimate the number of birds, we can divide the photograph into rectangles with equal areas, then select one area that appears to be representative of all the areas. Estimate (or count) the number of birds in this single area, and then multiply this number by the number of equal areas.

Let's divide the photo into 20 approximately equal areas. We will select the middle region in the bottom row as the representative region. We enlarge this region and count the birds in it. If half a bird is in the region, we count it (see enlargement). There are 13 birds in this region. Multiplying by 20 gives  $13 \times 20 = 260$ . Thus, there are about 260 birds in the photo.



In problems similar to that in Example 7, the number of regions or areas into which you choose to divide the total area is arbitrary. Generally, the more regions, the better the approximation, as long as the region selected is representative of the other regions in the map, diagram, or photo.

When you estimate an answer, the amount that your approximation differs from the actual answer will depend on how you round the numbers. Thus, in estimating the product of  $196,000 \times 0.02520$ , using the rounded values  $195,000 \times 0.025$  would yield an estimate much closer to the true answer than using the rounded values



$200,000 \times 0.03$ . Without a calculator, however, the product of  $195,000 \times 0.025$  might be more difficult to find than  $200,000 \times 0.03$ . When estimating, you need to determine the accuracy desired in your estimate and round the numbers accordingly.

## SECTION 1.2 EXERCISES

### Practice the Skills

In Exercises 1–57, your answers may vary from the answers given in the back of the text, depending on how you round your answers.

In Exercises 1–12, estimate the answer. There is no one correct estimate. Your answer, however, should be something near the answer given.

1.  $431 + 327.2 + 73.5 + 20.4 + 315.9$
2.  $3.89 + 402.8 + 156.9 + 189 + 0.23 + 416$
3.  $297,700 \times 4087$
4.  $1854 \times 0.0096$
5.  $\frac{405}{0.049}$
6.  $297.521 - 85.964$
7.  $0.049 \times 1989$
8. 9% of 2164
9.  $51,608 \times 6981$
10.  $\frac{0.0498}{0.00052}$
11.  $592 \times 2070 \times 992.62$
12.  $296.3 \div 0.0096$

### Problem Solving

In Exercises 13–24, estimate the answer.

13. The cost of 52 thirty-seven-cent stamps.
14. The income earned for 32 hours at \$7.95 per hour.
15. The total frequent flier mileage awarded for four trips of 1521, 1897, 2324, and 2817 miles.
16. The weight of six buckets of water if each bucket weighs 15.87 pounds (lb).
17. The cost of eight items purchased at a drugstore if the items cost \$2.29, \$12.16, \$4.97, \$6.69, \$49.76, \$0.47, \$3.49, and \$5.65.
18. The weight of one hamburger in a package of six hamburgers if the weight of the package is 3.12 lb.
19. One fifth of an annual profit of \$44,569.
20. The weight of the load of an 18-wheel truck if the weight of the truck when empty is 14,292 pounds and the weight of the truck when loaded is 32,798 lb.
21. The weight of nine identical packages of paper if each package weighs 5.12 lb.

22. An 8% sales tax on a car that cost \$14,876 before tax.
23. The average monthly distance traveled if Paul travels 23,663 miles in 1 year.
24. The cost of a pound of grapes if 3.2 lb of grapes cost \$10.87.
25. **Time Warner Bill** Dale Gray has both his cable TV and Road Runner internet service through Time Warner Communications. His cable TV costs \$29.17 per month and his Road Runner service costs \$39.95 per month. Both charges appear on the same monthly bill. Estimate the annual amount Dale pays to Time Warner.
26. **Estimating Weights** In a tug of war, the weight of the members of the two three-person teams is given below. Estimate the difference in the weights of the teams.

Team A	Team B
189	183
172	229
191	167

27. **Estimating the Tip** Ed and Dorothy Ruff go out for dinner and spend \$38.60 for their meal. If they want to leave a 15% tip, estimate the amount that they should leave.
28. **Estimating Area** Mrs. Sanchez determines that her lawn contains an average of 3.8 grubs per square foot ( $\text{ft}^2$ ). If her rectangular lawn measures 60 ft by 80.2 ft, estimate the total number of grubs in her lawn.
29. **Currency** Estimate the difference in the value of 100 Mexican pesos and 50 U.S. dollars. Assume that one Mexican peso is about 0.092 U.S. dollar.
30. **The Cost of a Vacation** The Kleins are planning a vacation in the Great Smokey Mountain National Park. Their round-trip airfare from Houston, Texas, to Knoxville, Tennessee, totals \$973. Car rental is \$41 per day, lodging is a total of \$97 per day, and they estimate a total of \$90 per day for food, gas, and other miscellaneous items. If they are planning to stay six full days and nights, estimate their total expenses.

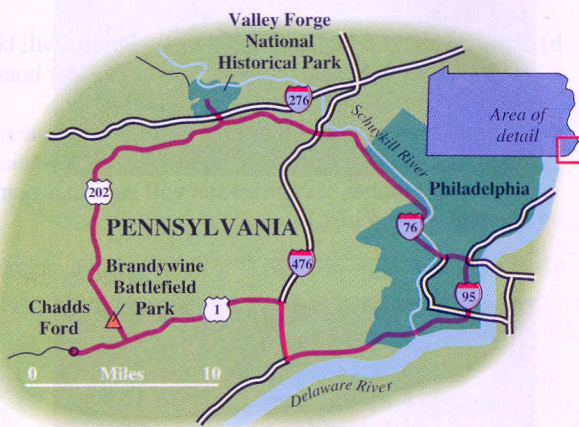


For Exercises 31 and 32, refer to the maps below. In the April 23, 2001, issue of U.S. News and World Report, an article by Marc Silver discusses great vacation drives. The maps that follow were selected from that article.

31. **A Drive in Mississippi** Using the scale on the map, estimate the distance of the route shown in red starting and ending in Oxford.



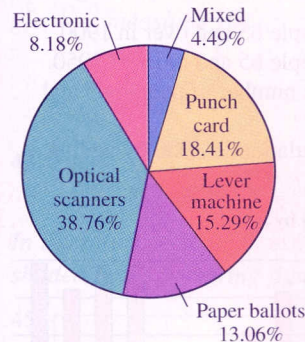
32. **A Drive in Pennsylvania** Using the scale on the map, estimate the distance of the route shown in red by starting and ending in Chadds Ford.



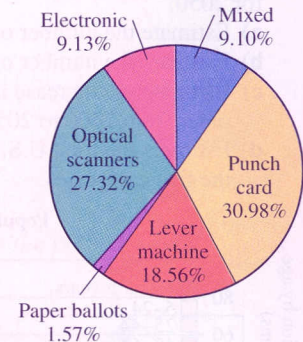
33. **Voting** The circle graphs above and to the right show the methods that Americans used to vote as of November 2000, according to Election Data Services, Inc. There were approximately 105 million Americans from all 3141 counties who voted in the 2000 presidential election.

Voting Systems in the United States

Percentage Use, by Counties



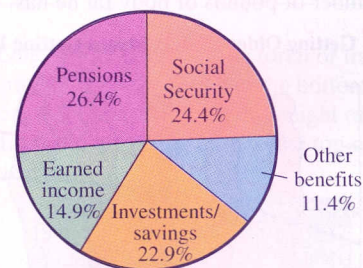
Percentage Use, by Voters



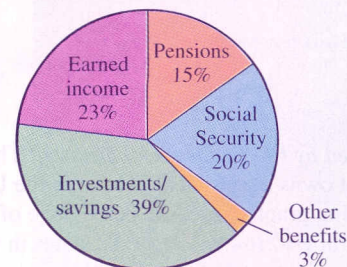
Source: Election Data Services Inc.

- Using the graphs, estimate the number of people who voted using punch cards.
  - Using the graphs, estimate the number of counties that voted using punch cards.
  - The percent of voters who use punch cards is greater than the percent of counties that use punch cards. Can you offer any suggestions as to how this is possible?
34. **Retirement** The circle graphs show sources of retirement income for retirees with at least \$20,000 in annual income for the years 1999 and 2002. Assume that Sheila Abbruzzo had a retirement income of \$40,075 from the various sources shown in the figures. Estimate her income from investments/savings in
- 1999.
  - 2002.

Source of Retirement Income 2002



Source of Retirement Income 1999

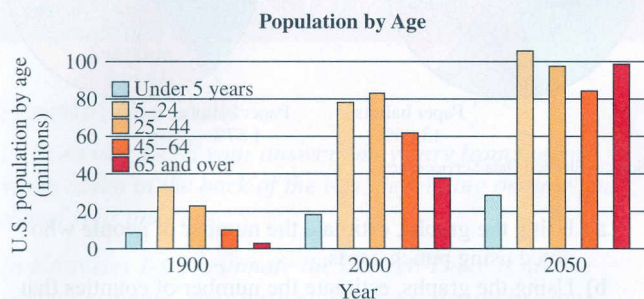


Source: Social Security Administration



35. **An Aging Population** The bar graph shows population figures for 1900 and 2000 and estimated population figures for 2050.

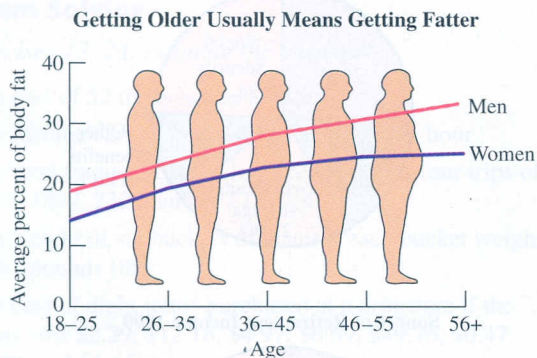
- Estimate the number of people 65 and over in 1900.
- Estimate the number of people 65 and older in 2050.
- Estimate the increase in the number of people 65 and older from 2000 to 2050.
- Estimate the total U.S. population in 2000 by adding the five categories.



Source: U.S. Census Bureau

36. **Gaining Weight** As the graph shows, as a society we tend to get heavier as we get older. Also, with age, the amount of muscle tends to drop, and fat accounts for a greater percentage of weight.

- Estimate the average percent of body fat for a male, age 18 to 25.
- Estimate the average percent of body fat for a female, age 56+.
- Greg, an average 40-year-old, weighs 179 lb. Estimate the number of pounds of body fat he has.

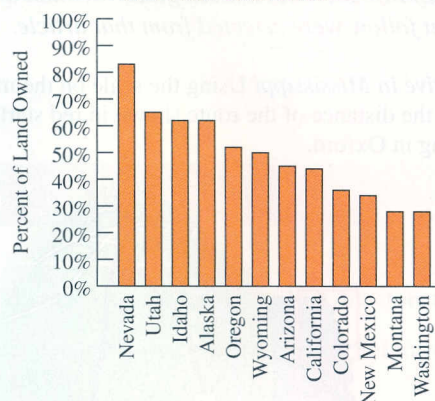


Source: Mayo Clinic Newsletter

37. **Land Owned by the Federal Government** The federal government owns a great deal of land in the United States. The following graph shows the percentage of land owned by the federal government in the 12 states in which it owns the greatest percentage of a state's land.

#### The Geography of Ownership

Percentage of Land Owned by the Federal Government



Source: Time Magazine, July 16, 2001

- Estimate the percent of land in Nevada owned by the federal government.
  - Estimate the difference in the percent of land owned by the federal government in Utah and in Arizona.
  - Nevada has a total area of 110,567 square miles. Estimate the number of acres owned by the federal government in Nevada.
  - By just looking at the graph, is it possible to determine whether the federal government owns more land in Nevada or Washington? Explain.
38. **Calories and Exercise** The chart shows the calories burned per hour for an average person who weighs 150 lb.
- Estimate the number of calories Phyllis Nye, who weighs 150 lb, burns in a week if she stair-climbs for 2 hours each week and jogs at 5 miles per hour 4 hours each week.
  - Estimate the difference in the calories Phyllis will burn each week if she runs for 4 hours at 8 miles per hour rather than does casual bike riding for 4 hours.
  - Assume Phyllis jogs at 5 miles per hour for 3 hours and bicycles at 13 miles per hour for 3 hours each week. Estimate the number of calories she will burn in a year from these exercises.

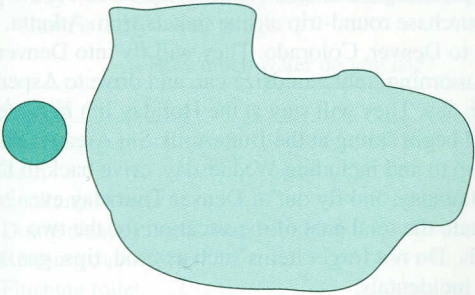
Activity	Calories* per hour
Running, 8 mph	920
Bicycling, 13 mph	545
Jogging, 5 mph	545
Air-walking	480
Stair-climbing	410
Weight-lifting	410
Walking, 4 mph	330
Casual bike riding	300

\*For a 150 lb person.

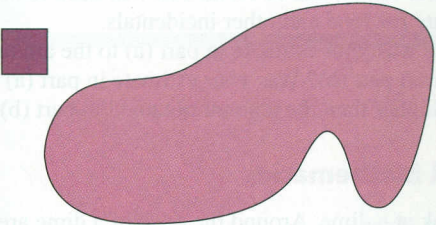


In Exercises 39 and 40, estimate the maximum number of smaller figures (at left) that can be placed in the larger figure (at right) without the small figures overlapping.

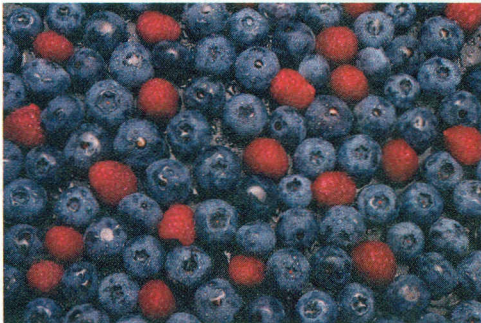
39.



40.



41. Estimate the number of berries shown in the photo.

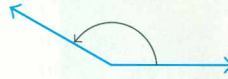


42. Estimate the number of leaves shown in the photo.

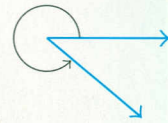


In Exercises 43 and 44, estimate, in degrees, the measure of the angles depicted. For comparison purposes a right angle,  $\angle$ , measures  $90^\circ$ .

43.

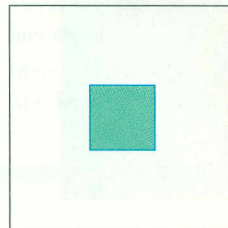


44.

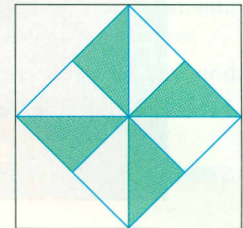


In Exercises 45 and 46, estimate the percent of area that is shaded in the following figures.

45.

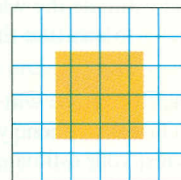


46.

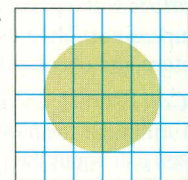


In Exercises 47 and 48, if each square represents one square unit, estimate the area of the shaded figure in square units.

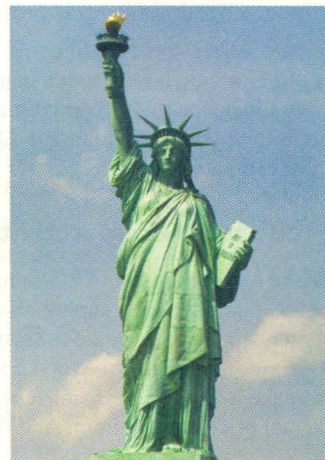
47.



48.

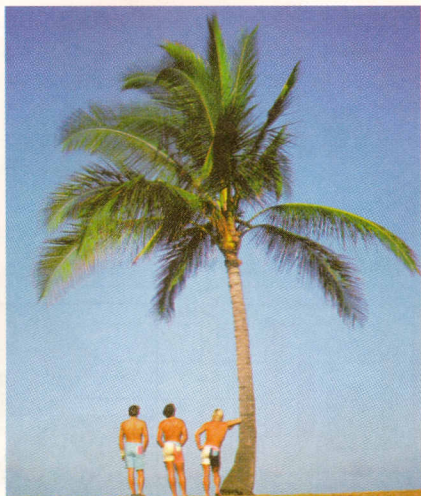


49. **Statue of Liberty** The length of the torch of the Statue of Liberty, from the tip of the flame to the bottom of the torch's baton, is 29 feet. Estimate the height of the Statue of Liberty from the top of the base to the top of the torch (the statue itself).





50. **Estimating Heights** If the height of the middle person in the photo is 62 in. tall, estimate the height of the tree.



51. **Weight** In a bag, place objects that you feel have a total weight of 10 lb. Weigh the bag to determine the accuracy of your estimate.
52. **Distance** Estimate, without a ruler, a distance of 12 in. Measure the distance. How good was your estimate?
53. **Temperature** Fill a glass with water and estimate the water's temperature. Then use a thermometer to measure the temperature and check your estimate.
54. **Phone Call** Estimate the number of times the phone will ring in 1 minute if unanswered. Have a classmate phone you so that you can count the rings and thus test your estimate.
55. **Pennies** Estimate the number of pennies that will fill a 3-ounce (oz) paper cup. Then actually fill a 3 oz paper cup with pennies, counting them to determine the accuracy of your estimate.
56. **Height** Estimate the ratio of your height to your neck size. Then have a friend measure your height and neck size. Determine the stated ratio and check the accuracy of your estimate.
57. **Walking Speed** Estimate how fast you can walk 60 ft. Then mark off a distance of 60 ft and use a watch with a second hand to time yourself walking it. Determine the accuracy of your estimate.

determine the actual total cost. How close was your estimate? (Don't forget to add tax on the taxable items.)

59. **A Ski Vacation** Two friends, Tiffany Connolly and Ana Pott, are planning a skiing vacation in the Rockies. They plan to purchase round-trip airline tickets from Atlanta, Georgia, to Denver, Colorado. They will fly into Denver on a Friday morning, rent a midsize car, and drive to Aspen that same day. They will stay at the Holiday Inn in Aspen. They will begin skiing at the Buttermilk Ski Area on Saturday, ski up to and including Wednesday, drive back to Denver on Thursday, and fly out of Denver Thursday evening.
- Estimate the total cost of the vacation for the two friends. Do not forget items such as food, tips, gas, and other incidentals.
  - Using informational sources, including the Internet, determine the airfare cost, hotel cost, cost of ski tickets, cost of a car rental, and so forth. You will need to make an estimate for food and other incidentals.
  - How close was your estimate in part (a) to the amount you found in part (b)? Was your estimate in part (a) lower or higher than the amount obtained in part (b)?

## Recreational Mathematics

60. **A Dime** Look at a dime. Around the edge of a dime are many lines. Estimate the number of lines there are around the edge of a dime.
61. **Golf Ball** Look at a golf ball. Estimate the number of dimples (depressed areas) on a golf ball.
62. **A Million Dollars**
- Estimate the time it would take, in days, to spend \$1 million if you spent \$1 a second until the \$1 million is used up.
  - Calculate the actual time it would take, in days, to spend \$1 million if you spent \$1 a second. How close was your estimate?
63. **The Middle Class** An article in the February 5, 2001, *U.S. News and World Report* indicated that many Americans believe that they are in the middle class (in terms of annual household income; see the chart), although according to U.S. government guidelines they are not. Estimate the annual household income range that you believe is classified as middle class according to the U.S. government.

Middle-Class Perception  
By income, households that call themselves middle class

Less than \$15,000	37%
\$15,000–\$35,000	29%
\$35,000–\$50,000	42%
\$50,000–\$75,000	57%
\$75,000–\$110,000	73%
More than \$110,000	70%

## Challenge Problems/Group Activities

58. **Shopping** Make a shopping list of 20 items you use regularly that can be purchased at a supermarket. Beside each item write down what you estimate to be its price. Add these price guesses to estimate the total cost of the 20 items. Next, make a trip to your local supermarket and record the actual price of each item. Add these prices to



## Internet/Research Activities

### 64. Water Usage

- a) About how much water does your household use per day? Use the following data to estimate your household's daily water usage.

How much water do you use?

Activity	Typical Use
Running clothes washer	40 gal
Bath	35 gal
5-minute shower	25 gal
Doing dishes in sink, water running	20 gal
Running dishwasher	11 gal
Flushing toilet	4 gal
Brushing teeth, water running	2 gal

Source: U.S. Environmental Protection Agency

- b) Determine from your water department (or company) your household's average daily usage by obtaining the total number of gallons used per year and dividing that amount by 365. How close was your estimate in part (a)?
- c) Current records indicate that the average household uses about 300 gal of water per day (the average daily usage is 110 gal per person). Based on the number of people in your household, do you believe your household uses more or less than the average amount of water? Explain your answer.
65. Develop a monthly budget by estimating your monthly income and your monthly expenditures. Your monthly income should equal your monthly expenditures.
66. Identify three ways that you use estimation in your daily life. Discuss each of them briefly and give examples.

## 1.3 PROBLEM SOLVING

Solving mathematical puzzles and real-life mathematical problems can be enjoyable. You should work as many exercises in this section as possible. By doing so, you will sample a variety of problem-solving techniques.

You can approach any problem by using a general procedure developed by George Polya. Before learning Polya's problem-solving procedure, let's consider an example.

### EXAMPLE 1 Saving Money When Purchasing Videotapes

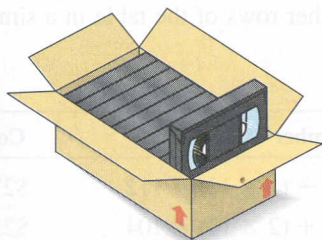
Businesses, to maximize their profits, try to keep their expenses down. We, as individuals, also try to keep our expenses down and we often look for "bargains" or the "best deal."

A video store owned by Roberto Santiago plans to purchase a large number of blank videotapes. One supplier, the Tashanna Miller Company, is selling boxes of 20 tapes for \$48 and boxes of 12 tapes for \$30. Only complete boxes of tapes are sold.

- a) Find the maximum number of tapes that can be purchased for \$280 or less. Indicate how many boxes of 20 and how many boxes of 12 will be purchased.
- b) If the maximum number of tapes determined in part (a) is purchased in the most economical way, how much will the tapes cost?

### SOLUTION:

- a) The first thing to do is to read the problem carefully. Read it at least twice and be sure you understand the facts given and what you are being asked to find. Next, make a list of the given facts and determine which are relevant to answering the question asked.





**Given information**

Store owner: Roberto Santiago

Supplier: Tashanna Miller Company

A box of 20 tapes costs \$48.

A box of 12 tapes costs \$30.

Only complete boxes of tapes can be purchased.

We need to determine the maximum number of tapes that the video store can purchase for \$280 or less. To determine this number, we need to know the number of tapes in each of the boxes and the cost of the boxes. We also need to know that only complete boxes of tapes may be purchased.

**Relevant information**

A box of 20 tapes costs \$48.

A box of 12 tapes costs \$30.

Only complete boxes of tapes may be purchased.

The next step is to determine the answer to the question. That is, we need to determine the maximum number of tapes that can be purchased for \$280 or less.

We now need a plan for solving the problem. One method is to set up a table or chart to compare costs of different combinations of boxes of tapes. Start by using the maximum number of boxes of 20 tapes. Then reduce the number of boxes of 20 tapes, and add more boxes of 12 tapes. In each case, we need to keep the cost at \$280 or less.

Since 1 box of 20 tapes costs \$48, we can determine the number of boxes of 20 tapes that can be purchased by dividing 280 by 48. Since the quotient is about 5.83, and since only whole boxes of tapes may be purchased, only 5 boxes of 20 may be purchased. Five boxes would cost  $5 \times 48 = 240$ . The remaining \$40 from the \$280 could be used to purchase boxes of 12 tapes. Since each box of 12 tapes costs \$30, only one box of 12 tapes could be purchased. Thus, for \$280 or less, one option is 5 boxes of 20 tapes and 1 box of 12 tapes. This option is indicated in the first row of the table below. Also given in the table is the cost of this option, which is \$270. We complete the other rows of the table in a similar manner.

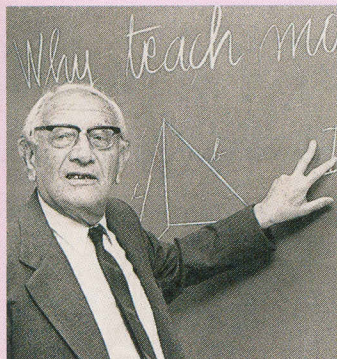
Boxes of 20 and Boxes of 12 Tapes	Number of Tapes	Cost
5 boxes of 20 and 1 box of 12	$(5 \times 20) + (1 \times 12) = 112$	\$270
4 boxes of 20 and 2 boxes of 12	$(4 \times 20) + (2 \times 12) = 104$	\$252
3 boxes of 20 and 4 boxes of 12	$(3 \times 20) + (4 \times 12) = 108$	\$264
2 boxes of 20 and 6 boxes of 12	$(2 \times 20) + (6 \times 12) = 112$	\$276
1 box of 20 and 7 boxes of 12	$(1 \times 20) + (7 \times 12) = 104$	\$258
0 boxes of 20 and 9 boxes of 12	$(0 \times 20) + (9 \times 12) = 108$	\$270

The question asks us to find the maximum number of tapes that can be purchased for \$280 or less. From the second column of the table, we see that the answer is 112 tapes. This result can be done in two different ways: either 5 boxes of 20 and 1 box of 12 tapes, or 2 boxes of 20 and 6 boxes of 12 tapes.



## PROFILE IN MATHEMATICS

### GEORGE POLYA



**G**eorge Polya (1887–1985) was educated in Europe and taught at Stanford University. In his book *How to Solve It*, Polya outlines four steps in problem solving. We will use Polya's four steps as guidelines for problem solving.

- b) When comparing the two possibilities for purchasing the 112 tapes discussed in part (a), we see that the most economical way to purchase the tapes is to purchase 5 boxes of 20 and 1 box of 12 tapes. The cost is \$270.

Following is a general procedure for problem solving as given by George Polya. Note that Example 1 demonstrates many of these guidelines.

### Guidelines for Problem Solving

#### 1. Understand the problem.

- Read the problem *carefully* at least twice. In the first reading, get a general overview of the problem. In the second reading, determine (a) exactly what you are being asked to find and (b) what information the problem provides.
- Try to make a sketch to illustrate the problem. Label the information given.
- Make a list of the given facts that are pertinent to the problem.
- Determine if the information you are given is sufficient to solve the problem.

#### 2. Devise a plan to solve the problem.

- Have you seen the problem or a similar problem before? Are the procedures you used to solve the similar problem applicable to the new problem?
- Can you express the problem in terms of an algebraic equation? (We explain how to write algebraic equations in Chapter 6.)
- Look for patterns or relationships in the problem that may help in solving it.
- Can you express the problem more simply?
- Can you substitute smaller or simpler numbers to make the problem more understandable?
- Will listing the information in a table help in solving the problem?
- Can you make an educated guess at the solution? Sometimes if you know an approximate solution, you can work backward and eventually determine the correct procedure to solve the problem.

#### 3. Carry out the plan.

Use the plan you devised in step 2 to solve the problem.

#### 4. Check the results.

- Ask yourself, “Does the answer make sense?” and “Is the answer reasonable?” If the answer is not reasonable, recheck your method for solving the problem and your calculations.
- Can you check the solution using the original statement?
- Is there an alternative method to arrive at the same conclusion?
- Can the results of this problem be used to solve other problems?

The following examples show how to apply the guidelines for problem solving.

### EXAMPLE 2 Hotel Cost

At the Courtyard by Marriot Hotel in Irving, Texas (near the Dallas/Fort Worth airport), the room rate is \$139 per day on weekdays and \$55 per day on weekends (Saturday and Sunday). In addition, a 13% sales tax is added to the cost of a room. In-room movies cost \$8.95 plus the 13% sales tax. Robin Ayers stays on the third floor of the Courtyard for four nights (Wednesday, Thursday, Friday, and Saturday) and watches three movies. Determine her hotel bill when she checks out.



## DID YOU KNOW

### *Archimedes Treatise, On Floating Bodies*

On October 14, 2000, the Associated Press reported that scientists are using problem-solving techniques and modern technology to unlock ancient secrets. Two teams of scientists, one from Johns Hopkins University and the other from the Rochester Institute of Technology, have deciphered five pages of the only known copy of a 2300-year-old Greek text by the mathematician Archimedes. The scientists hope to complete a translation of this only original copy of the 174-page treatise, *On Floating Bodies*, in the not too distant future. Scholars think the treatise was copied by a scribe in the 10th century from Archimedes' original Greek scrolls, written in the third century B.C. The treatise was erased 200 years later by a monk who reused the parchment for a prayer book, creating a twice-used parchment book.

The scientists are using digital cameras and processing techniques, as well as ultraviolet and infrared filters developed for medicine and space research, to reveal the hidden text. The text and diagrams also contain the roots of modern calculus and gravitational theory.



**SOLUTION:** We need to find the total cost of the hotel bill. Let's make a list of the information given and mark with an asterisk (\*) the information that is pertinent to solving the problem.

- \*Cost of room per day on Wednesday, Thursday, and Friday = \$139 + sales tax
- \*Cost of room on Saturday = \$55 + sales tax
- \*Days at hotel: one Wednesday, one Thursday, one Friday, one Saturday
- \*Sales tax is 13%
- \*Cost per movie = \$8.95 + sales tax
- \*Movies watched: 3
- Room on third floor

All the information is needed to solve the problem except for the floor on which Robin stayed.

Let's first determine the cost of the room, before tax, for the four days.

Day	Cost (before tax)
Wednesday	\$139
Thursday	\$139
Friday	\$139
Saturday	\$55
	\$472

The cost of 3 movies before tax is  $3 \times \$8.95 = \$26.85$ . Thus, the total amount of the bill before tax is  $\$472 + \$26.85 = \$498.85$ . Let's determine the sales tax to be added. We could find the sales tax for each individual item and add it to the items separately. However, since the same tax rate applies to each item, it is easier to just determine 13% of the total amount.

$$\begin{aligned}\text{Sales tax} &= 13\% \text{ of } \$498.85 \\ &= 0.13(498.85) = \$64.85\end{aligned}$$

The total hotel bill is determined by adding the tax to the pretax amount. Thus, the total hotel bill is  $\$498.85 + \$64.85 = \$563.70$ . ▲

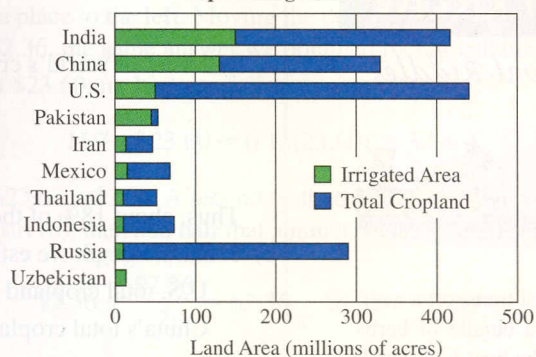
In Example 2, there are other ways that the total cost could have been determined. We presented the method we believe you would best understand.

### EXAMPLE 3 *More Food, Less Water*

We see graphs every day in newspapers and magazines. It is important to you as a consumer to be able to interpret the graphs. The following graph, which shows the top 10 irrigators worldwide and total cropland worldwide, was taken from the February 2001 issue of *Scientific American*. The article mentions that 6000 years ago, farmers in Mesopotamia dug a ditch to divert water from the Euphrates River to form the world's first irrigation-based civilization. Far more people today depend on irrigation than they did in Mesopotamia. Today, about 40% of the world's food grows in irrigated soil. The world's total irrigated cropland is about 670 million acres, and the world's total cropland is about 3722 million acres.



Top 10 Irrigators Worldwide



Source: UN FAO-AGROSTAT database

- Approximately what percent of the world's total irrigated cropland is found in the top four countries indicated on the graph?
- Determine the total percent of the area of the world's cropland that is irrigated.
- Estimate the difference in total cropland between the United States and China.

**SOLUTION:** A great deal of information is provided. The first thing to do is to read the question carefully to make sure you understand what is given and what you are asked to determine. You may need to use different information to answer different parts of the question.

- To determine the percent we are seeking, we need to divide the sum of the irrigated areas of the top four countries by the world's total irrigated area. From the graph, we determine that the top four countries for irrigated acreage are India, China, United States, and Pakistan. We are given that the world's total irrigated cropland is 670 million acres. Below we estimate the irrigated acreage of the top four countries, in millions of acres.

$$\begin{array}{rcl}
 \text{India} & \approx & 150 \\
 \text{China} & \approx & 130 \\
 \text{United States} & \approx & 50 \\
 \text{Pakistan} & \approx & 45 \\
 \hline
 & \approx & 375
 \end{array}$$

The total acreage of irrigated cropland of these four countries is about 375 million acres.

$$\begin{aligned}
 \text{Percent of irrigated areas of top four countries} &= \frac{\text{sum of irrigated areas of four countries}}{\text{total world area of irrigated land}} \\
 &\approx \frac{375}{670} \approx 0.560 \approx 56\%
 \end{aligned}$$

Thus, these four countries account for about 56% of the world's total irrigated cropland.

- Since not all countries are represented on the graph, we cannot use the graph to obtain our answer. The relevant information given is as follows.

The world's total irrigated cropland is about 670 million acres.

The world's total cropland is about 3722 million acres.

\*If you have forgotten how to change a decimal number to a percent, see Section 11.1.



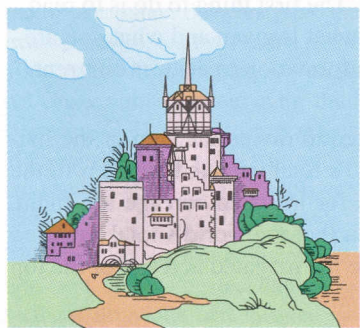
## DID YOU KNOW

## Two Medieval Riddles



A boatman must transport a wolf, a goat, and a bundle of herbs across a river. If the boat is so small he can carry only one of them at a time, how can he make the crossing so that the wolf will not devour the goat and the goat will not eat the herbs?

(Clue: The boatman must make multiple round trips.)



As I was going to St. Ives,  
I met a man with seven wives;  
Every wife had seven sacks,  
Every sack had seven cats,  
Every cat had seven kits.  
Kits, cats, sacks, and wives,  
How many were going to St. Ives?

To find the percent we are seeking, we use the following.

$$\begin{aligned}\text{Percent of world's cropland that is irrigated} &= \frac{\text{total irrigated cropland area}}{\text{total cropland area}} \\ &\approx \frac{670}{3722} \approx 0.18 \approx 18\%\end{aligned}$$

Thus, about 18% of the world's cropland is irrigated.

c) From the graph we estimate the following.

U.S. total cropland is about 440 million acres

China's total cropland is about 330 million acres

$$\begin{aligned}\text{Difference in cropland between U.S. and China} &= \text{U.S. acreage} - \text{China's acreage} \\ &= 440 - 330 = 110\end{aligned}$$

Thus, the difference in total cropland is about 110 million acres. ▲

## EXAMPLE 4 Determining a Tip

The cost of Sarah Decker's meal before tax is \$23.60.

- If a  $7\frac{1}{2}\%$  sales tax is added to her bill, determine the total cost of the meal including tax.
- If Sarah wants to leave a 10% tip on the *pretax* cost of the meal, how much should she leave?
- If she wants to leave a 15% tip on the *pretax* cost of the meal, how much should she leave?

## SOLUTION:

- The sales tax is  $7\frac{1}{2}\%$  of \$23.60. To determine the sales tax, first change the  $7\frac{1}{2}\%$  to a decimal number.  $7\frac{1}{2}\%$  when written as a decimal number is 0.075 (if you have forgotten how to change a percent to a decimal number, read Section 11.1). Next, multiply the decimal number, 0.075, by the amount, \$23.60.

$$\begin{aligned}\text{Sales tax} &= 7\frac{1}{2}\% \text{ of } 23.60 \\ &= 0.075(23.60) = 1.77\end{aligned}$$

The sales tax is \$1.77. The total bill is the cost of the meal plus the sales tax.

$$\begin{aligned}\text{Total bill} &= \text{cost of meal} + \text{sales tax} \\ &= 23.60 + 1.77 = 25.37\end{aligned}$$

Thus, the bill, including sales tax, is \$25.37.

- To find 10% of any number, we can multiply the number by 0.10.

$$\begin{aligned}10\% \text{ of pretax cost} &= 0.10(23.60) \\ &= 2.36\end{aligned}$$

Thus, a 10% tip is \$2.36.





A simple way to find 10% of any number is to simply move the decimal point in the number one place to the left. Moving the decimal point in \$23.60 one place to the left gives \$2.36, the same answer we obtained by our calculations.

- c) To find 15% of \$23.60, multiply as follows.

$$15\% \text{ of } 23.60 = 0.15(23.60) = 3.54$$

Thus, 15% of \$23.60 is \$3.54. A second method to find a 15% tip is to find 10% of the cost, as in part (b), then add half that amount. Following this procedure we get

$$\$2.36 + \frac{\$2.36}{2} = \$2.36 + \$1.18 = \$3.54$$

In most cases, tips are rounded. If the service is excellent, some people leave a 20% tip. Can you give two methods for determining a 20% tip on \$23.60? Determine the 20% tip now. ▲

### EXAMPLE 5 A Recipe for 6

The following chart shows the amount of each ingredient recommended to make 2, 4, and 8 servings of Potato Buds. Determine the amount of each ingredient necessary to make 6 servings of Potato Buds by using the following procedures.

- Multiply the amount for 2 servings by 3.\*
- Add the amounts for 2 servings to the amounts for 4 servings.
- Find the average of the amounts for 4 servings and for 8 servings.
- Subtract the amounts for 2 servings from the amounts for 8 servings.
- Compare the answers for parts (a) through (d). Are they the same? If not, explain why not.
- Which is the correct procedure for obtaining 6 servings?

Servings	2	4	8
Water	$\frac{2}{3}$ cup	$1\frac{1}{3}$ cups	$2\frac{2}{3}$ cups
Milk	2 tbsp	$\frac{1}{3}$ cup	$\frac{2}{3}$ cup
Butter or margarine	1 tbsp	2 tbsp	4 tbsp
Salt <sup>†</sup>	$\frac{1}{4}$ tsp	$\frac{1}{2}$ tsp	1 tsp
Potato Buds	$\frac{2}{3}$ cup	$1\frac{1}{3}$ cups	$2\frac{2}{3}$ cups

<sup>†</sup>Less salt can be used if desired.

### SOLUTION:

- a) We multiply the amounts for 2 servings by 3.

$$\text{Water: } 3\left(\frac{2}{3}\right) = 2 \text{ cups}$$

$$\text{Milk: } 3(2) = 6 \text{ tablespoons (tbsp)}$$

$$\text{Butter or margarine: } 3(1) = 3 \text{ tbsp}$$

$$\text{Salt: } 3\left(\frac{1}{4}\right) = \frac{3}{4} \text{ teaspoon (tsp)}$$

$$\text{Potato Buds: } 3\left(\frac{2}{3}\right) = 2 \text{ cups}$$

\*Addition, subtraction, multiplication, and division of fractions are discussed in detail in Section 5.3.



## MATHEMATICS *Everywhere*

### Decisions, Decisions

We deal with problem solving daily. There are aspects of problem solving to almost every decision we make. Can we afford to take that vacation in the mountains? Which vehicle should I purchase? What should I do with my tax return? What shall I major in in college? What college shall I attend? The list of questions we ask ourselves daily goes on and on. To make decisions, we often need to consider and weigh many factors, as well as consider all the possible consequences of our decisions.

Often, without consciously realizing it, different branches of mathematics are involved in our decision-making process. For example, we often use statistical data and consider the probability (or chance) of an event occurring or not occurring when we make decisions. Probability and statistics are two branches of mathematics covered in later chapters of the book.

For many people, problem solving is a recreational activity, as is evident by the great number of crossword puzzles and puzzle books sold daily. This is a very large and expanding market.

Today, as you go about your daily business, keep a record of all the problem-solving decisions you need to make! If you do this conscientiously, you will be amazed at the outcome.



A vacation in the mountains

- b) We find the amount of each ingredient by adding the amount for 2 and 4 servings.

$$\text{Water: } \frac{2}{3} \text{ cup} + 1\frac{1}{3} \text{ cup} = 2 \text{ cups}$$

$$\text{Milk: } 2 \text{ tbsp} + \frac{1}{3} \text{ cup}$$

To add these two amounts, we must convert one of them so that they have the same units. By looking in a cookbook or a book of conversion factors, we see that  $16 \text{ tbsp} = 1 \text{ cup}$ . The milk in part (a) was given in tablespoons, so we convert  $\frac{1}{3} \text{ cup}$  to tablespoons to compare answers. One third cup equals  $\frac{1}{3}(16) = \frac{16}{3}$  or  $5\frac{1}{3} \text{ tbsp}$ . Therefore,

$$\text{Milk: } 2 \text{ tbsp} + 5\frac{1}{3} \text{ tbsp} = 7\frac{1}{3} \text{ tbsp}$$

Let's continue with the rest of the ingredients:

$$\text{Butter: } 1 \text{ tbsp} + 2 \text{ tbsp} = 3 \text{ tbsp}$$

$$\text{Salt: } \frac{1}{4} \text{ tsp} + \frac{1}{2} \text{ tsp} = \frac{3}{4} \text{ tsp}$$

$$\text{Potato Buds: } \frac{2}{3} \text{ cup} + 1\frac{1}{3} \text{ cups} = 2 \text{ cups}$$

- c) We compute the amounts of the ingredients by finding the average of the amounts for 4 and 8 servings. We do so by adding the amounts for each ingredient and dividing the sum by 2.

$$\text{Water: } \frac{1\frac{1}{3} \text{ cups} + 2\frac{2}{3} \text{ cups}}{2} = \frac{4 \text{ cups}}{2} = 2 \text{ cups}$$

$$\text{Milk: } \frac{\frac{1}{3} \text{ cup} + \frac{2}{3} \text{ cup}}{2} = \frac{1 \text{ cup}}{2} = \frac{1}{2} \text{ cup (or 8 tbsp)}$$

$$\text{Butter: } \frac{2 \text{ tbsp} + 4 \text{ tbsp}}{2} = \frac{6 \text{ tbsp}}{2} = 3 \text{ tbsp}$$

$$\text{Salt: } \frac{\frac{1}{2} \text{ tsp} + 1 \text{ tsp}}{2} = \frac{\frac{3}{2} \text{ tsp}}{2} = \frac{3}{4} \text{ tsp}$$

$$\text{Potato Buds: } \frac{1\frac{1}{3} \text{ cups} + 2\frac{2}{3} \text{ cups}}{2} = \frac{4 \text{ cups}}{2} = 2 \text{ cups}$$

- d) We obtain the amounts of ingredients by subtracting the amounts for 2 servings from the amounts for 8 servings.

$$\text{Water: } 2\frac{2}{3} \text{ cups} - \frac{2}{3} \text{ cup} = 2 \text{ cups}$$

$$\begin{aligned} \text{Milk: } \frac{2}{3} \text{ cup} - 2 \text{ tbsp} &= \frac{2}{3}(16) \text{ tbsp} - 2 \text{ tbsp} \\ &= \frac{32}{3} \text{ tbsp} - \frac{6}{3} \text{ tbsp} \\ &= \frac{26}{3} \text{ tbsp, or } 8\frac{2}{3} \text{ tbsp} \end{aligned}$$

$$\text{Butter: } 4 \text{ tbsp} - 1 \text{ tbsp} = 3 \text{ tbsp}$$

$$\text{Salt: } 1 \text{ tsp} - \frac{1}{4} \text{ tsp} = \frac{3}{4} \text{ tsp}$$

$$\text{Potato Buds: } 2\frac{2}{3} \text{ cups} - \frac{2}{3} \text{ cup} = 2 \text{ cups}$$

- e) Comparing the answers in parts (a) through (d), we find that the amounts of all ingredients, except milk, are the same. For milk, we get the following results.

$$\text{Part (a): Milk} = 6 \text{ tbsp}$$

$$\text{Part (c): Milk} = 8 \text{ tbsp}$$

$$\text{Part (b): Milk} = 7\frac{1}{3} \text{ tbsp}$$

$$\text{Part (d): Milk} = 8\frac{2}{3} \text{ tbsp}$$

Why are all these answers different? After rechecking, we find that all our calculations are correct, so we must look deeper. Note that milk is the only ingredient that has different units for 2 servings and 4 servings. Let's check the relationship between  $2 \text{ tbsp}$  and  $\frac{1}{3} \text{ cup}$ . In going from 2 servings to 4 servings, we would expect that  $\frac{1}{3} \text{ cup}$  should be twice  $2 \text{ tbsp}$ . We know that  $1 \text{ cup} = 16 \text{ tbsp}$ , so

$$\frac{1}{3} \text{ cup} = \frac{1}{3}(16) = \frac{16}{3} = 5\frac{1}{3} \text{ tbsp}$$



Therefore, instead of the 4 tbsp of milk we expected for 4 servings, we get  $5\frac{1}{3}$  tbsp. This change causes all our calculations for milk to be different.

- f) Which is the correct answer? Because all our calculations for milk are correct, there is no single correct answer. All our answers are correct. Using 8 tbsp instead of  $5\frac{1}{3}$  tbsp might make the Potato Buds a little thinner. When we cook, we generally do not add the *exact* amount recommended. We rely on experience to alter the recommended amounts according to individual taste. ▲

Many real-life problems, such as the one in Example 6, can be solved by using proportions. A proportion is a statement of equality between two ratios (or fractions).\*

### EXAMPLE 6 Spraying Weed Killer

The instructions on the Ortho Weed-Be-Gone lawn weed killer indicates that to cover 1000 square feet ( $\text{ft}^2$ ) of lawn, 20 teaspoons (tsp) of the weed killer should be mixed in 5 gallons (gal) of water. Ron Haines wishes to spray his lawn with the weed killer using his pressurized sprayer.

- How much weed killer should be mixed with 8 gal of water to get the proper strength solution?
- How much weed killer is needed to cover an area of  $2820 \text{ ft}^2$  of lawn?

#### SOLUTION:

- Use the information that 20 teaspoons of weed killer is to be mixed with 5 gal of water.

$$\text{Given ratio } \left\{ \begin{array}{l} \frac{20 \text{ tsp}}{5 \text{ gal water}} = \frac{? \text{ tsp}}{8 \text{ gal}} \end{array} \right. \quad \begin{array}{l} \leftarrow \text{Item to be found} \\ \leftarrow \text{Other information given} \end{array}$$

Notice in the proportion that teaspoons and gallons are placed in the same relative positions. Often the unknown quantity is replaced with an  $x$ . The proportion may be written as follows and solved using cross multiplication.

$$\begin{aligned} \frac{20}{5} &= \frac{x}{8} \\ 20(8) &= 5x \\ 160 &= 5x \\ \frac{160}{5} &= \frac{5x}{5} && \text{Divide both sides of the equation by 5 to solve for } x. \\ 32 &= x \end{aligned}$$

Thus, Ron must mix 32 tsp [or  $10\frac{2}{3}$  tablespoons (tbsp) or  $\frac{2}{3}$  cup] of the weed killer with 8 gal of water. This answer seems reasonable since we would expect to get an answer greater than 20 tsp.

- To answer this question, we use the same procedure discussed in part (a). This time we will use the information that  $1000 \text{ ft}^2$  requires 20 tsp of weed killer. The areas may be placed either on the top or bottom of the fraction, as long as they are placed in the same relative position.

\*Proportions are discussed in greater detail in Section 6.2.





## DID YOU KNOW

East Meets West:  
Magic Squares

**A** Chinese myth says that in about 2200 B.C., a divine tortoise emerged from the Yellow River. On his back was a special diagram of numbers from which all mathematics was derived. The Chinese called this diagram Lo Shu. The Lo Shu diagram is the first known magic square.

Arab traders brought the Chinese magic square to Europe during the Middle Ages, when the plague was killing millions of people. Magic squares were considered strong talismans against evil, and possession of a magic square was thought to ensure health and wealth.

Given ratio  $\left\{ \begin{array}{l} \frac{1000 \text{ sq ft}}{20 \text{ tsp}} = \frac{2820 \text{ sq ft}}{? \text{ tsp}} \end{array} \right.$  ← Other information given  
← Item to be found

Now replace the question mark with an  $x$  and solve the proportion.

$$\begin{aligned} \frac{1000}{20} &= \frac{2820}{x} \\ 1000(x) &= 20(2820) \\ 1000x &= 56,400 \\ \frac{1000x}{1000} &= \frac{56,400}{1000} && \text{Divide both sides of the equation by} \\ &&& 1000 \text{ to solve for } x. \\ x &= 56.4 \end{aligned}$$

Thus, about 56.4 tsp are needed. This answer is reasonable for we would expect the answer to be more than twice the 20 tsp required for 1000 ft<sup>2</sup>. ▲

Most of the problems solved so far have been practical ones. Many people, however, enjoy solving brainteasers. One example of such a puzzle follows.

**EXAMPLE 7** Magic Squares

A magic square is a square array of numbers such that the numbers in all rows, columns, and diagonals have the same sum. Use the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 to construct a magic square.

**SOLUTION:** The first step is to create a figure with nine cells as in Fig. 1.1(a). We must place the nine numbers in the cells so that the same sum is obtained in each row, column, and diagonal. Common sense tells us that 7, 8, and 9 cannot be in the same row, column, or diagonal. We need some small and large numbers in the same row, column, and diagonal. To see a relationship, we list the numbers in order:

1, 2, 3, 4, 5, 6, 7, 8, 9

Note that the middle number is 5 and the smallest and largest numbers are 1 and 9, respectively. The sum of 1, 5, and 9 is 15. If the sum of 2 and 8 is added to 5, the sum is 15. Likewise 3, 5, 7, and 4, 5, 6 have sums of 15. We see that in each group of three numbers the sum is 15 and 5 is a member of the group.

9	5	1

(a)

		8
9	5	1
2		

(b)

4		8
9	5	1
2		6

(c)

4	3	8
9	5	1
2	7	6

(d)

**Figure 1.1**

Because 5 is the middle number in the list of numbers, place 5 in the center square. Place 9 and 1 to the left and right of 5 as in Fig. 1.1(a). Now we place the 2 and the 8. The 8 cannot be placed next to 9 because  $8 + 9 = 17$ , which is greater than 15. Place the smaller number 2 next to the larger number 9. We elected to place the 2 in the lower left-hand cell and the 8 in the upper right-hand cell as in Fig. 1.1(b). The



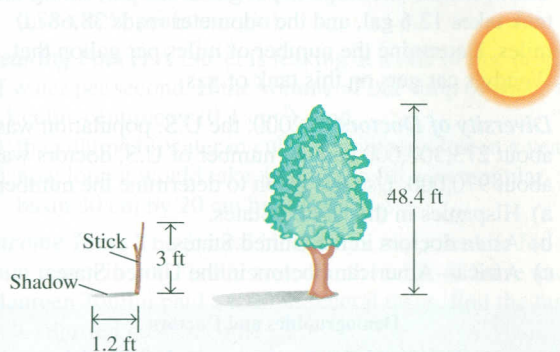
sum of 8 and 1 is 9. To arrive at a sum of 15, we place 6 in the lower right-hand cell as in Fig. 1.1(c). The sum of 9 and 2 is 11. To arrive at a sum of 15, we place 4 in the upper left-hand cell as in Fig. 1.1(c). Now the diagonals 2, 5, 8 and 4, 5, 6 have sums of 15. The numbers that remain to be placed in the empty cells are 3 and 7. Using arithmetic, we can see that 3 goes in the top middle cell and 7 goes in the bottom middle cell as in Fig. 1.1(d). A check shows that the sum in all the rows, columns, and diagonals is 15. ▲

The solution to Example 7 is not unique. Other arrangements of the nine numbers in the cells will produce a magic square. Also, other techniques of arriving at a solution for a magic square may be used. In fact, the process described will not work if the number of squares is even, for example, 16 instead of 9. Magic squares are not limited to the operation of addition or to the set of counting numbers.

## SECTION 1.3 EXERCISES

### Practice the Skills/Problem Solving

- Reading a Map** The scale on a map is 1 inch = 50 miles. How long a distance is a route on the map if it measures 3.75 in.?
- Blueprints** Chalon Bridges, an architect, is designing a shopping mall. The scale of her plan is 1 in. = 12 ft. If one store in the mall is to have a frontage of 82 ft, how long will the line representing that store's frontage be on the blueprint?
- Height of a Tree** At a given time of day, the ratio of the height of an object to the length of its shadow is the same for all objects. If a 3-ft stick in the ground casts a shadow of 1.2 ft, find the length of the shadow of a 48.4 ft tree.



- New York City Taxi** As of this writing, the fare for a New York City taxi is as follows:

\$2.00 upon entering the cab  
 \$0.30 for each  $\frac{1}{5}$  mile  
 \$0.30 for each 30 seconds of waiting (or moving less than 8 mph)

Determine the cost of a 12-mile ride with 2 minutes of standing still in traffic.

- Cost of Gas** The bar graph shows the average price for self-serve regular gasoline on August 1, 2001, in the states with the most expensive and least expensive gasoline cost. Assuming that the cost of the gasoline remained constant for the year, how much more would it cost to drive 20,000 miles in Hawaii than it would in South Carolina if your car's mileage was 20 miles per gallon?

A wide variety of pump prices

#### Costliest States for Gasoline

Self-serve regular, average price per gallon

1. Hawaii	\$2.02
2. California	1.73
3. Alaska	1.71
4. Connecticut	1.61
5. New York	1.58

#### Cheapest States for Gasoline

Self-serve regular, average price per gallon

46. North Carolina	\$1.26
47. Missouri	1.25
48. Oklahoma	1.24
49. Georgia	1.22
50. South Carolina	1.22

National average price: \$1.40 as of Monday, July 30, 2001

Source: AAA

- Use of Fertilizer** A bag of fertilizer covers 6000 ft<sup>2</sup>. How many bags are needed to cover an area of 26,000 ft<sup>2</sup>?
- Health Care Cost** According to Hewitt Associates, in 2000 the annual cost to companies that provided health care coverage to their employees averaged \$4222. The 2001 average cost per employee for employees' health care was 11.5% greater than the companies' 2000 cost. Determine the annual 2001 cost, per employee, for health care.
- Hours of Work** An article in the June 11, 2001, issue of *Business Week* indicates that in the United States we tend to put in more hours at work than many other countries, especially European countries, and each year the difference increases. The graph on the top of the next page shows a comparison of annual hours worked by employees in Germany and the United States from 1970 to 2001.





\*West Germany

Data: Organization for Economic Cooperation and Development

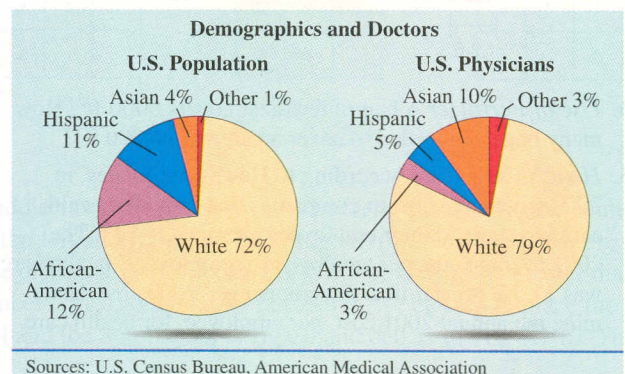
- b) Determine a procedure (or a formula) for converting a jacket from a U.S. size to a size in Japan.

U.S.	chest	34	36	38	40	42	44	46	48
Japan	chest	86.5	91.5	96.5	101.5	106.5	112	117	?

- a) Estimate the increase in annual hours worked by Americans from 1970 to 2001.
- b) Estimate the average *weekly* number of hours worked in the United States in 2001.
- c) Estimate the average *weekly* number of hours worked in Germany in 2001.
9. **Parking Costs** The Main Street Garage charges \$2.50 for the first hour of parking and \$1.00 for each additional hour or part thereof. Denise Tomey parks her car in the garage from 9 A.M. to 5 P.M., 5 days a week. How much money does she save by paying a weekly parking rate of \$35.00?
10. **Calling Person to Person** From a phone booth, Andrea Sheehan makes a person-to-person call from Houston, Texas, to Seattle, Washington. The call costs \$3.75 for the first 3 minutes and \$0.50 for each additional minute. How much did Andrea's 21-minute call cost?
11. **Buying a Computer** Emily Falcon wants to purchase a computer that sells for \$1250. She can either pay the total amount at the time of purchase, or she can agree to pay the store \$120 down and \$80 a month for 15 months. How much money can she save by paying the total amount at the time of purchase?
12. **Buying a House** The Suarezes want to purchase a house that costs \$120,000. With a \$20,000 down payment, their monthly cost would be \$699.99 for 30 years. With a \$40,000 down payment, their monthly cost would be \$559.20 for 30 years. Determine the total amount they would save on the cost of the house over 30 years if they used the \$40,000 down payment.
13. **Buying a House** The Browns want to purchase a house that costs \$130,000. They plan to take out a \$90,000 mortgage on the house and put \$40,000 as a down payment. The bank informs them that with a 20-year mortgage their monthly cost would be \$752.40, and with a 30-year mortgage their monthly cost would be \$660.60. Determine the amount they would save on the cost of the house if they selected the 20-year mortgage rather than the 30-year mortgage.
14. **Getting an 80 Average** On four exams, Wallace Memmer's grades were 77, 93, 90 and 76. What grade must he obtain on his fifth exam to have an 80 average?
15. **Japanese Sizes** The following chart shows men's jacket sizes as would be given in the United States and in Japan.
- a) Justin Smith is in Japan and finds a sports jacket that he wishes to buy. He is a size 48 in the United States. Determine the size of the jacket he should try on.
- b) Determine a procedure (or a formula) for converting a jacket from a U.S. size to a size in Japan.
16. **Playing a Lottery** In one state lottery game, you must select a four-digit number (digits may be repeated). If your number matches exactly the four-digit number selected by the lottery commission, you win.
- a) How many different numbers may be chosen?
- b) If you purchase one lottery ticket, what is your chance of winning?
17. **Energy Value and Energy Consumption** The table gives the approximate energy values of some foods, in kilojoules (kJ), and the energy requirements of some activities. How soon would you use up the energy from
- a) a fried egg by swimming?
- b) a hamburger by walking?
- c) a piece of strawberry shortcake by cycling?
- d) a hamburger and a chocolate milkshake by walking?

Food	Energy Value (kJ)	Activity	Energy Consumption (kJ/min)
Chocolate milkshake	2200	Walking	25
Fried egg	460	Cycling	35
Hamburger	1550	Swimming	50
Strawberry shortcake	1400	Running	80
Glass of skim milk	350		

18. **Gas Mileage** Wendy Weisner fills her gas tank completely and makes a note that the odometer reads 38,451.4 miles. The next time she stops to put gas in her car, filling the tank takes 12.6 gal, and the odometer reads 38,687.0 miles. Determine the number of miles per gallon that Wendy's car gets on this tank of gas.
19. **Diversity of Doctors** In 2000, the U.S. population was about 273,300,000 and the number of U.S. doctors was about 970,000. Use the graph to determine the number of
- a) Hispanics in the United States.
- b) Asian doctors in the United States.
- c) African-American doctors in the United States.





**20. Saving for a Stereo**

- a) Fernando Diez works 40 hours per week and makes \$8.50 per hour. How much money can he expect to earn in 1 year (52 weeks)?
- b) If he saves all the money that he earns, how long will he have to work to save for a stereo receiver that costs \$1275?

**21. Mail Order Purchase** Mary Liotta purchased 4 tires by mail order. She paid \$52.80 per tire plus \$5.60 per tire for shipping and handling. There is no sales tax on this purchase because they were purchased out of state. She also had to pay \$8.56 per tire for mounting and balancing. At a local tire store, her total for the 4 tires with mounting and balancing would be \$324 plus an 8% sales tax. How much did Mary save by purchasing the tires through the mail?

**22. Sealing a Gym Floor** A gymnasium floor has an area of 2400 square yards. Each gallon of floor sealant covers an area of 350 square feet. How many gallons of sealant are needed to cover the gymnasium floor?

**23. Profit Margins** The following chart shows retail stores' average percent profit margin on certain items.

Product Category	Average Profit Margin (%)
Video equipment	12
Audio components	14
Stereo speakers	20–25
Extended warranties	50–60

Source: *Consumer Reports*

- a) Determine the average profit of a store that has the list price on a camcorder of \$620.
- b) Determine the average profit of a store that has a list price on a pair of speakers for \$1200 (use a 22% profit margin).
- c) If you negotiate with the salesperson and get him or her to sell the speakers for \$1000, find the store's profit (use a 22% profit margin).
- 24. Leaking Faucet** A faucet is leaking at a rate of one drop of water per second. If the volume of one drop of water is 0.1 cubic centimeter ( $0.1 \text{ cm}^3$ ), find
- a) the volume of water in cubic centimeters lost in a year.
- b) how long it would take, in days, to fill a rectangular basin 30 cm by 20 cm by 20 cm.
- 25. Income Taxes** The federal income tax rate schedule for a joint return in 2002 is illustrated in the table. If Steve and Maureen Tomlin paid \$4590 in federal taxes, find the family's adjusted gross income.

Adjusted Gross Income	Taxes
\$0–\$12,000	10% of income
\$12,000–\$46,700	\$1200 + 15% in excess of \$12,000
\$46,700–\$112,850	\$6405 + 27% in excess of \$46,700
\$112,850–\$171,950	\$24,265.50 + 30% in excess of \$112,850
\$171,950–\$307,050	\$41,995.50 + 35% in excess of \$171,950
\$307,050 and above	\$89,280.50 + 38.6% in excess of \$307,050

- 26. Wasted Water** A faucet leaks 1 oz of water per minute.
- a) How many gallons of water are wasted in a year? (A gallon contains 128 oz.)

- b) If water costs \$11.20 per 1000 gal, how much additional money is being spent on the water bill?

**27. Water Restrictions** In 2000 and 2001, the entire state of Florida had a water shortage, and restrictions were placed on water usage throughout the state. The number of water restriction ordinance violations for Pinellas County, Florida, from March 2000 to March 2001 is shown in the table below.

Water Restriction Ordinance Violations  
March 2000–March 2001

Number of Violations	Number of Customers
7	2
6	0
5	1
4	29
3	201
2	1,408
1	10,352

Source: Pinellas County Utilities

Determine the total number of violations illustrated by the chart.

- 28. Tire Pressure** When a car's tire pressure is 30 pounds per square inch (psi), it averages 20.8 mpg of gasoline. If the tire pressure is increased to 35 psi, the car averages 21.6 mpg of gasoline.
- a) If Mr. Levy drives an average of 20,000 mi per year, how many gallons of gasoline will he save in a year by increasing his tire pressure from 30 to 35 psi?
- b) If gasoline costs \$1.60 per gallon, how much will he save in a year?
- c) If we assume that there are about 140 million cars in the United States and that these changes are typical of each car, how many gallons of gasoline would be saved if all drivers increased their cars' tire pressure?
- 29. Air Pollution** The graph illustrates the 10 countries that produced the most carbon dioxide, methane, and nitrous oxide in 2000. These gases make up the majority of the greenhouse gases. (The term *per capita* means per person.)

Producers of Major Greenhouse Gases

	TOTAL EMISSIONS (in millions of metric tons)	EMISSIONS PER CAPITA (in metric tons of carbon dioxide equivalent)
UNITED STATES	6503.8	24.3
CHINA	4964.8	4.0
INDIA	2081.7	2.2
RUSSIA	1980.3	13.4
JAPAN	1166.1	9.3
GERMANY	956.0	11.6
BRAZIL	695.7	4.2
SOUTH AFRICA	677.2	15.9
CANADA	634.0	20.9
BRITAIN	618.7	10.5

Source: Environmental Protection Agency, Census Bureau International Data Base



- a) By looking at the data provided, is it possible to determine the population of each country? If so, explain how to do so.
- b) Using the procedure you gave in part (a), determine the population of the United States.
- c) Determine the population of China.
30. **Adjusting for Inflation** Assume that the annual rate of inflation is 6% for each of the next 2 years. What will be the cost of goods 2 years from now, adjusted for inflation, if the goods cost \$450.00 today?
31. **Investing** You place \$1000 in a mutual fund. The first year the value of the fund increases by 10%. The second year the value of the fund decreases by 10%. Determine the value of the fund at the end of the second year. Is it greater than, less than, or equal to your initial investment?
32. **X-rays** With a certain medical insurance policy, the customer must first pay an annual \$100 deductible, then the policy covers 80% of the cost of x-rays. The first insurance claims for a specific year submitted by Yungchen Cheng are for two x-rays. The first x-ray cost \$640 and the second x-ray cost \$920. How much, in total, will Yungchen need to pay for these x-rays?
33. **A Photo Safari** Kelli Hammer is planning a trip to Africa where she will participate in a photo safari. She is planning on bringing a great deal of film. A photography store is selling 4 packs of film for \$17 and 10 packs of the same film for \$41.
- a) If she wishes to purchase only the 4 packs and 10 packs and wishes to spend a maximum of \$200 on film, what is the maximum number of rolls of film she can purchase?
- b) What will be the cost?
34. **Buying Film** Erika Gutierrez is planning a vacation to Australia and wishes to bring a large supply of film. At Wal-Mart, 4 packs of 24 exposure film costs \$4.08 and 4 packs of the same film with 36 exposures costs \$5.76.
- a) If she wishes to spend a maximum of \$50 on film and get the most exposures, how many 4 packs of 24 exposures and how many 4 packs of 36 exposures should she purchase?
- b) How many exposures will she get?
- c) What will be the cost? If there is more than one choice in part (a), give the minimum cost.

35. **Making Cream of Wheat** The following amounts of ingredients are recommended to make various servings of Nabisco Instant Cream of Wheat. *Note:* 16 tbsp = 1 cup.

Ingredient	1 Serving	2 Servings	4 Servings
Mix water or milk	1 cup	2 cups	$3\frac{3}{4}$ cups
With salt (optional)	$\frac{1}{8}$ tsp	$\frac{1}{4}$ tsp	$\frac{1}{2}$ tsp
Add Cream of Wheat	3 tbsp	$\frac{1}{2}$ cup	$\frac{3}{4}$ cup

Determine the amount of each ingredient needed to make 3 servings using the following procedures.

- a) Multiply the amounts for 1 serving by 3.

- b) Find the average of the amounts for 2 and 4 servings.
- c) Subtract the amounts for 1 serving from the amounts for 4 servings.
- d) Compare the answers obtained in parts (a) through (c) and explain any differences.

36. **Making Rice** Following are the amounts of ingredients recommended to make various servings of Uncle Ben's Original Converted Rice. *Note:* 1 tbsp = 3 tsp.

Ingredient	2 Servings	4 Servings	6 Servings	12 Servings
Rice (cups)	$\frac{1}{2}$	1	$1\frac{1}{2}$	3
Water (cups)	$1\frac{1}{3}$	$2\frac{1}{4}$	$3\frac{1}{3}$	6
Salt (teaspoons)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{2}$
Butter or margarine	1 tsp	2 tsp	1 tbsp	2 tbsp

Determine the amount of each ingredient needed to make 8 servings using the following procedures.

- a) Multiply the amount for 2 servings by 4.
- b) Multiply the amount for 4 servings by 2.
- c) Add the amounts for 2 and 6 servings.
- d) Subtract the amount for 4 servings from the amount for 12 servings.
- e) Compare the answers obtained in parts (a) through (d) and explain any differences.

*Solve the following problems.*

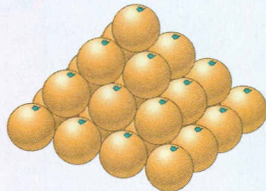
37. **One Square Foot** How many square inches, 1 in. by 1 in., fit in an area of 1 square foot, 1 ft by 1 ft?
38. **Cubic Inches** How many cubic inches fit in 1 cubic foot?
39. **Rectangle** If the length and width of a rectangle each double, what happens to the area of the rectangle?
40. **Cube** If the length, width, and height of a cube all double, what happens to the volume of the cube?
41. **Positive Numbers** Find two positive numbers that have a one-digit answer when multiplied and a two-digit answer when added.
42. **Buying Candy** How much do 10 pieces of candy cost if 1000 pieces cost \$10?
43. **A Balance** On the balance below, where should the one missing block ■ be placed so that the balance would balance on the triangle (the fulcrum). Assume that each block has the same weight.



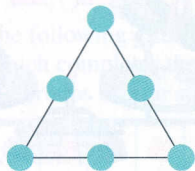
44. **Ties, Ties, Ties** All my ties are red except two. All my ties are blue except two. All my ties are brown except two. How many ties do I have?
45. **Birds and Lizards** A pet store had just received a supply of birds and lizards. Counting heads, the owner got 22. Counting feet, the owner got 68. How many birds and lizards are there?



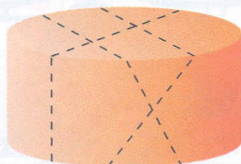
46. **Palindromes** A *palindrome* is a number (or word) that reads the same forward and backward. The numbers 1991 and 43234 are examples of palindromes. How many palindromes are there between the numbers 2000 and 3000? List them.
47. **Supermarket Display** The figure shows oranges in a supermarket display stacked in a *square pyramid* (the base is a square).



- a) How many oranges are in the pyramid shown if the base is 4 oranges by 4 oranges?
- b) How many oranges would be in a square pyramid if the base was 7 oranges by 7 oranges?
48. **Balancing a Scale** If you have a balance scale and only the four weights 1 gram (g), 3 g, 9 g, and 27 g, explain how you could show that an object had the following weights.
- a) 5 g      b) 16 g
- (Hint: Weights must be added to both sides of the balance scale.)
49. **Numbers in Circles** Place the numbers 1 through 6 in the circles below so that the sum along each of the three straight lines is the same. Each number must be used exactly once (Note: There is more than one correct answer.)



50. **Cuts in Cheese** If you make the three complete cuts in the cheese, as shown, how many pieces of cheese will you have?



51. **Magic Square** Create a magic square by using the numbers 2, 4, 6, 8, 10, 12, 14, 16, and 18. The sum of the numbers in every column, row, and diagonal must be 30.
52. **Magic Square** Create a magic square by using the numbers 1, 3, 5, 7, 9, 11, 13, 15, and 17. The sum of the numbers in every column, row, and diagonal must be 27.

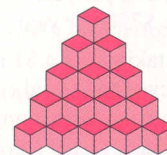
In Exercises 53–55, use the three magic squares illustrated to obtain the answers.

6	5	10
11	7	3
4	9	8

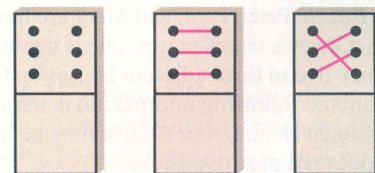
3	2	7
8	4	0
1	6	5

10	9	14
15	11	7
8	13	12

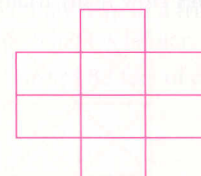
53. **Magic Square** Examine the 3 by 3 magic squares and find the sum of the four corner entries of each magic square. How can you determine the sum by using a key number in the magic square?
54. **Magic Square** For a 3 by 3 magic square, how can you determine the sum of the numbers in any particular row, column, or diagonal by using a key value in the magic square?
55. **Magic Square** For a 3 by 3 magic square, how can you determine the sum of all the numbers in the square by using a key value in the magic square?
56. **Stack of Cubes** Identical cubes are stacked in the corner of a room, as shown. How many of the cubes are not visible?



57. **Dominos** Consider a domino with six dots, as shown. Two ways of connecting the three dots on the left with the three dots on the right are illustrated. In how many ways can the three dots on the left be connected with the three dots on the right?

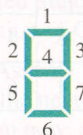


58. **Handshakes All Around** Five salespeople gather for a sales meeting. How many handshakes will each person make if each must shake hands with each of the four others?
59. **Consecutive Digits** Place the digits 1 through 8 in the eight boxes so that each digit is used exactly once and no two consecutive digits touch horizontally, vertically, or diagonally.





60. **A Digital Clock** Digital clocks display numerals by lighting some or all the seven parts of the pattern shown. If each digit 0 through 9 is displayed once, which of the seven parts is used least often? Which part is used most often?



61. **A Grid** Place five 1's, five 2's, five 3's, five 4's, and five 5's in a  $5 \times 5$  grid so that each digit—that is, 1, 2, 3, 4, 5—appears exactly once in each row and exactly once in each column.

### Challenge Problems/Group Activities

62. **Insurance Policies** Ray Kelley owns two cars (a Ford Mustang and a Ford Focus), a house, and a rental apartment. He has auto insurance for both cars, a homeowner's policy, and a policy for the rental property. The costs of the policies are
- Mustang: \$1648 per year
  - Focus: \$1530 per year
  - Homeowner's: \$640 per year
  - Rental property: \$750 per year

Ray is considering taking out a \$1 million personal umbrella liability policy. The annual cost of the umbrella policy would be \$450. If he has the umbrella policy, he can lower the limits on parts of his auto policies and still have equal or better protection. If Ray purchases the umbrella policy, he can reduce his premium on the Mustang by \$90 per year and his premium on the Focus by 12%. If he purchases the umbrella policy and reduces the amount he pays for auto insurance, what is the net amount he is actually paying for the umbrella policy?

63. **A Sports Puzzle** Peter, Paul, and Mary are three sports professionals. One is a tennis player, one is a golfer, and one is a skier. They live in three adjacent houses on City View Drive. From the following information determine which is the professional skier. (*Hint: A table may be helpful.*)

Mary does not play tennis.

Peter skis and plays tennis, but does not golf.

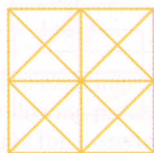
The golfer and the skier live next to each other.

Three years ago, Paul broke his leg skiing and has not tried it since.

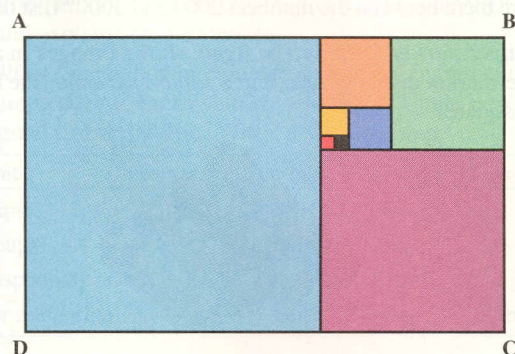
Mary lives in the last house.

The golfer and the tennis player share a common backyard swimming pool.

64. **Counting Triangles** How many triangles are in the figure?

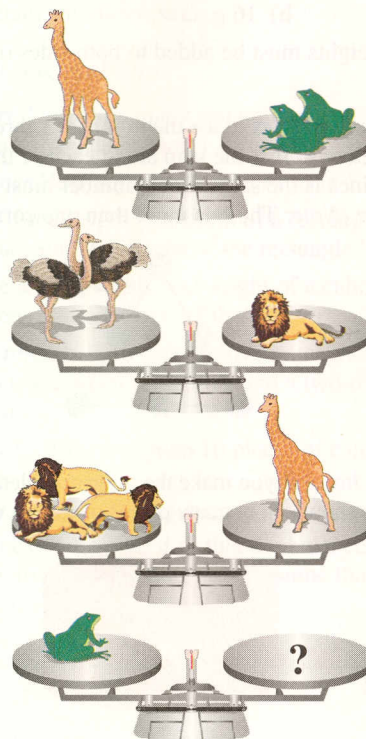


65. **Finding the Area** Rectangle ABCD is made up entirely of squares. The black square has a side of 1 unit. Find the area of ABCD.



### Recreational Math

66. **Ostriches** How many ostriches must replace the question mark to balance the fourth scale? Assume all animals of the same kind have the same weight. That is, all giraffes weigh the same, etc.



### Internet/Research Activity

67. **Puzzles** Many fun and interesting puzzle books and magazines are available. Using this chapter and puzzle books as a guide, construct five of your own puzzles and present them to your instructor.



## CHAPTER 1 SUMMARY

### IMPORTANT FACTS

The **natural numbers** or **counting numbers** are 1, 2, 3, 4, ...

A **conjecture** is a prediction based on specific observations.

A **counterexample** is a special case that satisfies all the conditions of a conjecture, but proves the conjecture false.

**Inductive reasoning** is the process of reasoning to a general conclusion through observations of specific cases.

**Deductive reasoning** is the process of reasoning to a specific conclusion from a general statement.

### GUIDELINES FOR PROBLEM SOLVING

1. Understand the problem.
2. Devise a plan to solve the problem.
3. Carry out the plan.
4. Check the results.

## CHAPTER 1 REVIEW EXERCISES

### 1.1\*

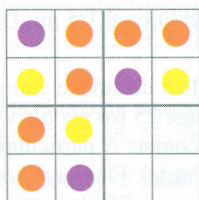
In Exercises 1–8, use inductive reasoning to predict the next three numbers or figures in the pattern.





1. 3, 8, 13, 18, ...
2. 1, 4, 9, 16, ...
3. -3, 6, -12, 24, ...
4. 5, 7, 10, 14, 19, ...
5. 30, 29, 27, 24, 20, ...
6. 6, 3,  $\frac{3}{2}$ ,  $\frac{3}{4}$ , ...

7.  , ...

8.  , ...

9. **Pattern** Examine the following grid for a pattern and then select the answer which completes the pattern. (*Hint: Think about rotating groups of four squares at a time.*)



- a) 
- b) 
- c) 
- d) 

10. Pick any number and multiply the number by 2. Add 10 to the product. Divide the sum by 2. Subtract 5 from the quotient.
  - a) What is the relationship between the number you started with and the final number?

- b) Arbitrarily select some different numbers and repeat the process, recording the original number and the results.
  - c) Make a conjecture about the original number and the final number.
  - d) Prove, using deductive reasoning, the conjecture you made in part (c).
11. Pick any number between 1 and 20. Add 5 to the number. Multiply the sum by 6. Subtract 12 from the product. Divide the difference by 2. Divide the quotient by 3. Subtract the number you started with from the quotient. What is your answer? Try this process with a different number. Make a conjecture as to what your final answer will always be.
  12. **Counterexample** Find a counterexample to the statement "The sum of two squares is an even number."

### 1.2

In Exercises 13–25, estimate the answer. Your answers may vary from those given in the back of the book, depending on how you round to arrive at the answer. However, your answer should be something near the answer given.

13.  $210,302 \times 1992$
14.  $346.2 + 96.402 + 1.04 + 897 + 821$
15. 21% of 1012
16. **Distance** Estimate the distance from your wrist to your elbow and estimate the length of your foot. Which do you think is greater? With the help of a friend, measure both lengths to determine which is longer.
17. **Cost** Estimate the cost of 82 feet of chain if the cost is 1.09 per foot.

\*The number in color indicates the section in which the material is covered.

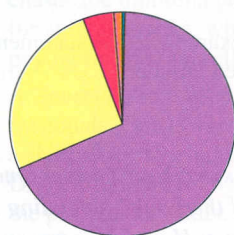


18. **Sales Tax** Estimate the amount of a 6% sales tax on a coat that costs \$202.
19. **Walking Speed** Estimate your average walking speed in miles per hour if you walked 1.1 mi in 22 min.
20. **Groceries** Estimate the total cost of six grocery items that cost \$2.49, \$0.79, \$1.89, \$0.10, \$2.19, \$6.75.
21. **A Walking Path** The scale of the map is  $\frac{1}{4}$  in. = 0.1 mi. Estimate the distance of the walking path indicated in red.



In Exercises 22 and 23, refer to the following graph, which illustrates the percent of people in the different tax brackets for the year 2000.

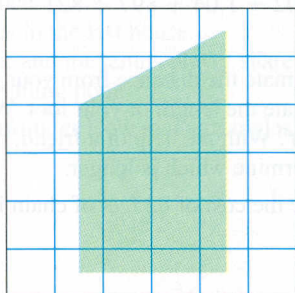
22. Estimate the percent of tax filers in the 15% tax bracket (the purple area of the circle graph).
23. Estimate the percent of tax filers in the 31% tax bracket (the red area of the circle graph).



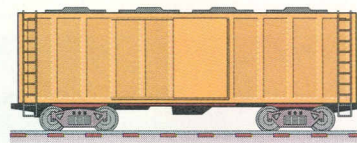
Bracket	Income
15%	\$0–\$41,199
28%	\$41,200–\$99,599
31%	\$99,600–\$151,749
36%	\$151,750–\$271,049
39.6%	\$271,050 and above

Source: Internal Revenue Service.

24. **Estimating an Area** If each square represents one square unit, estimate the size of the shaded area.



25. **Railroad Car Estimation** The scale of a model railroad is 1 in. = 12.5 ft. Estimate the size of an actual box car if this drawing is the same size as the model box car.



### 1.3

Solve the following problems.

26. **Change from a Twenty** Jeff Howard parked his car in a lot that charged \$2.00 for the first hour and \$1.50 for each additional hour. He left the car in the lot for 8 hr. How much change did he receive from a \$20 bill?
27. **Buying in Quantity** A six-pack of cola costs \$2.69. A carton of 4 six-packs costs \$9.60. How much will be saved by purchasing the carton rather than 4 individual six-packs?
28. **Jet Ski Rental** The rental cost of a jet ski from Nola Akala's Ski Rental is \$15 per 15 min, and the cost from Jill Berkman's Ski Rental is \$25 per half hour. If you plan to rent the jet ski for 2 hr, which is the better deal, and by how much?
29. **Oscars** In 2001, shortly before the Academy Awards show, many Oscars were lost by the shipping company. Fifty-two of the 55 Oscars were found, before the awards ceremony, in a dumpster by an Illinois man. The man was awarded \$50,000 (and two tickets to the ceremony). The actual cost to have each Oscar produced was \$327. How much more had the man been awarded then the actual cost to produce the 52 Oscars he found (disregarding the cost of the tickets)?
30. **Cab Fare** A taxicab charges \$1.50 for the first  $\frac{1}{5}$  mi and 30 cents for each additional  $\frac{1}{5}$  mi. Determine the cost of a 10 mi trip.
31. **Auto Insurance** Most insurance companies reduce premiums by 10% until age 25 for people who successfully pass a driver education course. A particular driver education course costs \$60. Patrick Flanigan, who just turned 18, has auto insurance that costs \$530 per year. By taking the driver education course, how much would he save in auto insurance, including the cost of the course, from the age of 18 until the age of 25?
32. **Pediatric Dosage** If 1.5 milligrams of a medicine is to be given for 10 lb of body weight, how many milligrams should be given to a child who weighs 47 lb?
33. **Qualifying for a Mortgage** Banks will grant an applicant a mortgage if the monthly payments are not greater than 28% of the person's take-home pay. What is the maximum monthly mortgage payment you can make if your gross salary is \$3800 a month and your payroll deductions are 30% of your gross salary?



34. **Flying West** New York City is on eastern standard time, St. Louis is on central standard time (1 hr earlier than eastern standard time), and Las Vegas is on Pacific standard time (3 hr earlier than eastern standard time). A flight leaves New York City at 9 A.M. eastern standard time, stops for 50 min in St. Louis, and arrives in Las Vegas at 1:35 P.M. Pacific time. How long is the plane actually flying?

35. **Crossing Time Zones** The international date line is an imaginary line of longitude (from the North Pole to the South Pole) on Earth's surface between Japan and Hawaii in the Pacific Ocean. Crossing the line east to west adds a day to the present date. Crossing the line west to east subtracts a day. At 3:00 P.M. on July 25 in Hawaii, what is the time and date in Tokyo, Japan, which is four time zones to the west?

36. **Conversions** 1 in. = 2.54 cm.

- How many square centimeters are in a square inch?
- How many cubic centimeters are in a cubic inch?
- How long is a centimeter in terms of inches?

37. **Dot Pattern** If the following pattern is continued, how many dots will be in the hundredth figure?



38. **Magic Square** The following magic square uses each number from 6 to 21 exactly once. Complete the magic square by using the unused numbers from 6 through 21 exactly once.

21	7		18
10		15	
14	12	11	17
9	19		

39. **Magic Square** Create a magic square by using the numbers 13, 15, 17, 19, 21, 23, 25, 27, and 29.

40. **Microbes in a Jar** A colony of microbes doubles in number every second. A single microbe is placed in a jar, and in an hour the jar is full. When was the jar half full?

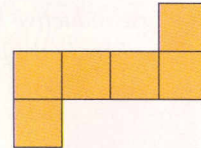
41. **Brothers and Sisters** Jim Carraway has four more brothers than sisters. How many more brothers than sisters does his sister Mary have?

42. **A Missing Dollar** Three friends check into a single room in a motel and pay \$10 apiece. The room costs \$25 instead of \$30, so a clerk is sent to the room to give \$5 back. The friends each take back \$1, and the clerk is given \$2 for his trouble. Now each of the friends paid \$9, a total of \$27, and the clerk received \$2. What happened to the missing dollar?

43. **The Average Weight** Four women in a room have an average weight of 130 lb. A fifth woman who weighs 180 lb enters the room. Find the average weight of all five women.

44. **Change for a Dollar** Could a person have \$1.15 worth of change in his pocket and still not be able to give someone change for a dollar bill? If so, what coins might he have?

45. **Volume of a Cube** Here is a flat pattern for a cube to be formed by folding. The sides of each square are 6 cm. Find the volume of the cube.



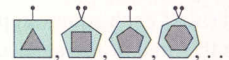
46. **The Heavier Coin** You have 13 coins, which all look alike. Twelve coins weigh exactly the same, but the other one is heavier. You have a pan balance. Tell how to find the heavier coin in just three weighings.

47. **The Sum of Numbers** Find the sum of the first 500 counting numbers. (Hint: Group in pairs.)

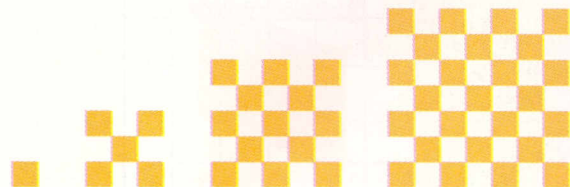
48. **Balancing a Scale** On a balance scale, three green balls balance six blue balls, two yellow balls balance five blue balls, and six blue balls balance four white balls. How many blue balls are needed to balance four green, two yellow, and two white balls?

49. **Palindromes** How many three-digit numbers greater than 100 are palindromes?

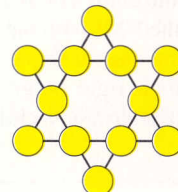
50. **Figures** Describe the fifth figure.



51. **Patterns** How many orange tiles will be required to build the sixth figure in this pattern?



52. **Sum of Numbers** Place the numbers 1 through 12 in the 12 circles so that the sum of the numbers in each of the six rows is 26. Use each number from 1 through 12 exactly once.





53. **People in a Line** In how many ways can

- two people stand in a line?
- three people stand in a line?
- four people stand in a line?

d) five people stand in a line?

e) Using the results from parts (a) through (d), make a conjecture about the number of ways in which  $n$  people can stand in a line.

## CHAPTER 1 TEST

In Exercises 1 and 2, use inductive reasoning to determine the next three numbers in the pattern.

1. 6, 9, 12, 15, ...

2.  $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

3. Pick any number, multiply the number by 5, and add 10 to the number. Divide the sum by 5. Subtract 1 from the quotient.

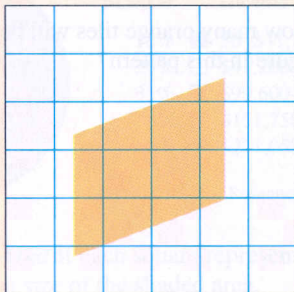
- What is the relationship between the number you started with and the final answer?
- Arbitrarily select some different numbers and repeat the process. Record the original number and the results.
- Make a conjecture about the relationship between the original number and the final answer.
- Prove, using deductive reasoning, the conjecture made in part (c).

In Exercises 4 and 5, estimate the answers.

4.  $0.06 \times 98,000$

5.  $\frac{102,000}{0.00302}$

6. **Estimating Area** If each square represents one square unit, estimate the area of the shaded figure.



7. **Body Mass Index** During the week of June 12, 2001, the federal government updated its method for determining if a child is overweight. To make this decision, first determine the child's body mass index (BMI). Then compare the BMI with one of the two charts, one for males and one for females, provided by the government. On the right we give the chart for males up to age 20. To determine a child's BMI:

1) Divide the child's weight (in pounds) by the child's height (in inches).

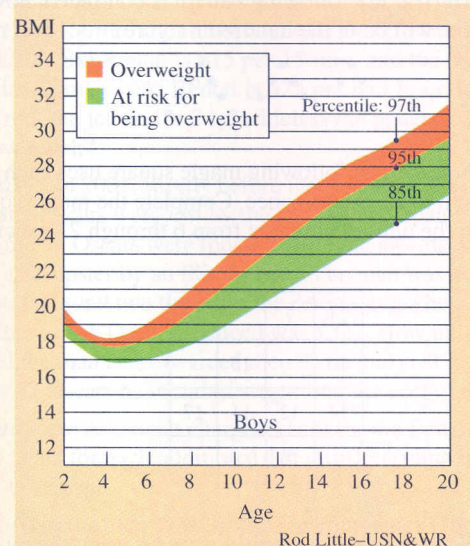
2) Divide the results from part 1 by the child's height again.

3) Multiply the result from part 2 by 703.

Richard is a 14-year-old male who weighs 130 lb and is 63 in. tall.

a) Determine his BMI.

b) Does he appear to be at risk for being overweight, or is he overweight? Explain.



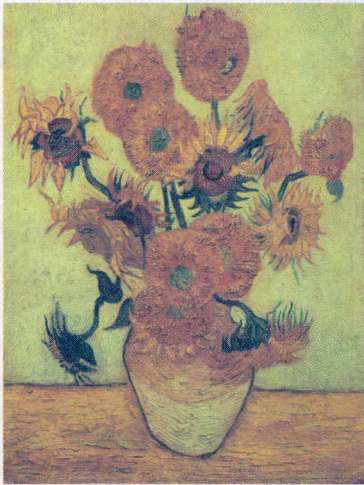
8. **Gas Usage** A gas company charges \$9.63 for the basic monthly fee, which includes the first 3 therms of gas used. It charges 72 cents for each additional therm used. If the Smiths' gas bill for December was \$122.13, how many therms of gas did they use during that month?

9. **Cans of Soda** At a local store a six-pack of soda costs \$2.59 and individual cans cost \$0.80. What is the maximum number of cans of soda that can be purchased for \$15?

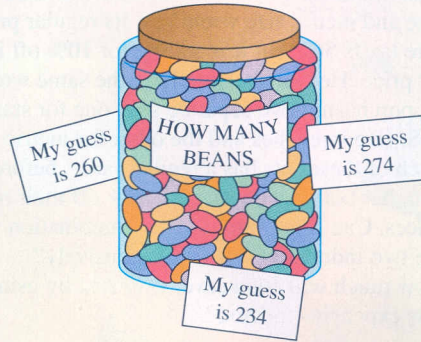
10. **Cutting Wood** How much time does it take Carla Knab, a carpenter, to cut a 10 ft length of wood into four equal pieces, if each cut takes  $2\frac{1}{2}$  min?



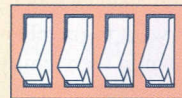
11. **Determining Size** In this photo of *Sunflowers*, 1889, by Vincent van Gogh, 1 in. equals 15.8 in. on the actual painting. Find the dimensions of the actual painting.



12. **Payment Shortfall** Monica Wilson gets \$12.75 per hour with time and a half for any time over 40 hours per week. If she works a 50 hr week and gets paid \$652.25, by how much was she underpaid?
13. **Magic Square** Create a magic square by using the numbers 5, 10, 15, 20, 25, 30, 35, 40, and 45. The sum of the numbers in every row, column, and diagonal must be 75.
14. **A Drive to the Beach** Mary Chin drove from her home to the beach that is 30 mi from her house. The first 15 mi she drove at 60 mph, and the next 15 mi she drove at 30 mph. Would the trip take more, less, or the same time if she traveled the entire 30 mi at a steady 45 mph?
15. **Pick Five Numbers** From the six numbers 2, 6, 8, 9, 11, and 13, pick five that, when multiplied, give 11,232.
16. **Jelly Bean Guess** One guess is off by 9, another guess is off by 17, and yet another guess is off by 31. How many jelly beans are in the jar?



17. **Buying Plants** David Mackin wants to purchase nine herb plants. Countryside Nursery has herbs that are on sale at three for \$3.99. David has a coupon for 25% off an unlimited number of herb plants at the original price of \$1.75 per plant.
- Determine the cost of purchasing nine plants at the sale price.
  - Determine the cost of purchasing nine plants if the coupon is used.
  - Which is the least expensive way to purchase the nine plants, and by how much?
18. **Setting the Switches** In how many different ways can a panel of four on-off switches be set if no two adjacent switches may be off?



## GROUP PROJECTS

### Holiday Shopping

- It is December 1 and John needs to begin his holiday shopping. He intends to purchase gifts for three people: his girlfriend, Melissa; his mother, Ruth; and his father, Don. He doesn't want to spend more than a total of \$325, including the 7% sales tax.
  - If John were to spend the \$325 equally among the three people, approximate the amount that would be spent on each person.
  - If John were to spend the \$325 equally among the three people, determine the maximum amount, *before tax*, that he could spend on each person and not exceed the maximum of \$325, including tax.
  - John decides to get a new set of wrenches for his father. He sees the specific set he wants on sale at Sears. He calls four Sears stores to see if they have the set of wrenches in stock. They all reply that the set is out of stock. He decides that calling additional Sears stores is useless, for he believes that



they will also tell him that the set of wrenches is out of stock. What type of reasoning did John use in arriving at his conclusion? Explain.

- d) John finds an equivalent set of wrenches at a True Value hardware store. The set he is considering is a combination set that contains both standard U.S. size and metric size wrenches. Its regular price before tax is \$62, but it is selling for 10% off its regular price. He can also purchase the same wrenches by purchasing two separate sets, one for standard U.S. size wrenches and the other for metric sizes. Each of these sets has a regular price, before tax, of \$36, but both are on sale for 20% off their regular prices. Can John purchase the combination set or the two individual sets less expensively?
- e) How much will John save, *after tax*, by using the less expensive method?

### Going on Vacation

2. Bob and Kristen Williams decide to go on a vacation. They live in San Francisco, California, and plan to drive to New Orleans, Louisiana.
  - a) Obtain a map that shows routes that they may take from San Francisco to New Orleans. Write directions for them from San Francisco to New Orleans via the shortest distance. Use major highways whenever possible.
  - b) Use the scale on the map to estimate the one-way distance to New Orleans.
  - c) If the Williams estimate that they will average 50 mph (including comfort stops), estimate the travel time, in hours, to New Orleans.
  - d) If the Williams want to travel about 400 miles per day, locate a town in the vicinity of where they will stop each evening.
  - e) If they begin each segment of the trip each day at 9 A.M., about what time will they look for a hotel each evening?

- f) Use the information provided in parts (a) through (e) to estimate the time of day they will arrive in New Orleans.
- g) Estimate the mileage of a typical midsize car and the cost per gallon of a gallon of regular unleaded gasoline. Then estimate the cost of gasoline for the Williams' trip.
- h) Estimate the cost of a typical breakfast, a typical lunch, and a typical dinner for two adults, and the cost of a typical motel room. Then estimate the total cost, including meals, gas, and lodging, for the Williams' trip from San Francisco to New Orleans (one way).

### Problem Solving

3. Four acrobats who bill themselves as the "Tumbling Tumbleweeds" finish up their act with the amazing "Human Pillar," in which the acrobats form a tower, each one standing on the shoulders of the one below. Each acrobat (Ernie, Jed, Tex, and Zeke Tumbleweed) wears a different distinctive item of western garb (chaps, holster, Stetson hat, or leather vest) in the act. Can you identify the members of the "Human Pillar," from top to bottom, by name and apparel?
  - a) Jed Tumbleweed is not on top, but he is somewhere above the man in the Stetson.
  - b) Zeke Tumbleweed does not wear the holster.
  - c) The man in the vest is not on top.
  - d) The man in the chaps is somewhere above Tex but somewhere below Zeke.

**Order**

**Name**

**Apparel**

_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____